Implicit Differentiation … Set 3

Implicit Differentiation

Finding a Derivative In Exercises 1–16, find dy/dx by implicit differentiation.

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$

1. $x^2 + y^2 = 9$

2. $x^2 - y^2 = 25$

4. $2x^3 + 3y^3 = 64$

5. $x^3 - xy + y^2 = 7$

6. $x^2y + y^2x = -2$

7. $x^3y^3 - y = x$

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Answers

Implicit Differentiation Worksheet

Finding a Derivative In Exercises 1–16, find dy/dx by implicit differentiation.

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'g + fg'$$

1. $x^2 + y^2 = 9$

$2x + 2y\left(\frac{dy}{dx}\right) = 0$

$2y\left(\frac{dy}{dx}\right) = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

2. $x^2 - y^2 = 25$

$2x - 2y\left(\frac{dy}{dx}\right) = 0$

$-2y\left(\frac{dy}{dx}\right) = -2x$

$\frac{dy}{dx} = \frac{-2x}{-2y}$

$\boxed{\frac{dy}{dx} = \frac{x}{y}}$

4. $2x^3 + 3y^3 = 64$

$6x^2 + 9y^2\left(\frac{dy}{dx}\right) = 0$

$9y^2\left(\frac{dy}{dx}\right) = -6x^2$

$\frac{dy}{dx} = \frac{-6x^2}{9y^2}$

$\boxed{\frac{dy}{dx} = \frac{-2x^2}{3y^2}}$

5. $x^3 - xy + y^2 = 7$ *product rule

$x^3 - \overset{f}{x}\overset{g}{y} + y^2 = 7$

$3x^2 - \left(\overset{f'}{(1)}\overset{g}{(y)} + \overset{f}{(x)}\overset{g'}{\left(\frac{dy}{dx}\right)}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$3x^2 - y - x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$-x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = y - 3x^2$

$\frac{dy}{dx}(-x + 2y) = y - 3x^2$

$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{-x + 2y}}$

6. $x^2y + y^2x = -2$

$\overset{f}{x^2}\overset{g}{y} + \overset{f}{y^2}\cdot\overset{g}{x} = -2$

$\overset{f'}{2x}\cdot\overset{g}{y} + \overset{f}{x^2}\cdot\overset{g'}{\frac{dy}{dx}} + \overset{f'}{2y}\left(\frac{dy}{dx}\right)\cdot\overset{g}{x} + \overset{f}{y^2}\cdot\overset{g'}{(1)} = 0$

$2xy + x^2\left(\frac{dy}{dx}\right) + 2xy\left(\frac{dy}{dx}\right) + y^2 = 0$

$x^2\left(\frac{dy}{dx}\right) + 2xy\left(\frac{dy}{dx}\right) = -2xy - y^2$

$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$

$\boxed{\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}}$

7. $x^3y^3 - y = x$

$\overset{f}{x^3}\overset{g}{y^3} - y = x$

$\overset{f'}{3x^2}\cdot\overset{g}{y^3} + \overset{f}{x^3}\cdot\overset{g'}{3y^2\left(\frac{dy}{dx}\right)} - 1\left(\frac{dy}{dx}\right) = 1$

$3x^2y^3 + 3x^3y^2\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 1$

$3x^3y^2\left(\frac{dy}{dx}\right) - 1\left(\frac{dy}{dx}\right) = 1 - 3x^2y^3$

$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$

$\boxed{\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}}$

Implicit Differentiation … Set 3

Finding and Evaluating a Derivative **In Exercises 21–28, find dy/dx by implicit differentiation and evaluate the derivative at the given point.**

21. $xy = 6, \quad (-6, -1)$

22. $y^3 - x^2 = 4, \quad (2, 2)$

24. $x^{2/3} + y^{2/3} = 5, \quad (8, 1)$

25)

$(x^2 + 4)y = 8$

Point: (2, 1)

Find Equation of tangent line:

51. $\sqrt{x} + \sqrt{y} = 5, \quad (9, 4)$

Implicit Differentiation … Set 3

Answers

Finding and Evaluating a Derivative **In Exercises 21–28, find dy/dx by implicit differentiation and evaluate the derivative at the given point.**

21. $xy = 6, \quad (-6, -1)$

$xy = 6$

$(1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$

$x\left(\frac{dy}{dx}\right) = -y$

$\frac{dy}{dx} = \frac{-y}{x}$

$\left.\frac{dy}{dx}\right|_{(-6,-1)} = \frac{-(-1)}{(-6)}$

$\left.\frac{dy}{dx}\right|_{(-6,-1)} = \frac{-1}{6}$

22. $y^3 - x^2 = 4, \quad (2, 2)$

$3y^2\left(\frac{dy}{dx}\right) - 2x = 0$

$3y^2\left(\frac{dy}{dx}\right) = 2x$

$\frac{dy}{dx} = \frac{2x}{3y^2}$

$\left.\frac{dy}{dx}\right|_{(2,2)} = \frac{2(2)}{3(2)^2} = \frac{1}{3}$

24. $x^{2/3} + y^{2/3} = 5, \quad (8, 1)$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\left(\frac{dy}{dx}\right) = 0$

$\frac{2}{3x^{1/3}} + \frac{2}{3y^{1/3}}\left(\frac{dy}{dx}\right) = 0$

$\frac{2}{3y^{1/3}}\left(\frac{dy}{dx}\right) = \frac{-2}{3x^{1/3}}$

$\frac{dy}{dx} = \frac{-2}{3x^{1/3}} \cdot \frac{3y^{1/3}}{2}$

$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}}$

$\left.\frac{dy}{dx}\right|_{(8,1)} = \frac{-(1)^{1/3}}{(8)^{1/3}}$

$\left.\frac{dy}{dx}\right|_{(8,1)} = \frac{-1}{2}$

25)

$(x^2 + 4)y = 8$

Point: (2, 1)

$(x^2+4)(y) = 8$

$(2x)(y) + (x^2+4)\left(\frac{dy}{dx}\right) = 0$

$2xy + (x^2+4)\left(\frac{dy}{dx}\right) = 0$

$(x^2+4)\left(\frac{dy}{dx}\right) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2+4}$

$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2xy}{x^2+4}$

$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2(2)(1)}{2^2+4}$

$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-4}{8}$

$= \frac{-1}{2}$

Find Equation of tangent line:

51. $\sqrt{x} + \sqrt{y} = 5, \quad (9, 4)$

$x^{1/2} + y^{1/2} = 5$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\left(\frac{dy}{dx}\right) = 0$

$\frac{1}{2x^{1/2}} + \frac{1}{2y^{1/2}}\left(\frac{dy}{dx}\right) = 0$

$\frac{1}{2y^{1/2}}\left(\frac{dy}{dx}\right) = \frac{-1}{2x^{1/2}}$

$\frac{dy}{dx} = \frac{-1}{2x^{1/2}} \cdot \frac{2y^{1/2}}{1}$

$\frac{dy}{dx} = \frac{-2y^{1/2}}{2x^{1/2}}$

$\frac{dy}{dx} = \frac{-y^{1/2}}{x^{1/2}}$

$\left.\frac{dy}{dx}\right|_{(9,4)} = \frac{-(4)^{1/2}}{(9)^{1/2}} = \frac{-2}{3}$

point: (9,4)

slope: $m = \frac{-2}{3}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{-2}{3}(x - 9)$