Tangent and Normal Lines … Practice Set 4

Tangent Lines and Normal Lines

To write the equation of a line, you need two things: the slope and a point

To find the slope of a tangent line, you would use the derivative.

Remember: the equation of a line in point-slope form is $y - y_1 = m(x - x_1)$.

Examples:

a) Find the equation of the tangent line to $f(x) = x^2 + x$ at x = 1.

using alternative def. of derivative

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x^2 + x) - 2}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= 1 + 2$$

$$= 3$$

Slope of tangent at $x = 1$ is 3.

$f(1) = 1^2 + 1 = 2$ so point of tangency is $(1, 2)$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 2 = 3(x - 1)}$$ This is the equation of the tangent line to f at x=1

b) Find the equation of the line tangent to $f(x)=\sqrt{x}$ at the point where the tangent line is parallel to $2x-y=4$

$-y = 4-2x$

$y = -4+2x$

$m = 2$

Need $f'(x) = \lim_{\Delta x \to 0} \dfrac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x} \cdot \dfrac{\sqrt{x+\Delta x}+\sqrt{x}}{\sqrt{x+\Delta x}+\sqrt{x}}$

$= \lim_{\Delta x \to 0} \dfrac{x+\Delta x - x}{\Delta x(\sqrt{x+\Delta x}+\sqrt{x})}$

$= \dfrac{1}{2\sqrt{x}}$

For what x-value is $f'(x)=2$?

$2 = \dfrac{1}{2\sqrt{x}}$

$4\sqrt{x} = 1$

$\sqrt{x} = \frac{1}{4}$

$x = \frac{1}{16}$

Point of tangency is $\left(\frac{1}{16}, \frac{1}{4}\right)$

$f\left(\frac{1}{16}\right)$

$$\boxed{y - \frac{1}{4} = 2\left(x - \frac{1}{16}\right)}$$

Equation of line tangent to f that is parallel to $2x-y=4$

c) Find the equations of any horizontal tangent lines on the graph of $f(x) = 2x^3 - 3x^2 - 12x$

$m = 0$

$$f'(x) = \lim_{\Delta x \to 0} \frac{2(x+\Delta x)^3 + {}^-3(x+\Delta x)^2 - 12(x+\Delta x) - 2x^3 + 3x^2 + 12x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3) - 3(x^2 + 2x\Delta x + \Delta x^2) - 12x - 12\Delta x - 2x^3 + 3x^2 + 12x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{2x^3} + 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 \cancel{- 3x^2} - 6x\Delta x - 3\Delta x^2 \cancel{- 12x} - 12\Delta x \cancel{- 2x^3} \cancel{+ 3x^2} \cancel{+ 12x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (6x^2 + 6x\Delta x + 2(\Delta x)^2 - 6x - 3\Delta x - 12)$$

$$f'(x) = 6x^2 - 6x - 12$$

$$0 = 6x^2 - 6x - 12$$

$$0 = 6(x^2 - x - 2)$$

$$0 = 6(x-2)(x+1)$$

$$x = 2, \quad x = -1$$

There are horizontal tangents at $x = 2, x = -1$

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A normal line is just another term for a perpendicular line.

Remember, lines that are perpendicular have opposite reciprocal **slopes.**

Example: Find the equation of the line normal to $f(x) = 3x^2 - 2x$ at x = 2.

$$f'(x) = \lim_{\Delta x \to 0} \frac{3(x+\Delta x)^2 - 2(x+\Delta x) - 3x^2 + 2x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3x^2} + 6x\Delta x + 3(\Delta x)^2 - \cancel{2x} - 2\Delta x - \cancel{3x^2} + \cancel{2x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (6x + 3\Delta x - 2)$$

$$f'(x) = 6x - 2$$

$f'(2) = 6(2) - 2 = 10$ slope of tan at $x = 2$

so $-\frac{1}{10}$ slope of normal line at $x = 2$

$f'(2) = 3(2)^2 - 2(2) = 8$

Normal line: $m = -\frac{1}{10}$ point of tangency: $(2, 8)$

$$\boxed{y - 8 = -\frac{1}{10}(x - 2)}$$ Equation of normal line