Differentiate these. The given answers are not simplified.

1.
$$f(x) = 4x^5 - 5x^4$$

$$2. \ f(x) = e^x \sin x$$

3.
$$f(x) = (x^4 + 3x)^{-1}$$

4.
$$f(x) = 3x^2(x^3 + 1)^7$$

5.
$$f(x) = \cos^4 x - 2x^2$$

4.
$$f(x) = 3x^2(x^3 + 1)^7$$
 5. $f(x) = \cos^4 x - 2x^2$ 6. $f(x) = \frac{x}{1 + x^2}$

7.
$$f(x) = \frac{x^2 - 1}{x}$$

8.
$$f(x) = (3x^2)(x^{\frac{1}{2}})$$
 9. $f(x) = \ln(xe^{7x})$

$$9. \ f(x) = \ln(xe^{7x})$$

10.
$$f(x) = \frac{2x^4 + 3x^2 - 1}{x^2}$$
 11. $f(x) = (x^3)\sqrt[5]{2 - x}$ 12. $f(x) = 2x - \frac{4}{\sqrt{x}}$

11.
$$f(x) = (x^3)\sqrt[5]{2-x}$$

12.
$$f(x) = 2x - \frac{4}{\sqrt{x}}$$

Answers

1.
$$f'(x) = 20x^4 - 20x^3$$

3.
$$f'(x) = -1(x^4 + 3x)^{-2}(4x^3 + 3)$$

5.
$$f'(x) = 4(\cos x)^3(-\sin x) - 4x$$

7.
$$f'(x) = 1 + x^{-2}$$
 (Simplify f first.)

9.
$$f'(x) = \frac{1}{x} + 7$$
 (Simplify f first.) 10. $f'(x) = 4x + 0 + 2x^{-3}$ (Simplify f first.)

11.
$$f'(x) = x^3 \cdot \frac{1}{5} (2-x)^{\frac{-4}{5}} (-1) + (2-x)^{\frac{1}{5}} (3x^2)$$

$$2. f'(x) = e^x \cos x + (\sin x)e^x$$

4.
$$f'(x) = 3x^2 \cdot 7(x^3 + 1)^6 (3x^2) + (x^3 + 1)^7 \cdot 6x$$

5.
$$f'(x) = 4(\cos x)^3(-\sin x) - 4x$$
 6. $f'(x) = \frac{(1+x^2)(1) - x(2x)}{(1+x^2)^2}$

7.
$$f'(x) = 1 + x^{-2}$$
 (Simplify f first.) 8. $f'(x) = 3 \cdot \frac{5}{2} x^{\frac{3}{2}}$ (Simplify f first.)

10.
$$f'(x) = 4x + 0 + 2x^{-3}$$
 (Simplify f first.)

12.
$$f'(x) = 2 + 2x^{\frac{-3}{2}}$$

Differentiate these. The given answers are not simplified.

13.
$$f(x) = \frac{4(3x-1)^2}{x^2+7^x}$$

14.
$$f(x) = \sqrt{x^2 + 8}$$

15.
$$f(x) = \frac{x}{\sqrt{1 - (\ln x)^2}}$$

16.
$$f(x) = \frac{6}{(3x^2 - \pi)^4}$$

17.
$$f(x) = \frac{(3x^2 - \pi x)^4}{6}$$

18.
$$f(x) = \frac{x}{(x^2 + \sqrt{3x})^5}$$

19.
$$f(x) = (xe^x)^{\pi}$$

20.
$$f(x) = \left[\arctan(2x)\right]^{10}$$
 21. $f(x) = (e^{2x} + e)^{\frac{1}{2}}$

21.
$$f(x) = (e^{2x} + e)^{\frac{1}{2}}$$

22.
$$f(x) = (x^6 + 1)^5 (4x + 7)^3$$

22.
$$f(x) = (x^6 + 1)^5 (4x + 7)^3$$
 23. $f(x) = (7x + \sqrt{x^2 + 3})^6$ 24. $f(x) = \frac{\frac{1}{x} + \frac{1}{x^2}}{x - 1}$

