Higher Order Derivatives

Higher Derivatives Rule … Set 2

1. Given that $f(x) = 2x^4 - 3x^3 + 4x^2$, what is $f'''(x)$?

2. Suppose $f(x) = x^7 - x^6 + x^5$. What is $f^{(4)}(x)$?

Higher Derivatives Rule … Set 2

Answers

1. Given that $f(x) = 2x^4 - 3x^3 + 4x^2$, what is $f'''(x)$?

Since $f(x)$ is a polynomial function, differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$f'(x) = (2 \cdot 4)x^{4-1} - (3 \cdot 3)x^{3-1} + (4 \cdot 2)x^{2-1}$$
$$f'(x) = 8x^3 - 9x^2 + 8x$$
$$f''(x) = (8 \cdot 3)x^{3-1} - (9 \cdot 2)x^{2-1} + (8 \cdot 1)x^{1-1}$$
$$f''(x) = 24x^2 - 18x + 8$$
$$f'''(x) = (24 \cdot 2)x^{2-1} - (18 \cdot 1)x^{1-1} + 0$$
$$f'''(x) = 48x - 18$$

Answer: $f'''(x) = 48x - 18$

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2. Suppose $f(x) = x^7 - x^6 + x^5$. What is $f^{(4)}(x)$?

Since $f(x)$ is a polynomial function, differentiate using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$f'(x) = 7x^{7-1} - 6x^{6-1} + 5x^{5-1}$$
$$f'(x) = 7x^6 - 6x^5 + 5x^4$$
$$f''(x) = (7 \cdot 6)x^{6-1} - (6 \cdot 5)x^{5-1} + (5 \cdot 4)x^{4-1}$$
$$f''(x) = 42x^5 - 30x^4 + 20x^3$$
$$f'''(x) = (42 \cdot 5)x^{5-1} - (30 \cdot 4)x^{4-1} + (20 \cdot 3)x^{3-1}$$
$$f'''(x) = 210x^4 - 120x^3 + 60x^2$$
$$f^{(4)}(x) = (210 \cdot 4)x^{4-1} - (120 \cdot 3)x^{3-1} + (60 \cdot 2)x^{2-1}$$
$$f^{(4)}(x) = 840x^3 - 360x^2 + 120x$$

Answer: $f^{(4)}(x) = 840x^3 - 360x^2 + 120x$

Higher Derivatives Rule … Set 2

3. Suppose $h(x) = 2x^6 - 3x^5 + 4x^4$. What is $h^{(4)}(x)$?

4. Find $k''(x)$ if $k(x) = 8x^{-3} - 24x^{-2} + 12x^{-1}$.

Higher Derivatives Rule … Set 2

Answers

3. Suppose $h(x) = 2x^6 - 3x^5 + 4x^4$. What is $h^{(4)}(x)$?

Since $h(x)$ is a polynomial, differentiate using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$h'(x) = 12x^5 - 15x^4 + 16x^3$$

$$h''(x) = 60x^4 - 60x^3 + 48x^2$$

$$h'''(x) = 240x^3 - 180x^2 + 96x$$

$$h^{(4)}(x) = 720x^2 - 360x + 96$$

Answer: $h^{(4)}(x) = 720x^2 - 360x + 96$

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4. Find $k''(x)$ if $k(x) = 8x^{-3} - 24x^{-2} + 12x^{-1}$. When differentiating negative exponents, use the same rule as with positive exponents. Differentiate using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$k'(x) = (8 \cdot -3)x^{-3-1} - (24 \cdot -2)x^{-2-1} + (12 \cdot -1)x^{-1-1}$$

$$k'(x) = -24x^{-4} + 48x^{-3} - 12x^{-2}$$

$$k''(x) = (-24 \cdot -4)x^{-4-1} + (48 \cdot -3)x^{-3-1} - (12 \cdot -2)x^{-2-1}$$

