Extrema on an Interval … Set 2

Extrema and the First Derivative Test

This chapter builds on the previous chapter, so we begin by reviewing the main ideas from Chapter 30. Given a function f,

- $f(x)$ increases where $f'(x)$ is positive.
- $f(x)$ decreases where $f'(x)$ is negative.
- A number c in the domain of f is called a **critical point** for f if either $f'(c) = 0$ or $f'(c)$ is not defined.
- If $f(x)$ stops increasing and starts decreasing (or stops decreasing and starts increasing) at $x=c$, then c is a critical point.

Our main definition formalizes the notion a function's hills and valleys.

> **Definition 31.1** Suppose c is a number in the domain of a function $f(x)$.
>
> We say f has a **local maximum** at $x=c$ if $f(x) \leq f(c)$ for all x near c.
> That is, the graph has a "hilltop" at $x=c$.
> Moving x immediately to the right or left of c will not make $f(x)$ larger.
>
> We say f has a **local minimum** at $x=c$ if $f(x) \geq f(c)$ for all x near c.
> That is, the graph has a "valley bottom" at $x=c$.
> Moving x imediately to the right or left of c will not make $f(x)$ smaller.

A local maximum or minimum is called a *local extremum*. The plurals are **local maxima**, **local minima** and **local extrema**. The function in Figure 31.1 has two local maxima, two local minima and four local extrema.

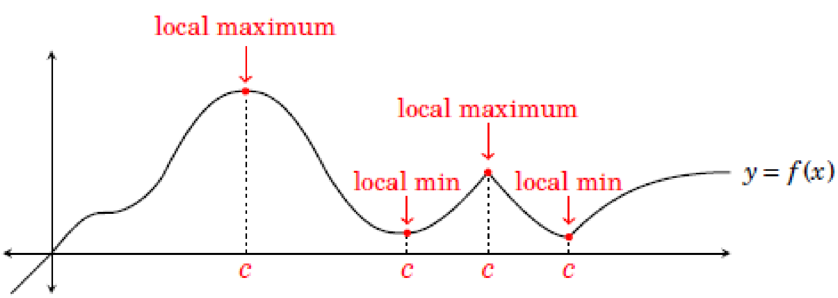

Figure 31.1. Local extrema a function.

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If f has a local maximum at c, then c gives the location of the hill, and $f(c)$ is the hill's height. Thus we may say that "*the local maximum is* $f(c)$" when we want to emphasize height over location. (The same remarks apply to local minima.) However, for now we will be concerned with where the extrema occur, not how high or low they happen to be.

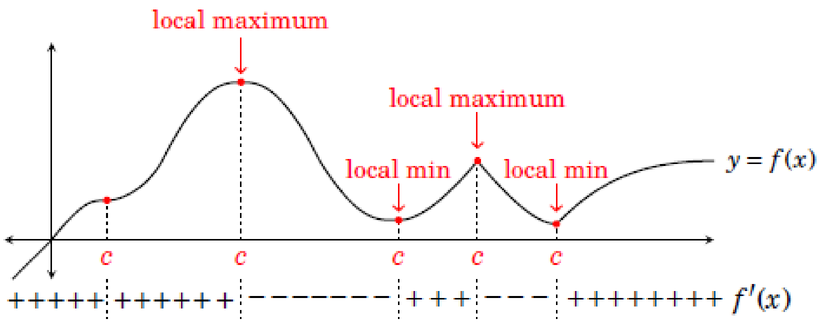
Our primary method for finding the locations of extrema is called the **first derivative test**. It is the simple observation that maxima occur where f stops increasing and starts decreasing, so $f'(x)$ changes sign from + to −. Minima occur where f stops decreasing and starts increasing, so $f'(x)$ goes from − to +. Either way, the sign change happens at a critical point.

Fact 31.1 The First Derivative Test (For finding local extrema.)

Suppose c is a critical point of $f(x)$.

1. If $f'(x)$ changes from + to − at c, then $f(x)$ has a **local maximum** at c.
2. If $f'(x)$ changes from − to + at c, then $f(x)$ has a **local minimum** at c.
3. If $f'(x)$ does not change sign at c, there is no local extremum at c.

The graph below illustrates this. Five critical points c of f are shown. There is a local maximum at c when $f'(x)$ changes sign from + to − at c (meaning f stops rising and starts falling). There is a local minimum at c if $f'(x)$ changes from − to + at c (meaning f stops falling and starts rising).



Notice that at the left-most critical point c (where $f'(c) = 0$), the sign of the derivative does not change. Here the graph increases before getting to c, levels out at c, and then continues increasing. The derivative has no sign-change at c, and this corresponds to f having no extremum at c.

We now have a procedure for finding all local extrema of a function.

