Trig Rules … Set 2

Finding a Derivative of a Trigonometric Function In Exercises 39–54, find the derivative of the trigonometric function.

39. $f(t) = t^2 \sin t$

42. $f(x) = \dfrac{\sin x}{x^3}$

43. $f(x) = -x + \tan x$

44. $y = x + \cot x$

51. $f(x) = x^2 \tan x$

52. $f(x) = \sin x \cos x$

53. $y = 2x \sin x + x^2 \cos x$

54. $h(\theta) = 5\theta \sec \theta + \theta \tan \theta$

Trig Rules … Set 2

Answers

39. $f(t) = t^2 \sin t$

*product rule
$f'g + fg'$

$$f'(t) = \overbrace{2t}^{f'} \cdot \overbrace{\sin t}^{g} + \overbrace{t^2}^{f} \cdot \overbrace{\cos(t)}^{g'}$$

$$\boxed{f'(t) = 2t\sin(t) + t^2\cos(t)}$$

42. $f(x) = \dfrac{\sin x}{x^3}$

*quotient rule
$\dfrac{f'g - fg'}{g^2}$

$$f'(x) = \frac{\overbrace{\cos x}^{f'} \cdot \overbrace{x^3}^{g} - \overbrace{\sin x}^{f} \cdot \overbrace{3x^2}^{g'}}{\underbrace{(x^3)^2}_{g^2}}$$

$$f'(x) = \frac{x^3\cos x - 3x^2\sin x}{x^6}$$

$$f'(x) = \frac{x^2(x\cos x - 3\sin x)}{x^6}$$

$$\boxed{f'(x) = \frac{x\cos x - 3\sin x}{x^4}}$$

43. $f(x) = -x + \tan x$

$$\boxed{f'(x) = -1 + \sec^2 x}$$

44. $y = x + \cot x$

$$\boxed{y' = 1 - \csc^2 x}$$

51. $f(x) = x^2 \tan x$

$$f'(x) = \overbrace{2x}^{f'} \ \overbrace{\tan x}^{g} + \overbrace{x^2}^{f} \cdot \overbrace{\sec^2 x}^{g'}$$

$$\boxed{f'(x) = 2x\tan x + x^2\sec^2 x}$$

52. $f(x) = \sin x \cos x$

$$f'(x) = \overbrace{(\cos x)}^{f'}\overbrace{(\cos x)}^{g} + \overbrace{(\sin x)}^{f}\overbrace{(-\sin x)}^{g'}$$

$$\boxed{f'(x) = \cos^2 x - \sin^2 x}$$

53. $y = 2x \sin x + x^2 \cos x$

$$y' = \overbrace{2}^{f'} \cdot \overbrace{\sin x}^{g} + \overbrace{2x}^{f} \overbrace{\cos x}^{g'} + \overbrace{2x}^{f'} \overbrace{\cos x}^{g} + \overbrace{x^2}^{f}\overbrace{(-\sin x)}^{g'}$$

$$\boxed{y' = 4x\cos x + 2\sin x - x^2\sin x}$$

54. $h(\theta) = 5\theta \sec\theta + \theta\tan\theta$

$$h'(\theta) = \overbrace{5}^{f'} \cdot \overbrace{\sec\theta}^{g} + \overbrace{5\theta}^{f} \overbrace{\sec\theta\tan\theta}^{g'} + \overbrace{1}^{f'} \overbrace{\tan\theta}^{g} + \overbrace{\theta}^{f} \cdot \overbrace{\sec^2\theta}^{g'}$$

$$\boxed{h'(\theta) = 5\sec\theta + 5\theta\sec\theta\tan\theta + \tan\theta + \theta\sec^2\theta}$$

Trig Rules … Set 2

43. $y = \cos 4x$

53. $y = 4\sec^2 x$

54) g(x) = $5\,cos^3 \pi x$

58. $h(t) = 2\cot^2(\pi t + 2)$

Finding a Derivative **In Exercises 1–16, find** dy/dx **by implicit differentiation.**

11. $\sin x + 2\cos 2y = 1$

13. $\sin x = x(1 + \tan y)$

14. $\cot y = x - y$

15. $y = \sin xy$

Trig Rules … Set 2

Answers

*chain rule

43. $y = \cos 4x$

$y' = -\sin(4x) \cdot 4$

$\boxed{y' = -4\sin(4x)}$

53. $y = 4\sec^2 x$

*rewrite expression

$y = 4[\sec x]^2$

out: $4[\ ]^2$
in: $\sec u$
inner: x

$y' = 8(\sec x) \cdot \sec x \tan x \cdot 1$

$\boxed{y' = 8\sec^2 x \tan x}$

57) $g(x) = 5\cos^3 \pi x$

out: $5(\ )^3$
in: $\cos u$
inner: πx

$g(x) = 5[\cos(\pi x)]^3$

$g'(x) = 15[\cos \pi x]^2 \cdot -\sin(\pi x) \cdot \pi$

$\boxed{g'(x) = -15\pi \cos^2 \pi x \sin \pi x}$

58. $h(t) = 2\cot^2(\pi t + 2)$

out: $2(\ )^2$
in: $\cot u$
inner: $\pi t + 2$

$h(t) = 2[\cot(\pi t + 2)]^2$

$h'(x) = 4(\cot(\pi t + 2)) \cdot -\csc^2(\pi t + 2) \cdot \pi$

$\boxed{h'(t) = -4\pi \cot(\pi t + 2)\csc^2(\pi t + 2)}$

Finding a Derivative **In Exercises 1–16, find** dy/dx **by implicit differentiation.**

11. $\sin x + 2\cos 2y = 1$

11) $\cos x - 2\sin(2y) \cdot 2\left(\frac{dy}{dx}\right) = 0$

$-4\sin 2y\left(\frac{dy}{dx}\right) = -\cos x$

$\frac{dy}{dx} = \frac{-\cos x}{-4\sin 2y} = \boxed{\frac{\cos x}{4\sin 2y}}$

13. $\sin x = x(1 + \tan y)$

*implicit
*product rule

$\sin x = x + x\tan y$

$\cos x = 1 + \overset{f'}{1} \cdot \overset{g}{\tan y} + \overset{f}{x} \cdot \overset{g'}{\sec^2 y\left(\frac{dy}{dx}\right)}$

$\cos x - 1 - \tan(y) = x\sec^2 y\left(\frac{dy}{dx}\right)$

$\boxed{\frac{\cos x - 1 - \tan(y)}{x\sec^2 y} = \frac{dy}{dx}}$

14. $\cot y = x - y$

$-\csc^2 y\left(\frac{dy}{dx}\right) = 1 - \left(\frac{dy}{dx}\right)$

$-\csc^2 y\left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 1$

$\frac{dy}{dx}(1 - \csc^2 y) = 1$

$\boxed{\frac{dy}{dx} = \frac{1}{1 - \csc^2 y}}$

15. $y = \sin xy$

$1\left(\frac{dy}{dx}\right) = \cos(xy) \cdot \left[1 \cdot y + x\left(\frac{dy}{dx}\right)\right]$

$1\left(\frac{dy}{dx}\right) = y\cos(xy) + x\cos(xy)\left(\frac{dy}{dx}\right)$

$1\left(\frac{dy}{dx}\right) - x\cos(xy)\left(\frac{dy}{dx}\right) = y\cos(xy)$

$\frac{dy}{dx}(1 - x\cos(xy)) = y\cos(xy)$

$\boxed{\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}}$