Tangent and Normal Lines with Implicit Differentiation … Set 2

Find $\frac{dy}{dx}$ if $y^2 = x$

Tangent and Normal Lines with Implicit Differentiation … Set 2

Answers

-To solve, differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

$y^2 = x$

$2y\frac{dy}{dx} = 1$ $\qquad$ $\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2) \bullet \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2y}$

-This is helpful in this case.

$y^2 = x$

$y = \sqrt{x}$

-This graph has 2 slopes for $x = 4$ one at $(4,2)$ and $(4,-2)$.

-Here our derivative is in terms of y so we can use y_1 and y_2 to find BOTH derivatives.

$\frac{1}{2y_1} = \frac{1}{4}$ $\qquad$ $\frac{1}{2y_2} = -\frac{1}{4}$

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Example-Finding Slope of a Circle

Find the slope of the circle $x^2 + y^2 = 25$ at $(3, -4)$

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Answers

-To graph this we would need to solve for y:

$$y_1 = \sqrt{25 - x^2} \qquad y_2 = -\sqrt{25 - x^2}$$

-Since $(3, -4)$ is on y_2 we could calculate explicitly

$$\frac{dy_2}{dx} = -\left(25 - x^2\right)^{1/2} = \frac{-1}{\sqrt{25 - x^2}}\left(2x\right)$$

$$= \left.\frac{-2x}{\sqrt{25 - x^2}}\right|_{x=3} = \frac{-6}{2\sqrt{25 - 9}} = \frac{3}{4}$$

-This can be done even easier using implicit differentiation and will
the entire circle.

$$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}\left(25\right)$$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$

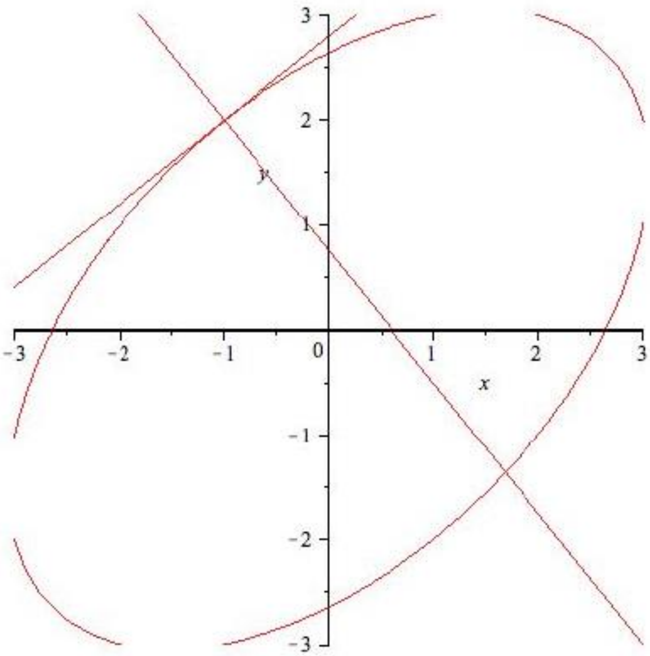
$$\frac{dy}{dx} = \frac{-x}{y}$$

At $(3, -4)$

$$= \frac{-3}{-4} = \frac{3}{4}$$

Tangent and Normal Lines with Implicit Differentiation … Set 2

-Find the tangent and normal to the ellipse $x^2 - xy + y^2 = 7$ at $\left(-1, 2\right)$.



Tangent and Normal Lines with Implicit Differentiation … Set 2

Answers

$$\frac{d}{dx}\left(x^2\right)-\frac{d}{dx}\left(xy\right)+\frac{d}{dx}\left(y^2\right)=\frac{d}{dx}\left(7\right)$$

$$2x-\left(x\frac{dy}{dx}+y\left(1\right)\right)+2y\frac{dy}{dx}=0$$ 'Treat xy as a product

$$\left(2y-x\right)\frac{dy}{dx}=y-2x$$

$$\frac{dy}{dx}=\frac{y-2x}{2y-x}$$

Evaluate at $x=-1, y=2$

$$\frac{dy}{dx}=\frac{2-2\left(-1\right)}{2\left(2\right)-\left(-1\right)}=\frac{4}{5}$$

Tangent at $x=-1, y=2$

$$y-2=\frac{4}{5}\left(x+1\right)$$

$$y-2=\frac{4}{5}x+\frac{4}{5}$$

$$y=\frac{4}{5}x+\frac{14}{5}$$

Normal at $x=-1, y=2$

$$y-2=-\frac{5}{4}\left(x+1\right)$$

$$y-2=-\frac{5}{4}x-\frac{5}{4}$$

$$y=-\frac{5}{4}x+\frac{3}{4}$$

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-Show the slope is defined everywhere on $2y = x^2 + \sin y$

$$2\frac{dy}{dx} = 2x + \cos y\frac{dy}{dx}$$

Tangent and Normal Lines with Implicit Differentiation … Set 2

Answers

$$2\frac{dy}{dx} - \cos y\frac{dy}{dx} = 2x$$

$$\left(2 - \cos y\right)\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{2x}{2 - \cos y}$$

Tangent and Normal Lines with Implicit Differentiation … Set 2

Example-Second Derivative Implicitly

Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 0$.

Tangent and Normal Lines with Implicit Differentiation … Set 2

Answers

Note: $y' = \dfrac{dy}{dx}$

$$6x^2 - 6yy' = 0$$

$$x^2 - yy' = 0$$

$$y' = \frac{x^2}{y}$$

To find y''.

$$y'' = \frac{2xy - x^2y'}{y^2} = \frac{2x}{y} - \frac{x^2y'}{y^2}$$

Don't forget we already solved $y' = \dfrac{x^2}{y}$

$$y'' = \frac{2x}{y} - \frac{x^2}{y^2}\left(\frac{x^2}{y}\right)$$

$$= \frac{2x}{y} - \frac{x^4}{y^3}$$