How to find all local extrema of a function

1. Find all critical points of the function.
2. Apply the First Derivative Test to each critical point.

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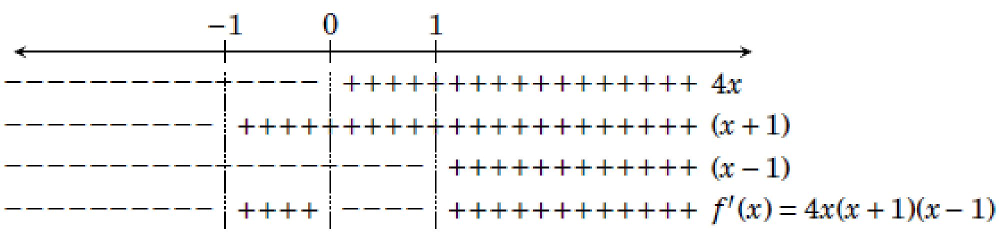
Example 31.1 Find all local extrema of the function $f(x) = x^4 - 2x^2 + 2$.
Solution The first step is to find all the critical points, the numbers in the domain of f for which the derivative is zero or undefined. The derivative $f'(x) = 4x^3 - 4x$ is defined for all x, so if there are any critical points, then they will make the derivative zero.

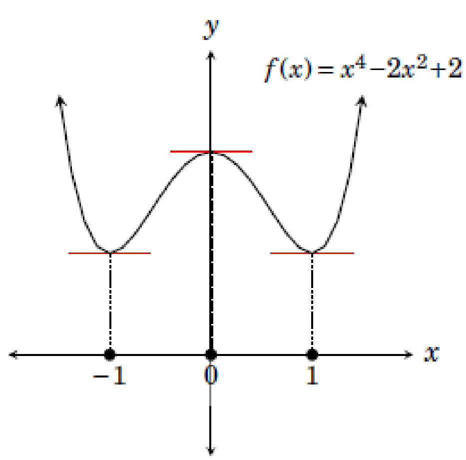
The derivative factors as $f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x+1)(x-1)$, so the solutions of $f'(x) = 0$ are -1, 0 and 1. These are the critical points of f, so if f has any local extrema, they will happen at these points.

We next apply the first derivative test to each of these critical points. This requires examining the sign change of $f'(x)$ at each critical point. We use our familiar procedure of examining the factors of $f'(x) = 4x(x+1)(x-1)$.



Because the derivative changes from $-$ to $+$ at -1 and 1, the first derivative test guarantees a local minimum for f at $x = -1$ and $x = 1$. And since the derivative changes from $+$ to $-$ at 0, there is a local maximum at $x = 0$.

Answer: $f(x) = x^4 - 2x^2 + 2$ has local minima at $x = -1$ and $x = 1$, and a local maximum at $x = 0$.



The graph of $f(x) = x^4 - 2x^2 + 2$ is sketched above.

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Example 31.2 Find all local extrema of $f(x) = \sqrt[3]{x}^2 - \dfrac{2}{3}x$ on $(-\infty, \infty)$.

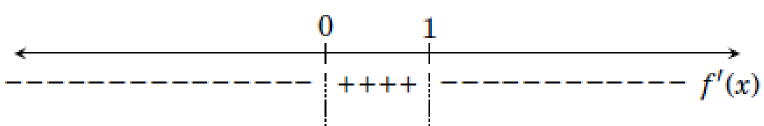
Solution The first step is to find this function's critical points, and that involves finding its derivative. Write it as $f(x) = x^{2/3} - \frac{2}{3}x$. Then

$$f'(x) = \frac{2}{3}x^{2/3-1} - \frac{2}{3} = \frac{2}{3}\left(x^{-1/3} - 1\right) = \frac{2}{3}\left(\frac{1}{\sqrt[3]{x}} - 1\right).$$

We immediately see that $x = 0$ is a critical point because $f'(0)$ is not defined (division by 0). To find any other critical points, solve the equation $f'(x) = 0$.

$$\begin{aligned} \frac{2}{3}\left(\frac{1}{\sqrt[3]{x}} - 1\right) &= 0 \\ \frac{1}{\sqrt[3]{x}} - 1 &= 0 \\ \frac{1}{\sqrt[3]{x}} &= 1 \\ 1 &= \sqrt[3]{x} \\ 1^3 &= \sqrt[3]{x}^3 \\ x &= 1 \end{aligned}$$

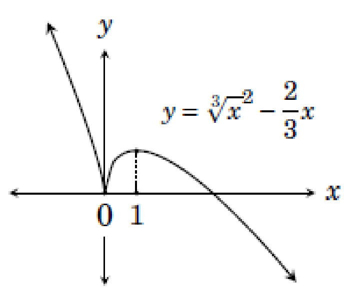
Therefore f has exactly two critical points 0 and 1, and these divide the function's domain into three intervals $(-\infty, 0)$, $(0, 1)$ and $(1, \infty)$.



To find the sign of $f'(x)$ on $(-\infty, 0)$ pick a test point, say $x = -1$, in this interval. Then $f'(-1) = \frac{2}{3}\left(\frac{1}{\sqrt[3]{-1}} - 1\right) = -\frac{4}{3}$, hence $f'(x)$ is negative on $(-\infty, 0)$.