$$k''(x) = 96x^{-5} - 144x^{-4} + 24x^{-3}$$

Answer: $k''(x) = 96x^{-5} - 144x^{-4} + 24x^{-3}$

Higher Derivatives Rule … Set 2

5. Given $g(x)$ below. Find $g'''(x)$.

$$g(x) = \frac{3}{x^2} - \frac{5}{x^4} + \frac{2}{x}$$

Higher Derivatives Rule … Set 2

Answers

5. Given $g(x)$ below. Find $g'''(x)$.

$$g(x) = \frac{3}{x^2} - \frac{5}{x^4} + \frac{2}{x}$$

First, change the function so you can differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$g(x) = \frac{3}{x^2} - \frac{5}{x^4} + \frac{2}{x} = 3x^{-2} - 5x^{-4} + 2x^{-1}$$

$$g'(x) = -6x^{-3} + 20x^{-5} - 2x^{-2}$$

$$g''(x) = 18x^{-4} - 100x^{-6} + 4x^{-3}$$

$$g'''(x) = -72x^{-5} + 600x^{-7} - 12x^{-4}$$

Convert $g'''(x)$ to the original form of the given function:

Answer:

$$g'''(x) = -\frac{72}{x^5} + \frac{600}{x^7} - \frac{12}{x^4}$$

Higher Derivatives Rule … Set 2

6. Given $g(x)$ below, Find $g''(x)$.

$$g(x) = \frac{1}{9x^5} + \frac{2}{3x^4} - \frac{1}{x}$$

Higher Derivatives Rule … Set 2

Answers

6. Given $g(x)$ below, Find $g''(x)$.

$$g(x) = \frac{1}{9x^5} + \frac{2}{3x^4} - \frac{1}{x}$$

First, change the function so you can differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$ and the chain rule.

$$g(x) = \frac{1}{9}x^{-5} + \frac{2}{3}x^{-4} - x^{-1}$$

$$g'(x) = \left(\frac{1}{9}\right)(-5)x^{-5-1} + \left(\frac{2}{3}\right)(-4)x^{-4-1} - (-1)x^{-1-1}$$

Simplify:

$$g'(x) = -\frac{5}{9}x^{-6} - \frac{8}{3}x^{-5} + x^{-2}$$

$$g''(x) = \left(-\frac{5}{9}\right)(-6)x^{-6-1} + \left(-\frac{8}{3}\right)(-5)x^{-5-1} + (-2)x^{-2-1}$$

Simplify:

$$g''(x) = \frac{10}{3}x^{-7} + \frac{40}{3}x^{-6} - 2x^{-3}$$

Convert back to the original form of the function.

Answer:

$$g''(x) = \frac{10}{3x^7} + \frac{40}{3x^6} - \frac{2}{x^3}$$

Higher Derivatives Rule … Set 2

7. Find $h''(x)$ if

$$h(x) = \frac{5}{9x + 4} + \frac{4}{3x + 2}$$

Higher Derivatives Rule … Set 2

Answers

7. Find $h''(x)$ if

$$h(x) = \frac{5}{9x+4} + \frac{4}{3x+2}$$

First, change the function so you can differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$ and the chain rule.

$$h(x) = 5(9x+4)^{-1} + 4(3x+2)^{-1}$$

$$h'(x) = -5(9x+4)^{-1-1} \cdot (9x+4)' - 4(3x+2)^{-1-1} \cdot (3x+2)'$$

$$h'(x) = -5(9x+4)^{-2} \cdot 9 - 4(3x+2)^{-2} \cdot 3$$

Simplify:

$$h'(x) = -45(9x+4)^{-2} - 12(3x+2)^{-2}$$

$$h''(x) = (-45)(-2)(9x+4)^{-2-1} \cdot (9x+4)' - (12)(-2)(3x+2)^{-2-1} \cdot (3x+2)'$$

$$h''(x) = (-45)(-2)(9x+4)^{-3} \cdot 9 - (12)(-2)(3x+2)^{-3} \cdot 3$$

Simplify:

$$h''(x) = 810(9x+4)^{-3} + 72(3x+2)^{-3}$$

Convert to the original form of $h(x)$:

