Formal Definition of Derivative … Practice Set 2

Definition of Derivative Practice

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Formal Definition of Derivative … Practice Set 2

Answers

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2(x+h)^2-3(x+h)+1-\left(2x^2-3x+1\right)}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{2(x^2+2xh+h^2)-3x-3h+1-2x^2+3x-1}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{2x^2}+4xh+2h^2-\cancel{3x}-3h\cancel{+1}-\cancel{2x^2}+\cancel{3x}\cancel{-1}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{4xh+2h^2-3h}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\cancel{h}(4x+2h-3)}{\cancel{h}} = 4x+2(0)-3$$

$$\boxed{f'(x) = 4x-3}$$

Formal Definition of Derivative … Practice Set 2

Definition of Derivative Practice

2) Use the Alternative definition of the derivative to find $f'(2)$ if $f(x) = \sqrt{2 - x}$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Formal Definition of Derivative … Practice Set 2

Answers

2) Use the Alternative definition of the derivative to find $H'(2)$ if $H(x) = \sqrt{3 - x}$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$c = 2$ $\quad h(2) = \sqrt{3-2} = 1$

$$h'(2) = \lim_{x \to 2} \frac{h(x) - h(2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{\sqrt{3-x} - 1}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{(\sqrt{3-x} - 1)}{(x - 2)} \cdot \frac{(\sqrt{3-x} + 1)}{(\sqrt{3-x} + 1)}$$

$$h'(2) = \lim_{x \to 2} \frac{3 - x - 1}{(x-2)(\sqrt{3-x} + 1)}$$

$$h'(2) = \lim_{x \to 2} \frac{\cancel{(2-x)}(-1)}{\cancel{(x-2)}(\sqrt{3-x} + 1)}$$

$$h'(2) = \frac{-1}{\sqrt{3-2} + 1} = \frac{-1}{1+1} = \boxed{-\frac{1}{2}}$$

Formal Definition of Derivative … Practice Set 2

Definition of Derivative Practice

3) Use the Limit Definition of a Derivative to find f'(x) if $f(x) = \sqrt{2x-1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Formal Definition of Derivative … Practice Set 2

3) Use the Limit Definition of a Derivative to find $f'(x)$ if $f(x) = \sqrt{2x-1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f(\ ) = \sqrt{2(\ )-1}$$

$$f(x+h) = \sqrt{2(x+h)-1}$$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{2x+2h-1}-\sqrt{2x-1}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{2x+2h-1}-\sqrt{2x-1}}{h} \cdot \frac{(\sqrt{2x+2h-1}+\sqrt{2x-1})}{\sqrt{2x+2h-1}+\sqrt{2x-1}}$$

$$f'(x) = \lim_{h\to 0} \frac{2x+2h-1-(2x-1)}{h[\sqrt{2x+2h-1}+\sqrt{2x-1}]}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x}+2h\cancel{-1}\cancel{-2x}\cancel{+1}}{h[\sqrt{2x+2h-1}+\sqrt{2x-1}]}$$

$$f'(x) = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2x+2h-1}+\sqrt{2x-1})}$$

$$= \frac{2}{\sqrt{2x-1}+\sqrt{2x-1}} = \frac{2}{2\sqrt{2x-1}} = \boxed{\frac{1}{\sqrt{2x-1}}}$$

Formal Definition of Derivative … Practice Set 2

4) Use the Limit Definition of a derivative to find $f'(3)$ if $f(x) = \frac{2}{5-x}$

Answers

4) Use the Limit Definition of a derivative to find f'(3) if $f(x) = \frac{2}{5-x}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{5-(x+h)} - \frac{2}{5-x}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{5-x-h} - \frac{2}{5-x}}{h} \cdot (5-x)(5-x-h)$$

$$\lim_{h\to 0} \frac{2(5-x)-2(5-x-h)}{h(5-x)(5-x-h)}$$

$$\lim_{h\to 0} \frac{\cancel{10}-\cancel{2x}-\cancel{10}+\cancel{2x}+2h}{h(5-x)(5-x-h)}$$

$$\lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}(5-x)(5-x-h)}$$

$$f'(x) = \lim_{h\to 0} \frac{2}{(5-x)(5-x-h)} = \frac{2}{(5-x)(5-x)}$$

$$\boxed{f'(x) = \frac{2}{(5-x)^2}}$$

$$f'(3) = \frac{2}{(5-3)^2} = \frac{2}{(2)^2}$$

$$\boxed{f'(3) = \frac{1}{2}}$$

Formal Definition of Derivative … Practice Set 2

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $y - y_1 = m(x - x_1)$

Answers

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $y - y_1 = m(x - x_1)$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(\ ) = 2(\ ) - 3(\ )^2$

$f(x+h) = 2(x+h) - 3(x+h)^2$

$f'(x) = \lim_{h \to 0} \frac{2(x+h) - 3(x+h)^2 - (2x - 3x^2)}{h}$

$f'(x) = \lim_{h \to 0} \frac{2x + 2h - 3(x^2 + 2xh + h^2) - 2x + 3x^2}{h}$

$f'(x) = \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{2x} + \cancel{3x^2}}{h}$

$\lim_{h \to 0} \frac{2h - 6xh - 3h^2}{h} \rightarrow \lim_{h \to 0} \frac{\cancel{h}(2 - 6x - 3h)}{\cancel{h}}$

$f'(x) = 2 - 6x - 0$

$\boxed{f'(x) = 2 - 6x}$

$f'(-1) = 2 - 6(-1)$

$= 2 + 6$

$f'(-1) = 8$

Alt. method $f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

$f'(-1) = \lim_{x \to -1} \frac{f(x) - f(-1)}{x - (-1)}$

$f'(-1) = \lim_{x \to -1} \frac{2x - 3x^2 - (-5)}{x + 1} = \frac{2x - 3x^2 + 5}{x + 1}$

$= \lim_{x \to -1} \frac{-1(3x^2 - 2x - 5)}{x + 1}$

$f'(-1) = \lim_{x \to -1} \frac{-1(3x - 5)\cancel{(x + 1)}}{\cancel{(x + 1)}}$

$f'(-1) = -1(-3 - 5) = 8$

point: $f(-1) = -2 - 3 = -5$

slope: $m = 8$

point: $(-1, -5)$

slope: $m = 8$

$\boxed{y + 5 = 8(x + 1)}$