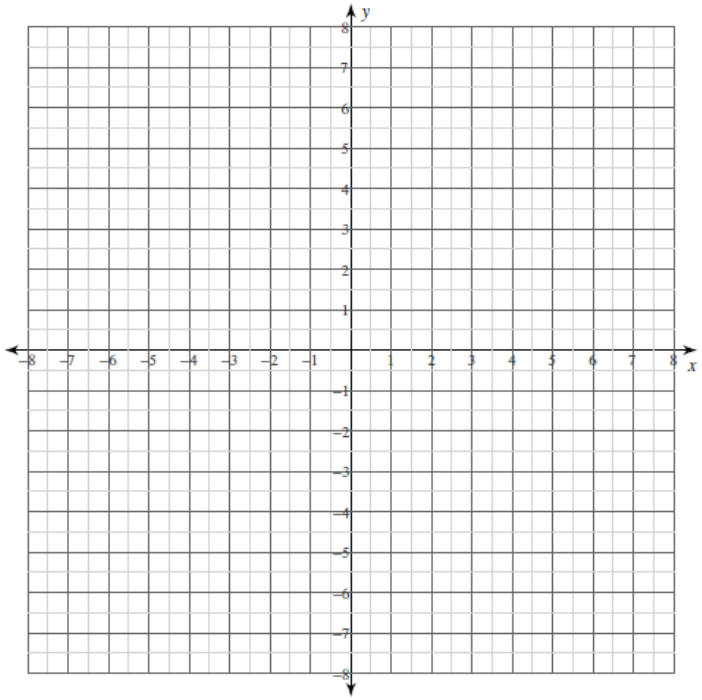
Curve Sketching … Set 2

Curve Sketching

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

1) $y = -\frac{x^3}{3} + x^2$



Curve Sketching … Set 2

Answers

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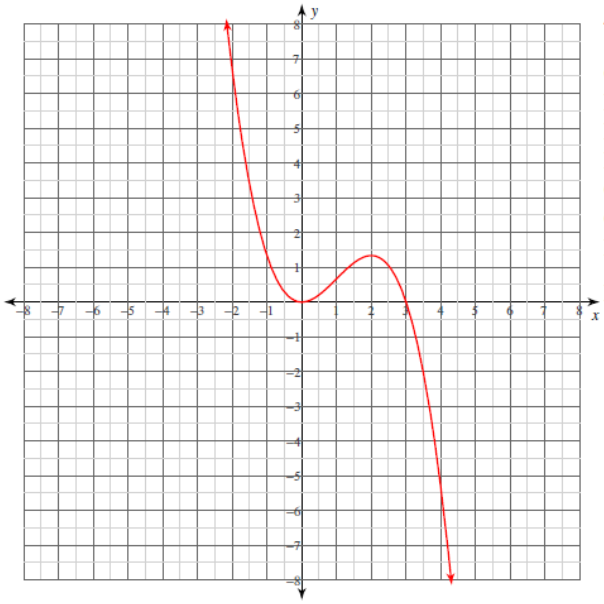
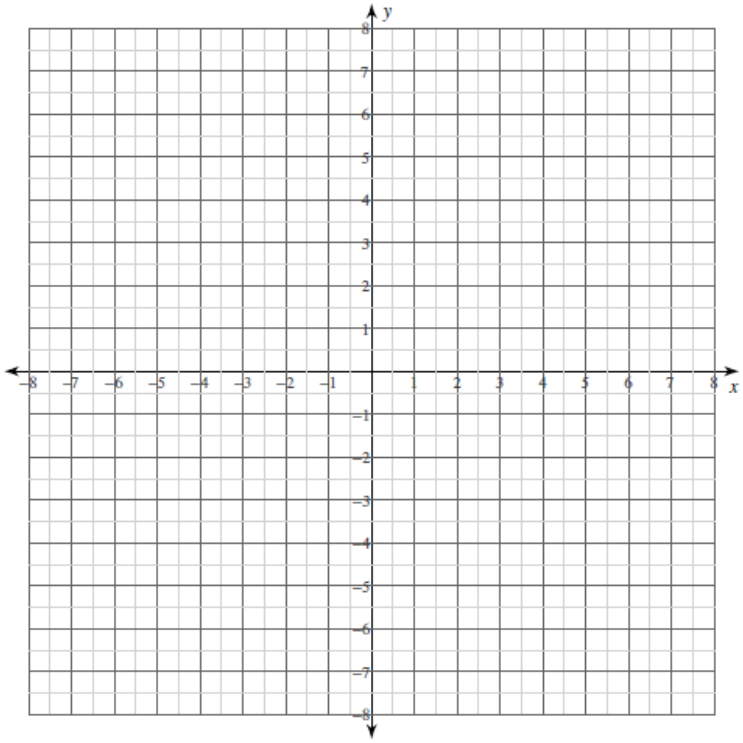


x-intercepts at $x = 0, 3$
y-intercept at $y = 0$
Critical points at: $x = 0, 2$
Increasing: $(0, 2)$
Decreasing: $(-\infty, 0), (2, \infty)$
Inflection point at: $x = 1$
Concave up: $(-\infty, 1)$
Concave down: $(1, \infty)$
Relative minimum: $(0, 0)$
Relative maximum: $\left(2, \frac{4}{3}\right)$