Answer:

$$h''(x) = \frac{810}{(9x+4)^3} + \frac{72}{(3x+2)^3}$$

Higher Derivatives Rule … Set 2

8. Find $f''(x)$ if

$$f(x) = \frac{2}{2x+3} - \frac{4}{2x-7}$$

Higher Derivatives Rule … Set 2

Answers

8. Find $f''(x)$ if

$$f(x) = \frac{2}{2x+3} - \frac{4}{2x-7}$$

First, change the function so you can differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$ and the chain rule.

$$f(x) = \frac{2}{2x+3} - \frac{4}{2x-7} = 2(2x+3)^{-1} - 4(2x-7)^{-1}$$

$$f'(x) = 2(-1)(2x+3)^{-1-1} \cdot (2x+3)' - 4(-1)(2x-7)^{-1-1} \cdot (2x-7)'$$

$$f'(x) = -2(2x+3)^{-2} \cdot 2 + 4(2x-7)^{-2} \cdot 2$$

Simplify: $f'(x) = -4(2x+3)^{-2} + 8(2x-7)^{-2}$

$$f''(x) = (-4)(-2)(2x+3)^{-2-1} \cdot (2x+3)' + (8)(-2)(2x-7)^{-2-1} \cdot (2x-7)'$$

$$f''(x) = (-4)(-2)(2x+3)^{-3} \cdot 2 + (8)(-2)(2x-7)^{-3} \cdot 2$$

Simplify: $f''(x) = 16(2x+3)^{-3} - 32(2x-7)^{-3}$

Convert to the form of the original function.

Answer:

$$f''(x) = \frac{16}{(2x+3)^3} - \frac{32}{(2x-7)^3}$$

Higher Derivatives Rule … Set 2

9. Suppose $h(x) = x^{\frac{5}{2}} - x^{\frac{3}{2}}$. What is $h''(x)$?

10. Suppose $h(x) = 6x^{\frac{1}{2}} - 12x^{\frac{3}{2}}$. What is $h''(x)$?

Higher Derivatives Rule … Set 2

Answers

9. Suppose $h(x) = x^{\frac{5}{2}} - x^{\frac{3}{2}}$. What is $h''(x)$?

Terms with rational exponents are differentiated the same way as terms with integer exponents. Differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$h'(x) = \frac{5}{2}x^{\frac{5}{2}-1} - \frac{3}{2}x^{\frac{3}{2}-1} = \frac{5}{2}x^{\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$h''(x) = \left(\frac{5}{2}\right)\left(\frac{3}{2}\right)x^{\frac{3}{2}-1} - \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}$$

$$h''(x) = \frac{15}{4}x^{\frac{1}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$$

Answer:

$$h''(x) = \frac{15}{4}x^{\frac{1}{2}} - \frac{3}{4}x^{-\frac{1}{2}}$$

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10. Suppose $h(x) = 6x^{\frac{1}{2}} - 12x^{\frac{3}{2}}$. What is $h''(x)$?

Terms with rational exponents are differentiated the same way as terms with integer exponents. Differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$h'(x) = (6)\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} - (12)\left(\frac{3}{2}\right)x^{\frac{3}{2}-1}$$

$$h'(x) = 3x^{-\frac{1}{2}} - 18x^{\frac{1}{2}}$$

$$h''(x) = (3)\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} - (18)\left(\frac{1}{2}\right)x^{\frac{1}{2}-1}$$

Simplify:

Answer:

$$h''(x) = -\frac{3}{2}x^{-\frac{3}{2}} - 9x^{-\frac{1}{2}}$$

Higher Derivatives Rule … Set 2

11. Given that $f(x) = \sqrt[3]{x^2 + x}$, what is $f''(x)$?

Higher Derivatives Rule … Set 2

Answers

11. Given that $f(x) = \sqrt[3]{x^2 + x}$, what is $f''(x)$?

First, change the function so you can differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$ and the chain rule, which states if $f(x) = u(v(x))$, then $f'(x) = u'\big(v(x)\big) \cdot v'(x)$.