To find the sign of $f'(x)$ on $(0, 1)$ let's use the test point $x = \frac{1}{8}$ because its a perfect cube, with cube root $\frac{1}{2}$. Then $f'\left(\frac{1}{8}\right) = \frac{2}{3}\left(\frac{1}{\sqrt[3]{1/8}} - 1\right) = \frac{2}{3}$, so $f'(x)$ is positive on $(0, 1)$. Finally, as $f'(8) = \frac{2}{3}\left(\frac{1}{\sqrt[3]{8}} - 1\right) = -\frac{1}{3}$, $f'(x)$ is negative on $(1, \infty)$.

Answer: By the first derivative test, f has a local minimum at $x = 0$ (where f' changes from $-$ to $+$) and a local maximum at $x = 1$ (where f' changes from $+$ to $-$).
The graph of $f(x) = \sqrt[3]{x}^2 - \frac{2}{3}x$ is shown on the right. ✍



Example 31.3 Find all local extrema of $y = x\sin(x) + \cos(x) - \dfrac{x^2}{4}$ on $(-\pi, \pi)$.

Solution The domain of this function is all real numbers, but we are only asked to find the extrema of it on the interval $(-\pi, \pi)$. As always, the first step is to take the derivative:

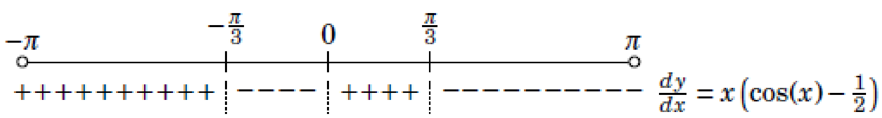
$$\frac{dy}{dx} = 1 \cdot \sin(x) + x\cos(x) - \sin(x) - \frac{x}{2} = x\left(\cos(x) - \frac{1}{2}\right).$$

This derivative is defined for all values of x, so there are no critical points that make the derivative undefined. To find any other critical points, we solve the equation $\frac{dy}{dx} = 0$:

$$x\left(\cos(x) - \frac{1}{2}\right) = 0.$$

Certainly $x = 0$ is one solution. Familiarity with the unit circle reveals the two other solutions in $(-\pi, \pi)$. Since $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ and $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$, the two values $x = \pm\frac{\pi}{3}$ make $\frac{dy}{dx}$ zero and are hence critical points.

The three critical points $-\frac{\pi}{3}$, 0 and $\frac{\pi}{3}$ divide the interval $(-\pi, \pi)$ into four sub-intervals, as diagramed below.



Let's now pick a test point in each interval. Use $-\frac{\pi}{2}$ for the first interval, and then $-\frac{\pi}{6}$ for the second. Use $\frac{\pi}{6}$ and $\frac{\pi}{2}$ for the third and fourth intervals.

Since $\frac{dy}{dx}\Big|_{-\pi/2} = -\frac{\pi}{2}\left(\cos\left(-\frac{\pi}{2}\right) - \frac{1}{2}\right) = -\frac{\pi}{2}\left(0 - \frac{1}{2}\right) = \frac{\pi}{4} > 0$, $\frac{dy}{dx}$ is positive on $\left(-\pi, -\frac{\pi}{3}\right)$.

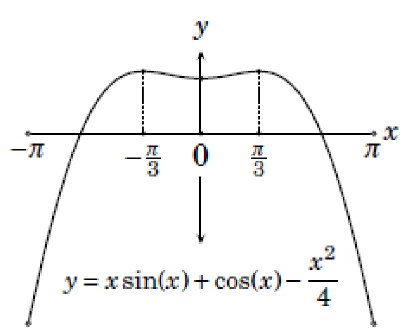
Since $\frac{dy}{dx}\Big|_{-\pi/6} = -\frac{\pi}{6}\left(\cos\left(-\frac{\pi}{6}\right) - \frac{1}{2}\right) = -\frac{\pi}{6}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) < 0$, $\frac{dy}{dx}$ is negative on $\left(-\frac{\pi}{3}, 0\right)$.

Since $\frac{dy}{dx}\Big|_{\pi/6} = \frac{\pi}{6}\left(\cos\left(\frac{\pi}{6}\right) - \frac{1}{2}\right) = \frac{\pi}{6}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) > 0$, $\frac{dy}{dx}$ is positive on $\left(0, \frac{\pi}{3}\right)$.

Since $\frac{dy}{dx}\Big|_{\pi/2} = \frac{\pi}{2}\left(\cos\left(\frac{\pi}{2}\right) - \frac{1}{2}\right) = \frac{\pi}{2}\left(0 - \frac{1}{2}\right) = -\frac{\pi}{4} < 0$, $\frac{dy}{dx}$ is negative on $\left(\frac{\pi}{3}, \pi\right)$.

Answer: By the first derivative test, $y = x\sin(x) + \cos(x) - \frac{x^2}{4}$ has two local maxima, at $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$. There is one local maximum, at $x = 0$.

