2^{nd} Derivative Test … Set 1

Critical Points and Classifying Local Maxima and Minima

To find and classify critical points of a function $f(x)$

First steps:

1. Take the derivative $f'(x)$.
2. Find the critical points by setting f' equal to 0, and solving for x .

To finish the job, use either the first derivative test or the second derivative test.

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Via First Derivative Test

3. Determine if f' is positive (so f is increasing) or negative (so f is decreasing) on both sides of each critical point.

   • Note that since all that matters is the sign, you can check *any* value on the side you want; so use values that make it easy!

   • Optional, but helpful in more complex situations: draw a *sign diagram.*

   For example, say we have critical points at 11/8 and 22/7. It's usually easier to evaluate functions at integer values than non-integers, and it's especially easy at 0. So, for a value to the left of 11/8, choose 0; for a value to the right of 11/8 and the left of 22/7, choose 2; and for a value to the right of 22/7, choose 4. Let's say $f'(0) = -1$; $f'(2) = 5/9$; and $f'(4) = 5$. Then we could draw this sign diagram:

Value of x		$\frac{11}{8}$		$\frac{22}{7}$	
Sign of $f'(x)$	negative		positive		positive

4. Apply the *first derivative test* (textbook, top of p. 172, restated in terms of the derivative):

   If f' changes from negative to positive: f has a local **minimum** at the critical point
   If f' changes from positive to negative: f has a local **maximum** at the critical point
   If f' is positive on both sides or negative on both sides: f has neither one

   For the above example, we could tell from the sign diagram that 11/8 is a local minimum of $f(x)$, while 22/7 is neither a local minimum nor a local maximum.

A simple but complete example: Let $y = f(x) = x^3$.

1. $f'(x) = 3x^2$
2. $3x^2 = 0$ ==> $x = 0$; this is the only critical point.
3. $f'(-1) = 3$ ==> sign for $f'(x)$ to the left of the critical point is positive; $f'(1) = 3$ ==> sign to the right is also positive.
4. So 0 is neither local maximum or minimum.

2^{nd} Derivative Test … Set 1

Via Second Derivative Test

3. Take the 2^{nd} derivative $f''(x)$.
4. Determine if $f''(x)$ is positive (so f is concave up), negative (so f is concave down), or zero at each critical point.

Apply the *second derivative test* (textbook, p. 172, restated in terms of the 2^{nd} derivative):

If f'' is positive: f has a local **minimum** at the critical point
If f'' is negative: f has a local **maximum** at the critical point
If f'' is zero: f has neither one

A simple but complete example: Let $y = f(x) = x^3 - 12x$.

1. $f'(x) = 3x^2 - 12$
2. $3x^2 = 12 = 0$ ==> $x = -2$, $x = 2$; these are the critical points.
3. $f''(x) = 6x$
4. $f''(-2) = -12$ ==> f has a local maximum at –2
   $f''(2) = 12$ ==> f has a local minimum at 2