$$f(x) = \sqrt[3]{x^2 + x} = (x^2 + x)^{\frac{1}{3}}$$

$$f'(x) = \frac{1}{3}(x^2 + x)^{\frac{1}{3}-1} \cdot (x^2 + x)'$$

$$f'(x) = \frac{1}{3}(x^2 + x)^{-\frac{2}{3}} \cdot (2x + 1)$$

Since $f'(x)$ is the product of two functions, differentiate using the product rule: $\frac{d}{df}(uv) = udv + vdu.$

$$f''(x) = udv + vdu$$

$$= \frac{1}{3}(x^2 + x)^{-\frac{2}{3}} \cdot (2x + 1)' + \left[\frac{1}{3}(x^2 + x)^{-\frac{2}{3}}\right]' \cdot (2x + 1)$$

$$= \frac{1}{3}(x^2 + x)^{-\frac{2}{3}} \cdot (2) + \left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)(x^2 + x)^{-\frac{5}{3}} \cdot (x^2 + x)' \cdot (2x + 1)$$

$$= \frac{2}{3}(x^2 + x)^{-\frac{2}{3}} - \frac{2}{9}(x^2 + x)^{-\frac{5}{3}} \cdot (2x + 1) \cdot (2x + 1)$$

Simplify:

$$f''(x) = \frac{2}{3}(x^2 + x)^{-\frac{2}{3}} - \frac{2}{9}(2x + 1)^2(x^2 + x)^{-\frac{5}{3}}$$

Convert to radical form:

$$f''(x) = \frac{2}{3(x^2 + x)^{\frac{2}{3}}} - \frac{2(2x + 1)^2}{9(x^2 + x)^{\frac{5}{3}}}$$

$$f''(x) = \frac{2}{3\sqrt[3]{(x^2 + x)^2}} - \frac{2(2x + 1)^2}{9\sqrt[3]{(x^2 + x)^5}}$$

Answer:

$$f''(x) = \frac{2}{3\sqrt[3]{(x^2 + x)^2}} - \frac{2(2x + 1)^2}{9\sqrt[3]{(x^2 + x)^5}}$$

Higher Derivatives Rule … Set 2

12. Suppose $h(x) = \sqrt{x^2 - 1}$. What is $h''(x)$?

Higher Derivatives Rule … Set 2

Answers

12. Suppose $h(x) = \sqrt{x^2 - 1}$. What is $h''(x)$?

First, change the function so you can differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$ and the chain rule, which states if $h(x) = u(v(x))$ then $h'(x) = u'\big(v(x)\big) \cdot v'(x)$.

$$h(x) = \sqrt{x^2 - 1} = (x^2 - 1)^{\frac{1}{2}}$$

$$h'(x) = \left(\frac{1}{2}\right)(x^2 - 1)^{\frac{1}{2}-1} \cdot (x^2 - 1)'$$

$$h'(x) = \frac{1}{2}(x^2 - 1)^{-\frac{1}{2}} \cdot (2x)$$

Simplify:

$$h'(x) = \frac{1}{2}(2x)(x^2 - 1)^{-\frac{1}{2}} = x(x^2 - 1)^{-\frac{1}{2}}$$

Now use the product rule, $\frac{d}{df}(uv) = udv + vdu$, and the chain rule to find the second derivative:

$$h'(x) = x(x^2 - 1)^{-\frac{1}{2}}$$

$$h''(x) = x \cdot \left[(x^2 - 1)^{-\frac{1}{2}}\right]' + (x^2 - 1)^{-\frac{1}{2}} \cdot [x]'$$

$$h''(x) = x \cdot \left[-\frac{1}{2}(x^2 - 1)^{-\frac{1}{2}-1} \cdot (x^2 - 1)'\right] + (x^2 - 1)^{-\frac{1}{2}} \cdot 1$$

$$h''(x) = x \cdot \left[-\frac{1}{2}(x^2 - 1)^{-\frac{3}{2}} \cdot (2x)\right] + (x^2 - 1)^{-\frac{1}{2}}$$

