Instantaneous Rate of Change ... Practice Set 3

Instantaneous rate of change example 1:

Find the instantaneous rate of change at the given value. Sketch a graph to model your answer. (You may use your calculator obtain the graph, be sure to label the necessary points.)

Use the instantaneous rate of change formula: $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$f(x) = 3x^2 - 5x + 4; x = 2$

Instantaneous Rate of Change … Practice Set 3

Create 2 points

Use $x = 2$ as the x-coordinate of the first point

Find the y-coordinate of the first point:

$$f(2) = 3(2)^2 - 5(2) + 4 = 6$$

First point: $(2, \ 6)$

Use $2 + h$ as the x-coordinate of the second point.

Find the y-coordinate of the second point:

$$f(2 + h) = 3(2 + h)^2 - 5(2 + h) + 4$$
$$= 3(2 + h)(2 + h) - 5(2 + h) + 4$$
$$= 3(4 + 4h + h^2) - 5(2 + h) + 4$$
$$= 12 + 12h + 3h^2 - 10 - 5h + 4$$
$$= 3h^2 + 7h - 5x + 6$$

Second point: $(2 + h, \ 3h^2 + 7h + 6)$

$$Instantaneous \ rate \ of \ change = f'(2) = \lim_{h \to 0} \frac{3h^2 + 7h + 6 - 6}{h}$$

$$f'(2) = \lim_{h \to 0} \frac{3h^2 + 7h}{h} = \lim_{h \to 0} \frac{h(3h + 7)}{h} = \lim_{h \to 0}(3h + 7) = 3(0) + 7 = 7$$

Answer: Instantaneous rate of change at $x = 2$ is 7: $f'(2) = 7$

Instantaneous rate of change example 2:

A pebble is dropped from a cliff, 288 $foot$ cliff. The pebble takes 3 seconds to hit the ground.

The formula: $f(t) = 288 - 32t^2$ Can be used to calculate the pebbles height of the ground in f feet $t - seconds$ after it is dropped.

a) Calculate the average rate of change (average speed) in feet per second of the pebble for the 3 seconds it takes to hit the ground.

We need to create two points.

Since we are asked to find the average rate of change (velocity) in feet per second

$x - coordinate$ of the points must be time in seconds (hours are mentioned second)

$y - coordinate$ of the points must be height in feet (feet are mentioned first)

$(seconds, feet)$

These are the points needed:

$(0, \ 288)$ (at 0 seconds the pebble is 288 feet high)

$(3, \ 0)$ (at 3 seconds the pebble is on the ground.

$$Average \ Rate \ of \ Change = \frac{0-288 \ (feet)}{3-0 \ (seconds)} = \frac{-288 \ feet}{3 \ seconds} =$$

$-96 \ feet \ per \ second$

Answer: Average Rate of Change (average speed):
$-96 \ feet \ per \ second$ **(answer is negative since the pebble is falling)**

b) Calculate the instantaneous rate of change in feet per second (velocity) of the pebble at t = 3 seconds.

Instantaneous Rate of Change ... Practice Set 3

Answers

Create 2 points

Use $x = 3$ as the x-coordinate of the first point

Find the y-coordinate of the first point:

$$f(3) = 288 - 32(3)^2 = 0$$

First point: $(3, \ 0)$

Use $3 + h$ as the x-coordinate of the second point.

Find the y-coordinate of the second point:

$$f(3 + h) = 288 - 32(3 + h)^2$$
$$= 288 - 32(3 + h)(3 + h)$$
$$= 288 - 32(9 + 6h + h^2)$$
$$= 288 - 288 - 192h - 32h^2$$
$$= -192h - 32h^2$$

Second point: $(3 + h, \ -192h - 32h^2)$

$$Instantaneous \ rate \ of \ change \ (velocity) = f'(3) =$$

$$\lim_{h \to 0} \frac{192h - 32h^2 - 0 \ (feet)}{h \ (seconds)}$$

$$f'(3) = \lim_{h \to 0} \frac{h(-192 - 32h)(feet)}{h \ (seconds)} = \lim_{h \to 0}(-192 - 32h) = -192 -$$
$$32(0) = -192 \ feet \ per \ second$$

Answer: The pebble's velocity (instantaneous rate of change) is $-192 \ feet \ per \ second$ when it hits the ground. (The negative sign indicates the pebble is falling.)