The graph of $y = x\sin(x) + \cos(x) - \frac{x^2}{4}$ is shown on the right.



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Example 31.4 Find all local extrema of $y = \frac{\ln(x^2)}{x^2}$.

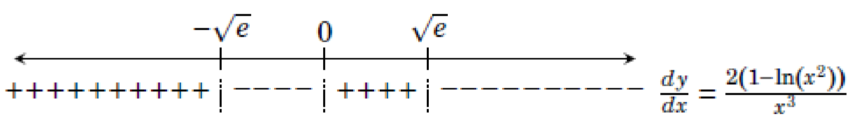
Solution: The domain of this function is the two intervals $(-\infty, 0) \cup (0, \infty)$. Any critical points will further split these intervals. The derivative is

$$\frac{dy}{dx} = \frac{\frac{2x}{x^2} \cdot x^2 - \ln(x^2) \cdot 2x}{(x^2)^2} = \frac{2x - 2x\ln(x^2)}{x^4} = \frac{2(1 - \ln(x^2))}{x^3}.$$

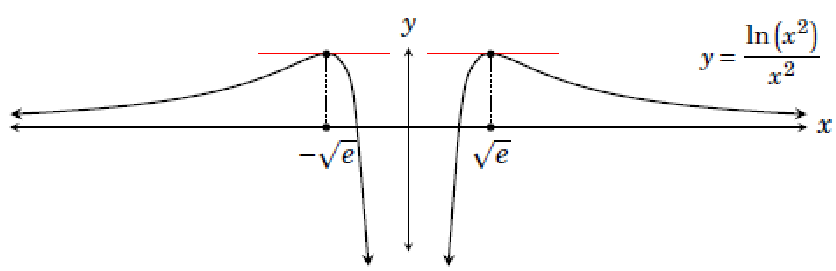
This is undefined only for $x = 0$, **but** 0 is not in the domain of $\frac{\ln(x^2)}{x^2}$, so 0 is **not a critical point**. Thus the critical points (if any) will be the x for which the derivative is zero. To find them we solve the equation $\frac{dy}{dx} = 0$.

$$\begin{aligned} \frac{2(1-\ln(x^2))}{x^3} &= 0 \\ 1 - \ln(x^2) &= 0 \\ \ln(x^2) &= 1 \\ e^{\ln(x^2)} &= e^1 \\ x^2 &= e \qquad \text{Thus } x = \pm\sqrt{e}. \end{aligned}$$

The two critical points $x \pm \sqrt{e}$ divide the intervals $(-\infty, 0)$ and $(0, \infty)$ as shown below. Picking test points, we determine the sign of $\frac{dy}{dx}$ on each interval.



Answer: By the first derivative test, $\frac{\ln(x^2)}{x^2}$ has two local maxima, at $x = -\sqrt{e}$ and $x = \sqrt{e}$, where $\frac{dy}{dx}$ changes from + to −. There is no local minimum. (See the graph below.)



Important: Since 0 is not in the functions's domain, there is no extremum at 0, even though $\frac{dy}{dx}$ changes sign there. If we mistakenly interpreted 0 as a critical point, then we would mistakenly get a local minimum at 0.

Extrema on an Interval … Set 2

Example 31.5 Find all local extrema of $f(x) = \sin(x) + x$ on $(-\infty, \infty)$.

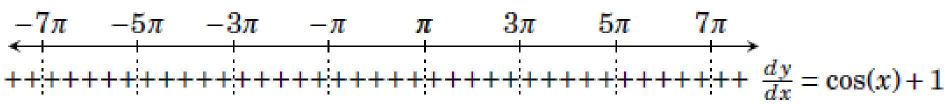
Solution The first step is to find all critical critical points. The derivative $f'(x) = \cos(x) + 1$ is defined for all real values of x, so there are no critical points c for which $f'(c)$ is not defined. Therefore all critical points of $f(x)$ will be solutions to the equation $f'(x) = 0$.

$$\begin{aligned} \cos(x) + 1 &= 0 \\ \cos(x) &= -1 \end{aligned}$$

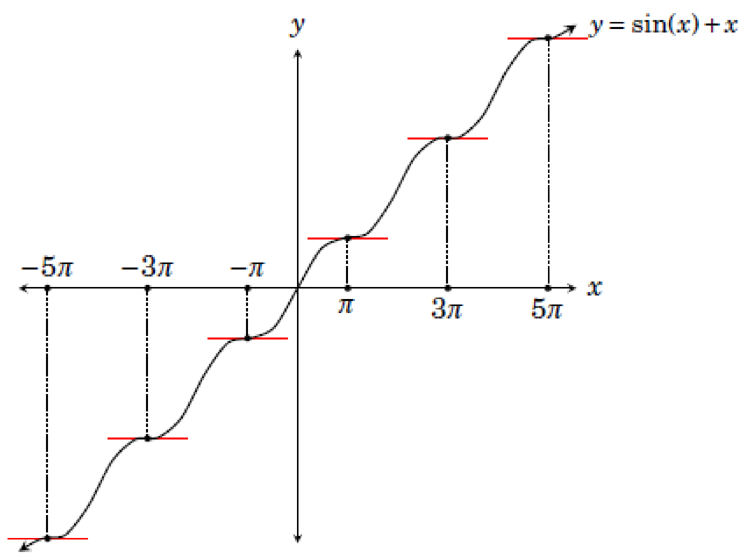
The values of x for which $\cos(x) = -1$ are the odd multiples of π (think of the unit circle), so f has infinitely many critical points, which we list as