Simplify:

$$h''(x) = -x^2(x^2 - 1)^{-\frac{3}{2}} + (x^2 - 1)^{-\frac{1}{2}}$$

$$h''(x) = -\frac{x^2}{(x^2 - 1)^{\frac{3}{2}}} + \frac{1}{(x^2 - 1)^{\frac{1}{2}}}$$

Convert to radical form:

Answer:

$$h''(x) = -\frac{x^2}{(x^2 - 1)\sqrt{x^2 - 1}} + \frac{1}{\sqrt{x^2 - 1}}$$

Higher Derivatives Rule … Set 2

13. Find $g^{(4)}(x)$ if $g(x) = (2x^5 + 1)(x^3 + 1)$.

14. Find $f''(x)$ if $f(x) = (5x^3 - x^2)(3x^4 + 7)$.

Higher Derivatives Rule … Set 2

Answers

13. Find $g^{(4)}(x)$ if $g(x) = (2x^5 + 1)(x^3 + 1)$.

Notice that $g(x)$ is the product of two binomials. You could differentiate the function using the product rule, which is possible, but tedious. Multiplying the binomials and differentiating the result term by term is a more direct approach. Then differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$g(x) = (2x^5 + 1)(x^3 + 1) = 2x^8 + 2x^5 + x^3 + 1$$

$$g'(x) = 16x^7 + 10x^4 + 3x^2$$

$$g''(x) = 112x^6 + 40x^3 + 6x$$

$$g'''(x) = 672x^5 + 120x^2 + 6$$

$$g^{(4)}(x) = 3360x^4 + 240x$$

Answer: $g^{(4)}(x) = 3360x^4 + 240x$

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14. Find $f''(x)$ if $f(x) = (5x^3 - x^2)(3x^4 - 7)$.

Notice that $f(x)$ is the product of two binomials. You could differentiate the function using the product rule, which is possible, but tedious. Multiplying the binomials and differentiating the result term by term is a more direct approach.

Then differentiate term by term using the power rule: $\frac{d}{dx}(ax^n) = (an)x^{n-1}$.

$$f(x) = (5x^3 - x^2)(3x^4 - 7) = 15x^7 - 3x^6 - 35x^3 + 7x^2$$

$$f'(x) = 105x^6 - 18x^5 - 105x^2 + 14x$$

$$f''(x) = 630x^5 - 90x^4 - 210x + 14$$

Answer: $f''(x) = 630x^5 - 90x^4 - 210x + 14$

Higher Derivatives Rule … Set 2

15. Suppose $h(x) = (5x^4)(\cos\ x)$. Find $h''(x)$.

Higher Derivatives Rule … Set 2

Answers

15. Suppose $h(x) = (5x^4)(\cos\ x)$. Find $h''(x)$.

The given function $h(x)$ contains the product of two functions. Differentiate using the product rule, $\frac{d}{df}(uv) = udv + vdu$.

$$h'(x) = (5x^4) \cdot (\cos x)' + (5x^4)' \cdot (\cos\ x)$$

$$h'(x) = (5x^4) \cdot -\sin x + (20x^3) \cdot (\cos\ x)$$

$$h'(x) = -5x^4 \sin x + 20x^3 \cos\ x$$

Now, $h'(x)$ is two products of two different functions. Differentiate again, using the product rule.

$$h''(x) = -5x^4 \cdot (\sin x)' + (-5x^4)' \cdot \sin x + 20x^3(\cos x)' + (20x^3)' \cdot \cos x$$

$$h''(x) = -5x^4 \cdot \cos x - 20x^3 \cdot \sin x + 20x^3 \cdot -\sin x + 60x^2 \cdot \cos x$$

$$h''(x) = -5x^4 \cos x - 20x^3 \sin x - 20x^3 \sin x + 60x^2 \cos x$$

Simplify:

