Product and Quotient Rules

Differentiate each function with respect to x.

1)
$$y = (2x^2 + 1) \cdot -4x^2$$

2)
$$y = (-2x^2 + 3) \cdot 2x^4$$

3)
$$y = 3x^3(4x^5 + 2)$$

4)
$$y = 2x^3(5x^4 + 5)$$

5)
$$v = (x^4 + 4)(x^5 + 2)$$

6)
$$y = (-x^3 - 4)(4x^3 + 3)$$

7)
$$y = (4x^3 + 1)(-3x^3 + 3)$$

8)
$$y = (5x^4 + 3)(5x^4 + 2)$$

9)
$$y = (x^5 + 3x^4 + 3)(x^3 - 5)$$

10)
$$y = (-4x^5 + 2x^3 + 1)(2x^4 + 5)$$

11)
$$y = (2x^2 + 3)(-5x^5 + x^3 - 5)$$

12)
$$y = (-2x^5 - 4)(4x^4 + 3x^2 + 4)$$

Answers

1)
$$\frac{dy}{dx} = (2x^2 + 1) \cdot -8x - 4x^2 \cdot 4x$$

= $-32x^3 - 8x$

3)
$$\frac{dy}{dx} = 3x^3 \cdot 20x^4 + (4x^5 + 2) \cdot 9x^2$$
$$= 96x^7 + 18x^2$$

5)
$$\frac{dy}{dx} = (x^4 + 4) \cdot 5x^4 + (x^5 + 2) \cdot 4x^3$$

= $9x^8 + 20x^4 + 8x^3$

7)
$$\frac{dy}{dx} = (4x^3 + 1) \cdot -9x^2 + (-3x^3 + 3) \cdot 12x^2$$
$$= -72x^5 + 27x^2$$

9)
$$\frac{dy}{dx} = (x^5 + 3x^4 + 3) \cdot 3x^2 + (x^3 - 5)(5x^4 + 12x^3)$$
$$= 8x^7 + 21x^6 - 25x^4 - 60x^3 + 9x^2$$

10)
$$\frac{dy}{dx} = (-4x^5 + 2x^3 + 1) \cdot 8x^3 + (2x^4 + 5)(-20x^4 + 6x^2)$$
$$= -72x^8 + 28x^6 - 100x^4 + 8x^3 + 30x^2$$

11)
$$\frac{dy}{dx} = (2x^2 + 3)(-25x^4 + 3x^2) + (-5x^5 + x^3 - 5) \cdot 4x$$
$$= -70x^6 - 65x^4 + 9x^2 - 20x$$

12)
$$\frac{dy}{dx} = (-2x^5 - 4)(16x^3 + 6x) + (4x^4 + 3x^2 + 4) \cdot -10x^4$$

2)
$$\frac{dy}{dx} = (-2x^2 + 3) \cdot 8x^3 + 2x^4 \cdot -4x$$
$$= -24x^5 + 24x^3$$

4)
$$\frac{dy}{dx} = 2x^3 \cdot 20x^3 + (5x^4 + 5) \cdot 6x^2$$
$$= 70x^6 + 30x^2$$

6)
$$\frac{dy}{dx} = (-x^3 - 4) \cdot 12x^2 + (4x^3 + 3) \cdot -3x^2$$
$$= -24x^5 - 57x^2$$

8)
$$\frac{dy}{dx} = (5x^4 + 3) \cdot 20x^3 + (5x^4 + 2) \cdot 20x^3$$
$$= 200x^7 + 100x^3$$

Quotient Rule

Differentiate each function with respect to x.

13)
$$y = \frac{5x^5}{3x^5 + 3}$$

14)
$$y = \frac{5x^2}{2x^4 + 2}$$

15)
$$y = \frac{2x^3}{4x^4 + 5}$$

16)
$$y = \frac{4x^3}{x^5 + 3}$$

17)
$$y = \frac{x^5 + 5}{x^3 + 5}$$

18)
$$y = \frac{x^4 + 4x^2}{2x^4 + 2}$$

19)
$$y = \frac{x^5 - 3}{3x^3 + 3}$$

20)
$$y = \frac{2x^5 + 5x^3}{x^5 + 4}$$

Answers

$$= -72x^{8} - 42x^{6} - 40x^{4} - 64x^{3} - 24x$$

$$13) \frac{dy}{dx} = \frac{(3x^{5} + 3) \cdot 25x^{4} - 5x^{5} \cdot 15x^{4}}{(3x^{5} + 3)^{2}}$$

