Differentiability… Facts 1

Differentiability

def: the ability to take a derivative

First: Limits
exists @ $x=a$ if $LHL = RHL$

Second: Continuity
continuous @ $x=a$ if $LHL = RHL = f(a)$

Third: Differentiability
→ $f(x)$ is differentiable @ $x=a$ if $f(x)$ is continuous @ $x=a$ and $f'(a^+) = f'(a^-)$
(derivative from both sides must be =)

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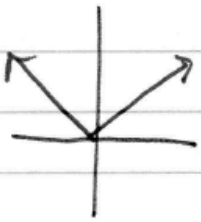
Ex: $f(x) = |x|$ → is it diff @ $x = 0$?



→ $f(x)$ is continuous @ $x = 0$

→ $f'(0^-) = -1$

→ $f'(0^+) = 1$

Since $f'(0^-) \neq f'(0^+)$, $f(x)$ is not diff @ $x = 0$

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Ex: (a) Determine if $f(x)$ is continuous or discont.
(b) Determine if $f(x)$ is differentiable or not diff.

$$f(x) = \begin{cases} x-2 & x \geq 2 \\ -x+2 & x < 2 \end{cases}$$

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(a) $\lim_{x \to 2^-} f(x) = -2+2 = \boxed{0}$ $\lim_{x \to 2^+} f(x) = 2-2 = \boxed{0}$ $f(2) = \boxed{0}$

$\therefore f(x)$ is continuous @ $x = 2$

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(b) $f'(x) = \begin{cases} 1 & x > 2 \\ -1 & x < 2 \end{cases}$

← do not include = b/c we do not know yet if it is diff.

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$f'(2^-) = -1$ $\qquad$ $f'(2^+) = 1$

Since $f'(2^-) \neq f'(2^+)$, $f(x)$ is not diff @ $x=2$.