Answer: $h''(x) = (60x^2 - 5x^4)\cos x - 40x^3 \sin x$

Higher Derivatives Rule … Set 2

16. Find $g''(x)$ if $g(x) = (\tan x)(6x^2)$.

Higher Derivatives Rule … Set 2

Answers

16. Find $g''(x)$ if $g(x) = (\tan x)(6x^2)$.

The given function $g(x)$ contains the product of two functions. Differentiate such a function using the product rule: $\frac{d}{df}(uv) = udv + vdu$.

$$g'(x) = (\tan x) \cdot (6x^2)' + (\tan x)' \cdot (6x^2)$$

$$g'(x) = (\tan x) \cdot (12x) + (\sec^2 x) \cdot (6x^2)$$

$$g'(x) = 12x(\tan x) + 6x^2(\sec^2 x)$$

Now, $g'(x)$ is two products of two different functions. Differentiate again, using the product rule.

$$g''(x) = 12x \cdot (\tan x)' + (12x)' \cdot (\tan x) + 6x^2 \cdot (\sec^2 x)' + (6x^2)' \cdot (\sec^2 x)$$

$$g''(x) = 12x\sec^2 x + 12\tan x + 6x^2 \cdot 2\sec x\sec x\tan x + 12x\sec^2 x$$

Simplify:

$$g''(x) = 24x\sec^2 x + 12\tan x + 12x^2\sec^2 x\tan x$$

Answer: $g''(x) = 24x\sec^2 x + 12\tan x + 12x^2\sec^2 x\tan x$

Higher Derivatives Rule … Set 2

17. Given that $k(x) = (3x^2)(\cos(3x))$. What is $k''(x)$?

Higher Derivatives Rule … Set 2

Answers

17. Given that $k(x) = (3x^2)(\cos(3x))$. What is $k''(x)$?

Note that $k(x)$ is the product of two functions, so differentiate using the product rule: $\frac{d}{dk}(uv) = udv + vdu$ and the chain rule.

$$k'(x) = 3x^2 \cdot (\cos(3x))' + (3x^2)' \cdot (\cos(3x))$$

$$k'(x) = 3x^2 \cdot -\sin(3x)(3) + 6x(\cos(3x))$$

Simplify:

$$k'(x) = -9x^2 \sin(3x) + 6x\cos(3x)$$

Now, $k'(x)$ is the sum of two products. Differentiate using the product rule.

$$k''^{(x)} = -9x^2 \cdot [\sin(3x)]' + (-9x^2)' \cdot \sin(3x) + 6x[\cos(3x)]' + (6x)' \cdot \cos(3x)$$

$$k''(x) = -9x^2 \cdot \cos(3x)(3) - 18x \cdot \sin(3x) + 6x \cdot -\sin(3x)(3) + 6 \cdot \cos(3x)$$

Simplify:

$$k''(x) = -27x^2 \cos(3x) - 18x \sin(3x) - 18x \sin(3x) + 6\cos(3x)$$

$$k''(x) = (-27x^2 + 6)\cos(3x) - (18x + 18x)\sin(3x)$$

Answer: $k''(x) = (-27x^2 + 6)\cos(3x) - 36x\sin(3x)$

Higher Derivatives Rule … Set 2

18. Find $k''(x)$ given that

$$k(x) = \frac{2x+5}{7x+6}$$

Higher Derivatives Rule … Set 2

Answers

18. Find $k''(x)$ given that

$$k(x) = \frac{2x+5}{7x+6}$$

Also use the quotient rule which states if $k(x) = \frac{f(x)}{g(x)}$ then,

$$k'(x) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

$$k'(x) = \frac{(7x+6)\cdot(2x+5)' - (2x+5)\cdot(7x+6)'}{(7x+6)^2}$$

$$k'(x) = \frac{(7x+6)\cdot 2 - (2x+5)\cdot 7}{(7x+6)^2} = \frac{14x+12-14x-35}{(7x+6)^2}$$

$$h'(x) = \frac{-23}{(7x+6)^2}$$

To find $k''(x)$, change $k'(x)$ so you can differentiate using the power rule and the chain rule.