24.
$$f(x) = \frac{\frac{1}{x} + \frac{1}{x^2}}{x - 1}$$

Answers

$$13. \ f'(x) = \frac{(x^2 + 7^x) \left[4 \cdot 2(3x - 1)(3) \right] - 4(3x - 1)^2 (2x + 7^x \ln 7)}{(x^2 + 7^x)^2} \qquad 14. \ f'(x) = \frac{1}{2} (x^2 + 8)^{\frac{-1}{2}} (2x)$$

$$15. \ f'(x) = \frac{\left(1 - (\ln x)^2 \right)^{\frac{1}{2}} (1) - x \cdot \frac{1}{2} \left(1 - (\ln x)^2 \right)^{\frac{-1}{2}} \left(-2(\ln x) \cdot \frac{1}{x} \right)}{1 - (\ln x)^2} \qquad 16. \ f'(x) = -24(3x^2 - \pi)^{-5} (6x)$$

$$17. \ f'(x) = \frac{1}{6} \left[4(3x^2 - \pi x)^3 (6x - \pi) \right] \qquad 18. \ f'(x) = \frac{(x^2 + \sqrt{3x})^5 (1) - x \left[5(x^2 + \sqrt{3x})^4 \left(2x + \frac{1}{2} (3x)^{\frac{-1}{2}} \cdot 3 \right) \right]}{(x^2 + \sqrt{3x})^{10}}$$

$$19. \ f'(x) = \pi (xe^x)^{(\pi - 1)} \left[xe^x + e^x \right] \qquad 20. \ f'(x) = 10 \left[\arctan(2x) \right]^9 \cdot \frac{1}{1 + (2x)^2} \cdot 2$$

$$21. \ f'(x) = \frac{1}{2} (e^{2x} + e)^{\frac{-1}{2}} (e^{2x} \cdot 2 + 0) \qquad 22. \ f'(x) = (x^6 + 1)^5 \left[3(4x + 7)^2 (4) \right] + (4x + 7)^3 \left[5(x^6 + 1)^4 (6x^5) \right]$$

$$23. \ f'(x) = 6(7x + \sqrt{x^2 + 3})^5 \left(7 + \frac{1}{2} (x^2 + 3)^{\frac{-1}{2}} \cdot 2x \right) \qquad 24. \ f'(x) = \frac{(x - 1)(-x^{-2} - 2x^{-3}) - (x^{-1} + x^{-2})(1)}{(x - 1)^2}$$

Differentiate these. The given answers are not simplified.

25.
$$f(x) = \sqrt[3]{x^2} - \frac{1}{\sqrt{x^3}}$$
 26. $f(x) = \sqrt{\frac{2x+5}{7x-9}}$

26.
$$f(x) = \sqrt{\frac{2x+5}{7x-9}}$$

$$27. \ f(x) = \frac{\sin x}{\cos x}$$

28.
$$f(x) = e^x(x^2 + 3)(x^3 + 4)$$
 29. $f(x) = \frac{5x^2 - 7x}{x^2 + 2}$

29.
$$f(x) = \frac{5x^2 - 7x}{x^2 + 2}$$

30.
$$f(x) = \left[\ln(5x^2 + 9)\right]^3$$

31.
$$f(x) = \ln(5x^2 + 9)^3$$
 32. $f(x) = \cot(6x)$ 33. $f(x) = \sec^2 x \cdot \tan x$

$$32. \ f(x) = \cot(6x)$$

33.
$$f(x) = \sec^2 x \cdot \tan x$$

34.
$$f(x) = \arcsin(2^x)$$

35.
$$f(x) = \tan(\cos x)$$

34.
$$f(x) = \arcsin(2^x)$$
 35. $f(x) = \tan(\cos x)$ 36. $f(x) = [(x^2 - 1)^5 - x]^3$

