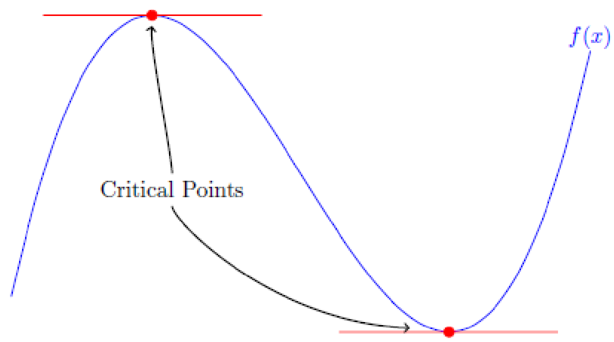
Critical Values … Rules Set 1

A **critical point** (or **stationary point**) of $f(x)$ is a point $(a, f(a))$ such that $f'(a) = 0$.

Recall that, geometrically, these are points on the graph of $f(x)$ who have a "flat" tangent line, i.e. a *constant* tangent line.



Example 1:

Find all critical points of $f(x) = x^3 - 3x^2 - 9x + 5$.

We see that the derivative is $f'(x) = 3x^2 - 6x - 9$. We need to solve $f'(x) = 0$.

$$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x+1)(x-3) = 0 \Longrightarrow x = -1, x = 3$$

Thus the critical points of $f(x)$ are $(-1, f(-1)) = (-1, 10)$ and $(3, f(3)) = (3, -22)$.

Example 2:

Find all critical points of $f(t) = e^{-3t} + 2t$.

Differentiating yields $f'(t) = -3e^{-3t} + 2$. Now we solve $f'(t) = 0$.

$$f'(t) = -3e^{-3t} + 2 = 0 \Longrightarrow 2 = 3e^{-3t} \Longrightarrow \frac{2}{3} \Longrightarrow \ln\left(\frac{2}{3}\right) = -3t \Longrightarrow \frac{1}{3}\ln\left(\frac{3}{2}\right) = t$$

Thus the only critical point of $f(t)$ is $\left(\frac{1}{3}\ln(1.5), f\left(\frac{1}{3}\ln(1.5)\right)\right) = \left(\frac{1}{3}\ln(1.5), \frac{2}{3}(1 + \ln(1.5))\right) \approx (0.135, 0.937)$.

Critical Values … Rules Set 1

You may notice, particularly from the graph on page 1, that the critical points seem to coincide with the peaks of the graph. These is *almost* true. In fact we have the following definition:

Suppose $(a, f(a))$ is a critical point of $f(x)$. Then,

$$(a, f(a)) \text{ is a } \textbf{local minimum} \iff f''(a) > 0$$
$$(a, f(a)) \text{ is a } \textbf{local maximum} \iff f''(a) < 0$$
$$(a, f(a)) \text{ is a } \textbf{point of inflection} \iff f''(a) = 0$$

We can think of this definition as a test to identify the local maximum and local minimum points of a function $f(x)$. If $f''(a) < 0$ or $f''(a) > 0$ then we have a local maximum or minimum, respectively, and if $f''(a) = 0$ then we know nothing. Different cases of $f''(a) = 0$ will be explored later.