$$k'(x) = -23(7x+6)^{-2}$$

$$k''(x) = (-23)(-2)(7x+6)^{-2-1}\cdot(7x+6)'$$

$$k''(x) = 46(7x+6)^{-3}\cdot 7$$

$$k''(x) = 322(7x+6)^{-3} = \frac{322}{(7x+6)^3}$$

Answer:

$$k''(x) = \frac{322}{(7x+6)^3}$$

Higher Derivatives Rule … Set 2

19. Find $g''(x)$ given that

$$g(x) = \frac{3x^2}{\sin x}$$

Higher Derivatives Rule … Set 2

Answers

19. Find $g''(x)$ given that

$$g(x) = \frac{3x^2}{\sin x}$$

Use the quotient rule which states if $g(x) = \frac{f(x)}{k(x)}$ then,

$$g'(x) = \frac{k(x) \cdot f'(x) - f(x) \cdot k'(x)}{[k(x)]^2}$$

$$g'(x) = \frac{\sin x \cdot [3x^2]' - (3x^2) \cdot (\sin x)'}{(\sin x)^2}$$

$$g'(x) = \frac{\sin x \cdot (6x) - 3x^2 \cdot \cos x}{\sin^2 x}$$

Simplify:

$$g'(x) = \frac{6x\sin x - 3x^2\cos x}{\sin^2 x}$$

Differentiate using the product rule and the quotient rule.

$$g''(x) = \frac{\sin^2 x \cdot [6x\sin x - 3x^2\cos x]' - (6x\sin x - 3x^2\cos x) \cdot [\sin^2 x]'}{(\sin^2 x)^2}$$

$$\frac{d}{dx}[6x\sin x - 3x^2\cos x]$$
$$= 6x \cdot [\sin x]' + (6x)'\sin x - 3x^2 \cdot [\cos x]' - (3x^2)' \cdot \cos x$$

$$\frac{d}{dx}[6x\sin x - 3x^2\cos x] = 6x \cdot \cos x + 6\sin x - 3x^2 \cdot -\sin x - 6x \cdot \cos x$$

$$\frac{d}{dx}[6x\sin x - 3x^2\cos x] = 6x\cos x + 6\sin x + 3x^2\sin x - 6x\cos x$$

$$\frac{d}{dx}[6x\sin x - 3x^2\cos x] = 6\sin x + 3x^2\sin x$$

Higher Derivatives Rule … Set 2

$$g''(x) = \frac{\sin^2 x \cdot (6\sin x + 3x^2 \sin x) - (6x\sin x - 3x^2 \cos x) \cdot 2\sin x \cos x}{(\sin^2 x)^2}$$

$$g''(x) = \frac{\sin^2 x(6\sin x + 3x^2 \sin x) - 2(6x\sin x - 3x^2 \cos x)\sin x \cos x}{(\sin^2 x)^2}$$

$$g''(x) = \frac{6\sin^3 x + 3x^2 \sin^3 x - 12x\sin^2 x\cos x + 6x^2 \cos^2 x\sin x}{(\sin^2 x)^2}$$

$$g''(x) = \frac{6\sin^2 x + 3x^2 \sin^2 x - 12x\sin x\cos x + 6x^2 \cos^2 x}{\sin^3 x}$$

Answer:

$$g''(x) = \frac{6\sin^2 x + 3x^2 \sin^2 x - 12x\sin x\cos x + 6x^2 \cos^2 x}{\sin^3 x}$$

Higher Derivatives Rule … Set 2

20. Find $h''(x)$ given that

$$h(x) = \frac{5x-3}{4x-9}$$

Higher Derivatives Rule … Set 2

Answers

20. Find $h''(x)$ given that

$$h(x) = \frac{5x-3}{4x-9}$$

Use the quotient rule which states if $h(x) = \frac{f(x)}{g(x)}$ then,

$$h'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$h'(x) = \frac{(4x-9) \cdot (5x-3)' - (5x-3) \cdot (4x-9)'}{(4x-9)^2}$$

$$h'(x) = \frac{(4x-9) \cdot 5 - (5x-3) \cdot 4}{(4x-9)^2}$$

$$h'(x) = \frac{5(4x-9) - 4(5x-3)}{(4x-9)^2} = \frac{20x - 45 - 20x + 12}{(4x-9)^2}$$

$$h'(x) = \frac{-33}{(4x-9)^2}$$

To find $h''(x)$ change $h'(x)$ so you can differentiate using the power rule and the chain rule.