Answers

25.
$$f'(x) = \frac{2}{3}x^{\frac{-1}{3}} + \frac{3}{2}x^{\frac{-5}{2}}$$
 26. $f'(x) = \frac{1}{2}\left(\frac{2x+5}{7x-9}\right)^{\frac{-1}{2}}\left[\frac{(7x-9)(2)-(2x+5)(7)}{(7x-9)^2}\right]$ 27. $f'(x) = \sec^2 x$ 28. $f'(x) = \left[e^x(x^2+3)\right](3x^2) + (x^3+4)\left[e^x(2x) + (x^2+3)e^x\right]$ 29. $f'(x) = \frac{(x^2+2)(10x-7)-(5x^2-7x)(2x)}{(x^2+2)^2}$ 30. $f'(x) = 3\left[\ln(5x^2+9)\right]^2 \cdot \frac{1}{5x^2+9}(10x+0)$ 31. $f'(x) = \frac{1}{(5x^2+9)^3} \cdot \left[3(5x^2+9)^2(10x+0)\right]$ 32. $f'(x) = -\csc^2(6x) \cdot 6$ 33. $f'(x) = \sec^2 x(\sec^2 x) + \tan x \left[2 \cdot \sec x(\sec x \tan x)\right]$ 34. $f'(x) = \frac{1}{\sqrt{1-(2^x)^2}} \cdot 2^x \ln 2$ 35. $f'(x) = \left(\sec^2(\cos x)\right)(-\sin x)$ 36. $f'(x) = 3\left[(x^2-1)^5-x\right]^2\left(5(x^2-1)^4 \cdot 2x-1\right)$

Differentiate these. The given answers are not simplified.

$$37. \ f(x) = \sec x \cdot \sin(3x)$$

38.
$$f(x) = \frac{(x-1)^3}{x(x+3)^4}$$

39.
$$f(x) = \log_5(3x^2 + 4x)$$

In problems 40 – 42, find $\frac{dy}{dx}$. Assume y is a differentiable function of x.

40.
$$3y = xe^{5y}$$

41.
$$xy + y^2 + x^3 = 7$$

42.
$$\frac{\sin y}{y^2 + 1} = 3x$$

If f and g are differentiable functions such that f(2)=3, f'(2)=-1, f'(3)=7, g(2)=-5 and g'(2)=2, find the numbers indicated in problems 43-48.

43.
$$(g-f)'(2)$$

44.
$$(fg)'(2)$$

45.
$$\left(\frac{f}{g}\right)'(2)$$

46.
$$(5f + 3g)'(2)$$

47.
$$(f \circ f)'(2)$$

48.
$$\left(\frac{f}{f+g}\right)'(2)$$

Answers

37. $f'(x) = \sec x \left(\cos(3x) \cdot 3\right) + \sin(3x) \left(\sec x \tan x\right)$

38.
$$f'(x) = \frac{x(x+3)^4 [3(x-1)^2(1)] - (x-1)^3 [x \cdot 4(x+3)^3(1) + (x+3)^4(1)]}{x^2(x+3)^8}$$

39.
$$f'(x) = \frac{1}{(3x^2 + 4x) \cdot \ln 5} \cdot (6x + 4)$$
 40. $\frac{dy}{dx} = \frac{e^{5y}}{3 - 5xe^{5y}}$

$$40. \ \frac{dy}{dx} = \frac{e^{5y}}{3 - 5xe^{5y}}$$

41.
$$\frac{dy}{dx} = \frac{-3x^2 - y}{x + 2y}$$

41.
$$\frac{dy}{dx} = \frac{-3x^2 - y}{x + 2y}$$
 42. $\frac{dy}{dx} = \frac{3(y^2 + 1)^2}{(y^2 + 1)(\cos y) - 2y \sin y}$ 43. 3 44. 11 45. $\frac{-1}{25}$ 46. 1 47. -7 48. $\frac{-1}{4}$

45.
$$\frac{-1}{25}$$

$$47. -7$$

48.
$$\frac{-1}{4}$$

Differentiate these. The given answers are not simplified.

1.
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5.
$$f(x) = \cos^4 x - 2x^2$$

6.
$$f(x) = \frac{x}{1 + x^2}$$

7.
$$f(x) = \frac{x^2 - 1}{x}$$

8.
$$f(x) = (3x^2)(x^{\frac{1}{2}})$$

$$9. \ f(x) = \ln(xe^{7x})$$

10.
$$f(x) = \frac{2x^4 + 3x^2 - 1}{x^2}$$

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$$f(x) = (x^3)\sqrt[5]{2-x}$$

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$$\left(\frac{f}{a}\right)'(2)$$

46.
$$(5f + 3g)'(2)$$

47.
$$(f \circ f)'(2)$$

$$48. \left(\frac{f}{f+g}\right)'(2)$$

Power. Product, and Quotient Rules Worksheet

Find the derivative of each function.