$$\ldots -7\pi,\ -5\pi,\ -3\pi,\ -\pi,\ \pi,\ 3\pi,\ 5\pi,\ 7\pi, \ldots$$

Consider the derivative $f'(x) = \cos(x) + 1$. Because $\cos(x)$ is never less than -1, it follows that $f'(x) = \cos(x) + 1$ is never negative. (Though it equals 0 at each critical point.) Therefore $f'(x)$ never changes sign. Unless x is a critical point, $f'(x)$ is positive. This information is tallied on our usual chart, below.



Answer: The function $f(x) = \sin(x) + x$ has no local extrema at all.



The above graph of $y = \sin(x) + x$ may help make sense of our answer. Notice that the graph is continually rising, though it levels out at each critical point, where the tangent to the graph is horizontal. ✍

Extrema on an Interval … Set 2

1. Find all local extrema of $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$.

2. Find all local extrema of $f(x) = \frac{3}{2}x^4 - x^6$.

3. Find all local extrema of $g(x) = x\sqrt{8 - x^2}$.

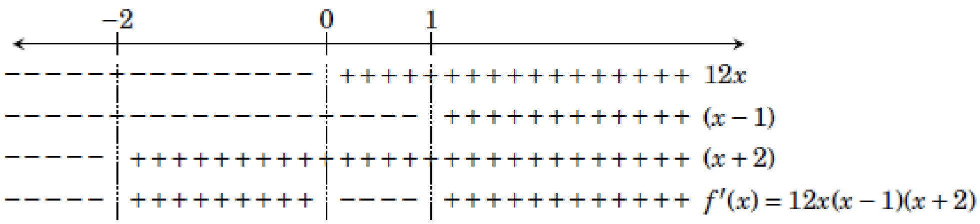
4. Find all local extrema of $f(x) = 5x^4 + 20x^3 + 10$.

Extrema on an Interval … Set 2

Answers

1. Find all local extrema of $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$.

The derivative is $f'(x) = 12x^3 + 12x^2 - 24x = 12x\left(x^2 + x - 2\right) = 12x(x-1)(x+2)$. The critical points are $x = 0$, $x = 1$ and $x = -2$.



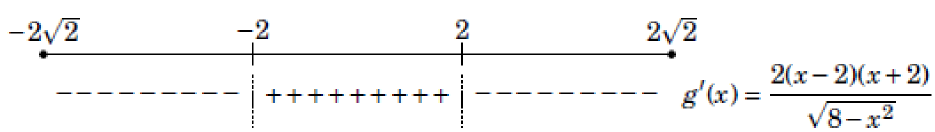
By the first derivative test, f has local minima at $x = -2$ and $x = 1$, and a local maximum at $x = 0$.

3. Find all local extrema of $g(x) = x\sqrt{8 - x^2}$.

Notice that $x - 8^2 \geq 0$ on $[-\sqrt{8}, \sqrt{8}] = [-2\sqrt{2}, 2\sqrt{2}]$, and $x - 8^2$ is negative elsewhere. Thus the domain of g is the interval $[-2\sqrt{2}, 2\sqrt{2}]$. By the product rule,

$$\begin{aligned} g'(x) &= 1 \cdot \sqrt{8 - x^2} + x \cdot \frac{-x}{\sqrt{8 - x^2}} \\ &= \sqrt{8 - x^2} - \frac{x^2}{\sqrt{8 - x^2}} \\ &= \frac{\left(8 - x^2\right) - x^2}{\sqrt{8 - x^2}} \\ &= \frac{8 - 2x^2}{\sqrt{8 - x^2}} \\ &= \frac{2(x-2)(x+2)}{\sqrt{8 - x^2}} \end{aligned}$$

This is undefined at $x = \pm\sqrt{8} = \pm 2\sqrt{2}$, but these are not critical points because the are not in the domain of g. Thus the only critical points of g are the values of x for which $g'(x) = 0$, namely $x = -2$ and $x = 2$. These divide the domain $[-2\sqrt{2}, 2\sqrt{2}]$ of g into three smaller intervals, as shown on the diagram below.



Looking at the sign of g' on these intervals, we see that the first derivative test gives a local minimum of $g(-2) = -2\sqrt{8 - (-2)^2} = -4$ at $x = -2$, and a local maximum of $g(2) = 2\sqrt{8 - 2^2} = 4$ at $x = 2$.

