Higher Order Implicit Derivatives … Set 3

IMPLICIT DIFFERENTIATION

Implicit differentiation is nothing more than a special case of the chain rule for derivatives. It is used to solve equations such as $x^3 - y^3 + 7x = 0$, which does not let us solve for y in terms of x easily.

The majority of differentiation problems in first-year calculus involve functions y written EXPLICITLY as functions of x. For example, if

$$y = 3x^2 - \sin(7x+5) \qquad \text{OR} \qquad y = 3x^2 + 4x$$

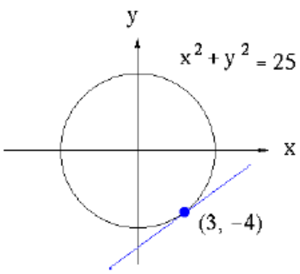
then, the derivatives are

$$y' = 6x - 7\cos(7x+5) \qquad\qquad y' = 6x + 4$$

However, some functions y are written IMPLICITLY as functions of x. An example of this is the equation:

$$x^2 + y^2 = 25$$

which represents a circle of radius five centered at the origin. Suppose that we wish to find the slope of the line tangent to the graph of this equation at the point (3, -4).



- How could we find the derivative of y in this instance?

  One way is to first write y explicitly as a function of x.

  $x^2 + y^2 = 25$ <u>Note</u>: the positive square root represents the top semi-circle and the negative square root represents the bottom semi-circle.

The derivative of y is:

Thus, the slope of the line tangent to the graph at the point (3, -4) is

Unfortunately, not every equation involving x and y can be solved explicitly for y. So let's do the same problem implicitly.

$$x^2 + y^2 = 25$$

Thus, the slope of the line tangent to the graph at the point (3, -4) is

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The second method illustrates the process of **implicit differentiation**. It is important to note that the derivative expression for explicit differentiation involves x only, while the derivative expression for implicit differentiation may involve both x and y.

Steps for Implicit Differentiation

1) Differentiate both sides of the equation with respect to x.
2) Collect the terms with $\frac{dy}{dx}$ on one side of the equation
3) Factor out $\frac{dy}{dx}$ (when there is more than one y)
4) Solve for $\frac{dy}{dx}$ by dividing.

Examples: Find $\frac{dy}{dx}$ of the each of the following using implicit differentiation.

1) $3x^3 - 4y^2 = 2x$

2) $x = \tan y$

3) $3x^2y + 2x = 4x^3$

4) $y^3 + 7y = x^3$

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5) $y^4 + 3xy = 2x^3 + 1$

6) $x^2y^3 + 3y^7 - 5x^2 = 7$

7) $y^3 - xy^2 + \cos(xy) = 2$

8) Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 7$