1.
$$f(x) = 3x^2 + 5x - 2$$

2.
$$g(x) = -4x^4 + 5x^3 - 2x + 3$$

3.
$$f(x) = 2\sqrt{x} + 7\sqrt{x^3} - \frac{2}{x^2}$$

4.
$$g(x) = 8\sqrt{x^5} - 7x^4 + \frac{5}{\sqrt{x}}$$

5.
$$f(x) = \frac{x^2 + 7x - 18}{x + 9}$$

6.
$$f(x) = \frac{x^2 - 5x - 24}{x - 8}$$

7.
$$f(x) = \frac{x^{-3} + 7\sqrt{x^3} - 4x^2}{2\sqrt{x}}$$

8.
$$h(x) = \frac{\frac{2}{x^3} + 5x^2 - 8\sqrt{x^7}}{-3\sqrt{x}}$$

9.
$$s(x) = 2x^{-3} \sec(x)$$

10.
$$f(x) = 3x^4 e^x$$

11.
$$f(x) = -7x^3e^x$$

$$12. \quad f(x) = 5x^2 \cos(x)$$

13.
$$h(x) = 2e^x \sqrt{x}$$

14.
$$f(x) = 4x^4 - 5x^3 + 2x^2e^x$$

15.
$$f(x) = \frac{\tan(x)}{2x^2 + 1}$$

16.
$$g(x) = \frac{\sin(x)}{e^x + 5}$$

17.
$$f(x) = \frac{x^2 - 3x + 2}{x + 3}$$

18.
$$f(x) = \frac{2e^x}{x - 2e^x}$$

19.
$$h(x) = \frac{2x^4 \cot(x)}{3x^2}$$

$$20. \quad f(x) = \frac{\csc(x)}{-4xe^x}$$

Chain Rule Worksheet

Find the derivative of each function.

1.
$$f(x) = (2x^2 - 5x)^3$$

2.
$$f(x) = \sqrt{5x^3 - 2x}$$

3.
$$y = 3\sin(x-3)$$

4.
$$y = -2\cos(x^2 + 2)$$

5.
$$g(x) = \sin^2(3x^2)$$

6.
$$h(x) = \sec^3(x^2 - 5)$$

7.
$$f(x) = 3x^3e^{2x-5}$$

8.
$$g(x) = -5x^2e^{x^2+3x}$$

9.
$$y = 3x^2 \sqrt{4x^2 - 5x + 1}$$

10.
$$h(t) = \frac{2}{3}t^3\sqrt{3t^3 - 5t}$$

11.
$$y = \frac{1}{\sqrt[3]{x^3 - 4x^2 + 1}}$$

12.
$$g(t) = \frac{-3}{\sqrt[4]{2t^3 + 5t - 3}}$$

13.
$$g(m) = \sin(\cos(m))$$

14.
$$f(x) = \cos(\tan x)$$

15.
$$h(x) = \sqrt{x^3 + 2(x^2 - 1)^4}$$

16.
$$h(m) = \sqrt{m^2 + 1}(m^2 + 1)^3$$

17.
$$f(t) = \sqrt[3]{\frac{t^2 + 2}{t^2 - 2}}$$

18.
$$f(t) = \sqrt[4]{\frac{t^3 + 8}{t^3 - 8}}$$

19.
$$h(x) = (2x+5)^7 (3x^4-8)^5$$

20.
$$g(n) = (3x^2 - 2)(4x^3 + 1)$$

21.
$$f(t) = \csc^2(t^3)$$

22.
$$f(t) = \cot^4(2t^2)$$

23.
$$h(x) = e^{\sqrt{2x^3-x^2}}$$

24.
$$f(x) = e^{\sqrt{4x^2 - 3x}}$$

25.
$$h(x) = \frac{3x}{\sqrt[3]{5 + 2x^2}}$$

26.
$$f(s) = \frac{2s^3}{\sqrt[4]{s^2 - 5s}}$$

27.
$$f(x) = 5^{\sin x^3}$$

28.
$$f(x) = 2^{e^{4x}}$$