Extrema on an Interval … Set 2

5. Find all local extrema of $f(x) = x^2 e^{-x}$.

6. Find all local extrema of $f(x) = x^2 e^x$.

7. Find all local extrema of $f(x) = x \ln|x|$.

8. Find all local extrema of $f(x) = e^{x^3 - 12x}$.

9. Find all local extrema of $y = \tan^{-1}(x^2 + x - 2)$.

10. Find all local extrema of $y = x \tan^{-1}(x)$.

11. Find all local extrema of $y = \dfrac{1}{1 + x^2}$

12. Find all local extrema of $y = \dfrac{x}{1 + x^2}$

13. Find all local extrema of $y = \sqrt[3]{8 - x^3}$

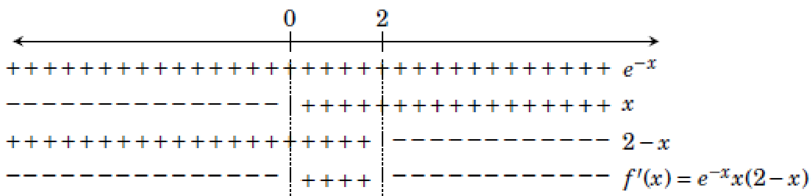
14. Find all local extrema of $z = \ln|w^2 + 10w + 24|$

Extrema on an Interval … Set 2

Answers

5. Find all local extrema of $f(x)=x^2e^{-x}$.

By the product rule, $f'(x)=2xe^{-x}+x^2(-e^{-x})=e^{-x}x(2-x)$. There are two critical points, 0 and -2. Here are the signs on the resulting intervals.



By the first derivative test. f has a local minimum of $f(0)=0$ at $x=0$, and a local maximum of $f(2)=\frac{4}{e^2}$ at $x=2$.

7. Find all local extrema of $f(x)=x\ln|x|$.

By the product rule, the derivative is $f'(x)=1\cdot\ln|x|+x\frac{1}{x}=\ln|x|+1$. This is undefined at $x=0$, but 0 is not in the domain of f (which is all real numbers *except* 0), so 0 is not a critical point. To find the critical points, we solve $f'(x)=0$, that is, $\ln|x|+1=0$, or $\ln|x|=-1$. From this, $e^{\ln|x|}=e^{-1}$, so $|x|=\frac{1}{e}$, and the solutions are $\pm\frac{1}{e}$. Therefore there are two critical points $\pm\frac{1}{e}$. These divide the domain $(-\infty,0)\cup(0,\infty)$ into four intervals as indicated in the chart below.

Interval	$\left(-\infty,-\frac{1}{e}\right)$	$\left(-\frac{1}{e},0\right)$	$\left(0,\frac{1}{e}\right)$	$\left(\frac{1}{e},\infty\right)$
Test point a	$-e$	$-\frac{1}{e^2}$	$\frac{1}{e^2}$	e
$f'(a)$	$f'(-e)=\ln\|-e\|+1=2$	$f'(-1/e^2)=-1$	$f'(1/e^2)=-1$	$f'(e)=2$
Sign of $f'(a)$	+	−	−	+
f is	increasing	decreasing	decreasing	increasing

By the first derivative test, f has a local maximum of $f(-1/e)=\frac{1}{e}$ at $x=-\frac{1}{e}$. Also there is a local minimum of $f(1/e)=-\frac{1}{e}$ at $x=\frac{1}{e}$.

9. Find all local extrema of $y=\tan^{-1}\left(x^2+x-2\right)$.

The derivative is $f'(x)=\dfrac{2x+1}{1+\left(x^2+x-2\right)^2}$. This is defined for all x and equals 0 only for $x=-\frac{1}{2}$. Thus $x=-\frac{1}{2}$ is the only critical point. Note that the denominator of $f'(x)$ is always positive, to the sign of $f'(x)$ is determined by the numerator $2x+1$. Thus $f'(x)<0$ on $(\infty,-1/2)$, and Thus $f'(x)>0$ on $(-1/2,\infty)$. Therefore f has a local maximum of $f(-1/2)$ at $x=-\frac{1}{2}$. There is no local minimum.

11. Find all local extrema of $y=\dfrac{1}{1+x^2}$. The derivative is $y'=\dfrac{-2x}{\left(1+x^2\right)^2}$.

This is positive when x is negative, and negative when x is positive. Thus the derivative changes from positive to negative at $x=0$, so the function has a local maximum at $x=0$. There is no local minimum.

13. Find all local extrema of $y=\sqrt[3]{8-x^3}$.

The derivative is $y'=\dfrac{1}{3}(8-x^3)^{-2/3}(-3x^2)=\dfrac{-x^2}{\sqrt[3]{8-x^3}^2}$. This is zero when $x=0$ and undefined when $x=2$. For all other x the derivative is defined and negative. Thus there are exactly two critical points, 0 and 2. But since the derivative is negative for all x, there are no local extrema.