$$= \frac{25x^{4}}{3x^{10} + 6x^{5} + 3}$$

$$15) \frac{dy}{dx} = \frac{(4x^{4} + 5) \cdot 6x^{2} - 2x^{3} \cdot 16x^{3}}{(4x^{4} + 5)^{2}}$$

$$= \frac{-8x^{6} + 30x^{2}}{16x^{8} + 40x^{4} + 25}$$

$$17) \frac{dy}{dx} = \frac{(x^{3} + 5) \cdot 5x^{4} - (x^{5} + 5) \cdot 3x^{2}}{(x^{3} + 5)^{2}}$$

$$= \frac{2x^{7} + 25x^{4} - 15x^{2}}{x^{6} + 10x^{3} + 25}$$

$$19) \frac{dy}{dx} = \frac{(3x^{3} + 3) \cdot 5x^{4} - (x^{5} - 3) \cdot 9x^{2}}{(3x^{3} + 3)^{2}}$$

$$= \frac{2x^{7} + 5x^{4} + 9x^{2}}{3x^{6} + 6x^{3} + 3}$$

$$20) \frac{dy}{dx} = \frac{(x^{5} + 4)(10x^{4} + 15x^{2}) - (2x^{5} + 5x^{3}) \cdot 5x^{4}}{(x^{5} + 4)^{2}}$$

$$= \frac{-10x^{7} + 40x^{4} + 60x^{2}}{x^{10} + 8x^{5} + 16}$$

14)
$$\frac{dy}{dx} = \frac{(2x^4 + 2) \cdot 10x - 5x^2 \cdot 8x^3}{(2x^4 + 2)^2}$$

$$= \frac{-5x^5 + 5x}{x^8 + 2x^4 + 1}$$
16)
$$\frac{dy}{dx} = \frac{(x^5 + 3) \cdot 12x^2 - 4x^3 \cdot 5x^4}{(x^5 + 3)^2}$$

$$= \frac{-8x^7 + 36x^2}{x^{10} + 6x^5 + 9}$$
18)
$$\frac{dy}{dx} = \frac{(2x^4 + 2)(4x^3 + 8x) - (x^4 + 4x^2) \cdot 8x^3}{(2x^4 + 2)^2}$$

$$= \frac{-4x^5 + 2x^3 + 4x}{x^8 + 2x^4 + 1}$$

Quotient Rule

Differentiate each function with respect to x.

21)
$$y = \frac{5x^4 + 2x^3 + x^2}{2x^5 + 2}$$

22)
$$y = \frac{2x^4 + 4x^3 + 5x^2}{5x^3 + 5}$$

23)
$$y = \frac{x^3 + 4x^2 + 4}{4x^3 + 3}$$

24)
$$y = \frac{x^5 + 5x^3 + 5x^2}{3x^2 + 3}$$

Answers

21)
$$\frac{dy}{dx} = \frac{(2x^5 + 2)(20x^3 + 6x^2 + 2x) - (5x^4 + 2x^3 + x^2) \cdot 10x^4}{(2x^5 + 2)^2}$$

$$= \frac{-5x^8 - 4x^7 - 3x^6 + 20x^3 + 6x^2 + 2x}{2x^{10} + 4x^5 + 2}$$
22)
$$\frac{dy}{dx} = \frac{(5x^3 + 5)(8x^3 + 12x^2 + 10x) - (2x^4 + 4x^3 + 5x^2) \cdot 15x^2}{(5x^3 + 5)^2}$$

$$= \frac{2x^6 - 5x^4 + 8x^3 + 12x^2 + 10x}{5x^6 + 10x^3 + 5}$$
23)
$$\frac{dy}{dx} = \frac{(4x^3 + 3)(3x^2 + 8x) - (x^3 + 4x^2 + 4) \cdot 12x^2}{(4x^3 + 3)^2}$$

$$= \frac{-16x^4 - 39x^2 + 24x}{16x^6 + 24x^3 + 9}$$
24)
$$\frac{dy}{dx} = \frac{(3x^2 + 3)(5x^4 + 15x^2 + 10x) - (x^5 + 5x^3 + 5x^2) \cdot 6x}{(3x^2 + 3)^2}$$

$$= \frac{3x^6 + 10x^4 + 15x^2 + 10x}{3x^4 + 6x^2 + 3}$$