Motion Along a Line … Set 3

Examples – Rectilinear Motion

1. A car is driven along a straight track with position given by $s(t) = 150t - 300$ ft (t in seconds).

   (a) Find $v(t)$ and $a(t)$.

   Solution: We are given that $s(t) = 150t - 300$ ft, so $v(t) = s'(t) = 150$ ft/s, and $a(t) = v'(t) = 0$ ft/s^2.

   (b) Use calculus to find the displacement and total distance traveled over the interval [1, 4].

   Solution: The displacement on [1, 4] is simply the definite integral of velocity on [1, 4]:

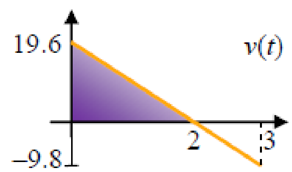
   $$\text{Displacement } = \int_1^4 150\, dt = (150t)\Big|_1^4 = 150 \cdot 4 - 150 \cdot 1 = 450 \text{ ft.}$$

   Since the velocity of the car is constant, the car is always moving in the same direction. Therefore, the total distance traveled is the same as the displacement *in this case*.

2. A projectile is fired upward from a 15.3 m cliff at a speed of 19.6 m/s and allowed to fall into a valley below. The acceleration g due to Earth's gravity is about 9.8 m/s^2, or about 32 ft/s^2, downward.

   (a) Given that $a(t) = -9.8$ m/s^2, find $v(t)$ and use it to find the time at which the projectile reaches its maximum height. Find the maximum height of the projectile using geometry.

   Solution: If $a(t) = -9.8$, then $v(t) = -9.8t + C$ for some constant C. Note that $v(0) = C$ in this problem, so that C is the initial velocity. Therefore, $v(t) = -9.8t + 19.6$ m/s. The maximum height occurs when the velocity is zero, so $-9.8t + 19.6 = 0$ implies that the maximum height occurs at $t = 2$ seconds. Although we do not have a position function, we can find the maximum height using geometry. Since the maximum height in this problem is simply the displacement over the first 2 seconds, and the displacement is the net area bounded by the velocity curve, we see that the maximum height is the area of the shaded triangle, which is (1/2)(2)(19.6) = 19.6 meters.



   (b) Use geometry to find the displacement and total distance traveled over the interval [0, 3].

   Solution: The displacement is the net area bounded by $v(t)$, and the total distance traveled is the total area. In this case, we can use the two triangles in the figure to compute displacement on [0, 3] as (1/2)(2)(19.6) – (1/2)(1)(9.8) = 14.7 meters, and the total distance traveled on [0, 3] as (1/2)(2)(19.6) + (1/2)(1)(9.8) = 24.5 meters.