Extrema on an Interval … Set 2

15. Find all local extrema of $f(x)=\ln\left(x^2e^x+1\right)$.

16. Find all local extrema of $f(x)=x^2+\dfrac{16}{x}$ on its domain.

17. Find all local extrema of $f(x)=\ln(x)+\dfrac{3}{x}-\dfrac{1}{x^2}$ on its domain.

18. Find all local extrema of $f(x)=\sin^{-1}(x)-2x$ on its domain.

Extrema on an Interval … Set 2

Answers

15. Find all local extrema of $f(x) = \ln\left(x^2e^x + 1\right)$.

To find the critical points, we compute $f'(x)$ and examine what x values would make it undefined or zero.

$$f'(x) = \frac{2xe^x + x^2e^x \ + 0}{x^2e^x + 1} = \frac{e^x(2x + x^2)}{x^2e^x + 1} = \frac{e^x(2+x)x}{x^2e^x + 1}$$

The domain of the derivative is all real numbers, so no x values make it undefined. But it will be zero for $x = 0$ and $x = -2$, so these are the critical points. Next we examine the signs of the various factors of $f'(x)$.

	$x < -2$	$-2 < x < 0$	$x > 0$
e^x	+ + + + + + + + + +	+ + + + + + + + + +	+ + + + + + + + + + + +
$(2+x)$	− − − − − − − − − −	+ + + + + + + + + +	+ + + + + + + + + + + +
x	− − − − − − − − − −	− − − − − − − − − −	+ + + + + + + + + + + +
$x^2e^x + 1$	+ + + + + + + + + +	+ + + + + + + + + +	+ + + + + + + + + + + +
$f'(x) = e^x(2+x)x/(x^2e^x + 1)$	+ + + + + + + + + +	− − − − − − − − − −	+ + + + + + + + + + + +

By the first derivative test, f has a local maximum at $x = -2$ and a local minimum at $x = 0$.

17. Find all local extrema of $f(x) = \ln(x) + \frac{3}{x} - \frac{1}{x^2}$ on its domain.

The domain of this function is the interval $(0, \infty)$. To find the critical points, look at $f'(x) = \frac{1}{x} - \frac{3}{x^2} + \frac{2}{x^3}$. This is defined for all x in the domain of f, so the only critical points we will find are those we will find by solving $f'(x) = 0$.

$$\frac{1}{x} - \frac{3}{x^2} + \frac{2}{x^3} = 0$$

$$x^3\left(\frac{1}{x} - \frac{3}{x^2} + \frac{2}{x^3}\right) = 0 \cdot x^3$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

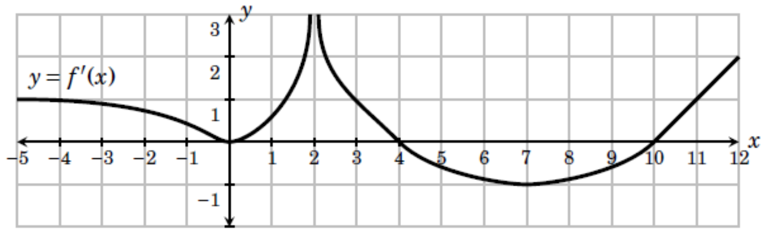
Therefore the critical points are $x = 1$ and $x = 2$. This breaks the domain $(0, \infty)$ of f into three intervals, namely $(0,1), (1,2)$ and $(2, \infty)$. We will decide the sign of $f'(x)$ on these three intervals with test points.

Interval	$(0,1)$	$(1,2)$	$(2,\infty)$
Test point a	$1/2$	$3/2$	3
$f'(a)$	$f'(1/2) = 2$	$f'(3/2) = -2/27$	$f'(3) = 2/27$
Sign of $f'(a)$	+	−	+

By the first derivative test, f has a local maximum at $x = 1$ and a local minimum at $x = 2$.

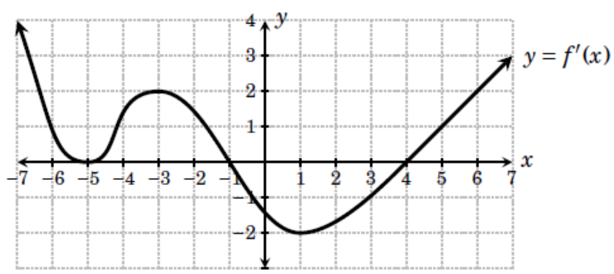
Extrema on an Interval … Set 2

19. The **derivative** $f'(x)$ of a function $f(x)$ with domain $(-5, 12)$ is graphed below. Answer the questions about $f(x)$.