$$h'(x) = \frac{-33}{(4x-9)^2} = -33(4x-9)^{-2}$$

$$h''(x) = (-33)(-2)(4x-9)^{-2-1} \cdot (4x-9)'$$

$$h''(x) = 66(4x-9)^{-3} \cdot 4$$

Simplify:

$$h''(x) = 264(4x-9)^{-3}$$

Convert $h''(x)$ into the form of the original function.

Answer:

$$h''(x) = \frac{264}{(4x-9)^3}$$

Higher Derivatives Rule … Set 2

21. Find $k''(x)$ given that

$$k(x) = \frac{8x^3}{5x + 2}$$

Higher Derivatives Rule … Set 2

Answers

21. Find $k''(x)$ given that

$$k(x) = \frac{8x^3}{5x+2}$$

You have the option to the quotient rule or, to avoid the quotient rule, you can rewrite the function as a product and use the product rule: $\frac{d}{dk}(uv) = udv + vdu$. We will use the product rule.

$$k(x) = \frac{8x^3}{5x+2} = (8x^3)(5x+2)^{-1}$$

$$k'(x) = (8x^3) \cdot [(5x+2)^{-1}]' + (8x^3)' \cdot (5x+2)^{-1}$$

$$k'(x) = (8x^3) \cdot (-1)(5x+2)^{-1-1}(5) + 24x^2 \cdot (5x+2)^{-1}$$

Simplify:

$$k'(x) = -40x^3(5x+2)^{-2} + 24x^2(5x+2)^{-1}$$

Find the derivatives of the two products separately.

First find $\frac{d}{dx}[(-40x^3)(5x+2)^{-2}]$

$$\frac{d}{dx}[(-40x^3)(5x+2)^{-2}] = (-40x^3) \cdot [(5x+2)^{-2}]' + (-40x^3)' \cdot (5x+2)^{-2}$$

$$\frac{d}{dx}[(-40x^3)(5x+2)^{-2}]$$
$$= (-40x^3) \cdot (-2)(5x+2)^{-2-1}(5) - 120x^2(5x+2)^{-2}$$

Simplify:

$$\frac{d}{dx}[(-40x^3)(5x+2)^{-2}] = 400x^3(5x+2)^{-3} - 120x^2(5x+2)^{-2}$$

Next find $\frac{d}{dx}[(24x^2)(5x+2)^{-1}]$

Higher Derivatives Rule … Set 2

$$\frac{d}{dx}[(24x^2)(5x+2)^{-1}] = (24x^2)\cdot[(5x+2)^{-1}]' + (24x^2)'\cdot(5x+2)^{-1}$$

$$\frac{d}{dx}[(24x^2)(5x+2)^{-1}] = (24x^2)(-1)(5x+2)^{-1-1}(5) + (48x)(5x+2)^{-1}$$

Simplify:

$$\frac{d}{dx}[(24x^2)(5x+2)^{-1}] = -120x^2(5x+2)^{-2} + 48x(5x+2)^{-1}$$

Put the two derivatives together:

$k''(x) = 400x^3(5x+2)^{-3} - 120x^2(5x+2)^{-2} - 120x^2(5x+2)^{-2} + 48x(5x+2)^{-1}$

Simplify.

$$k''(x) = 400x^3(5x+2)^{-3} - 240x^2(5x+2)^{-2} + 48x(5x+2)^{-1}$$

Convert to the original form of the function:

Answer:

$$k''(x) = \frac{400x^3}{(5x+2)^3} - \frac{240x^2}{(5x+2)^2} + \frac{48x}{5x+2}$$