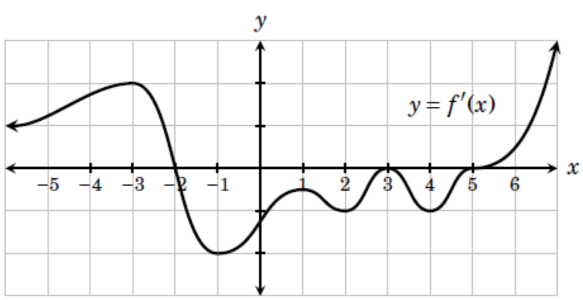
(a) State the intervals on which $f(x)$ increases.
(b) State the intervals on which $f(x)$ decreases.
(c) List all critical points of $f(x)$.
(d) At which of its critical points does $f(x)$ have a local maximum?
(e) At which of its critical points does $f(x)$ have a local minimum?
(f) Based on this information, sketch a possible graph of $f(x)$.

20. The **derivative** $f'(x)$ of a function $f(x)$ is graphed below. Answer the questions about $f(x)$.



(a) State the intervals on which $f(x)$ increases.
(b) State the intervals on which $f(x)$ decreases.
(c) List all critical points of $f(x)$.
(d) At which of its critical points does $f(x)$ have a local maximum?
(e) At which of its critical points does $f(x)$ have a local minimum?
(f) Based on this information, sketch a possible graph of $f(x)$.

21. The **derivative** $f'(x)$ of a function $f(x)$ is graphed below. Answer the questions about $f(x)$.

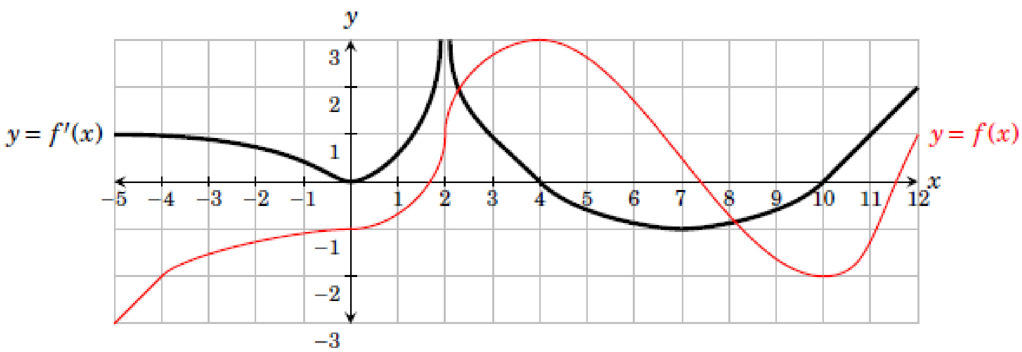


(a) State the critical points of f.
(b) State the interval(s) on which f decreases.
(c) Does f have a local maximum? Where?
(d) Does f have a local minimum? Where?

Extrema on an Interval … Set 2

Answers

19. The **derivative** $f'(x)$ of a function $f(x)$ with domain $(-5, 12)$ is graphed below. Answer the questions about $f(x)$.



(a) $f(x)$ increases on $(-5,0)$, $(0,2)$, $(2,4)$ and $(10,12)$ where $f'(x) > 0$.

(b) $f(x)$ decreases on $(4,10)$ because $f'(x)$ is negative there.

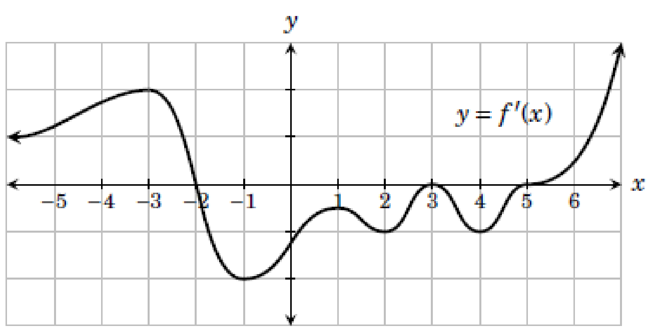
(c) The critical points of $f(x)$ are the values for which $f'(x)$ equals 0 or is undefined. These values are $x = 0$, $x = 2$, $x = 4$ and $x = 5$.

(d) $f(x)$ has a local max at $x = 4$ because $f'(x)$ changes sign from + to − there.

(e) $f(x)$ has a local min at $x = 10$ because $f'(x)$ changes sign from − to + there.

(f) A possible graph of $f(x)$ is sketched in red.

21. The **derivative** $f'(x)$ of a function $f(x)$ is graphed below. Answer the questions about $f(x)$.



(a) The critical points of f are $\boxed{-2,\ 3 \text{ and } 5}$ because $f'(x)$=0 for these x values.

(b) f increases on the intervals $\boxed{(-\infty,-2)\ \&\ (5,\infty)}$ since $f'(x)$>0 there.

(c) f decreases on the intervals $\boxed{(-2,3)\ \&\ (3,5)}$ since $f'(x)$<0 there

(d) Does f have a local maximum? $\boxed{\text{Yes, at } x = -2}$ as $f'(x)$ goes + to − there.

(e) Does f have a local minimum? $\boxed{\text{Yes, at } x = 5}$ as $f'(x)$ goes − to + there.