Curve Sketching … Set 2

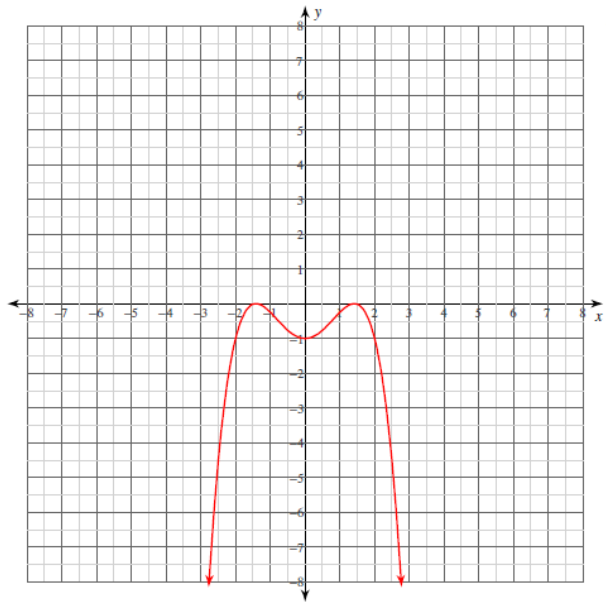
2) $y = -\frac{x^4}{4} + x^2 - 1$



Curve Sketching … Set 2

Answers

2) $y = -\frac{x^4}{4} + x^2 - 1$



x-intercepts at $x = -\sqrt{2},\ \sqrt{2}$

y-intercept at $y = -1$

Critical points at: $x = -\sqrt{2},\ 0,\ \sqrt{2}$

Increasing: $(-\infty, -\sqrt{2}),\ (0, \sqrt{2})$

Decreasing: $(-\sqrt{2}, 0),\ (\sqrt{2}, \infty)$

Inflection points at: $x = -\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$

Concave up: $\left(-\frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}\right)$

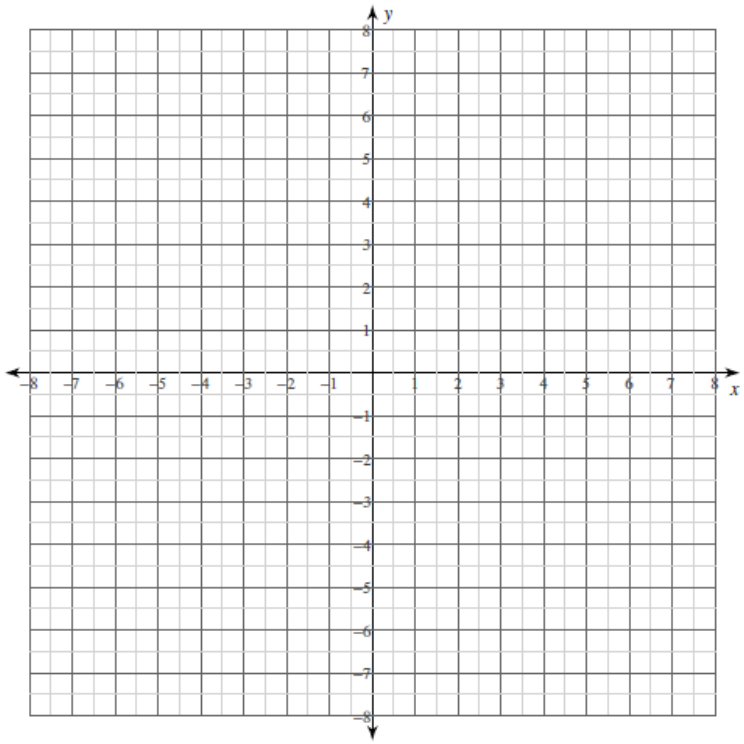
Concave down: $\left(-\infty, -\frac{\sqrt{6}}{3}\right),\ \left(\frac{\sqrt{6}}{3}, \infty\right)$

Relative minimum: $(0, -1)$

Relative maxima: $(-\sqrt{2}, 0),\ (\sqrt{2}, 0)$

Curve Sketching … Set 2

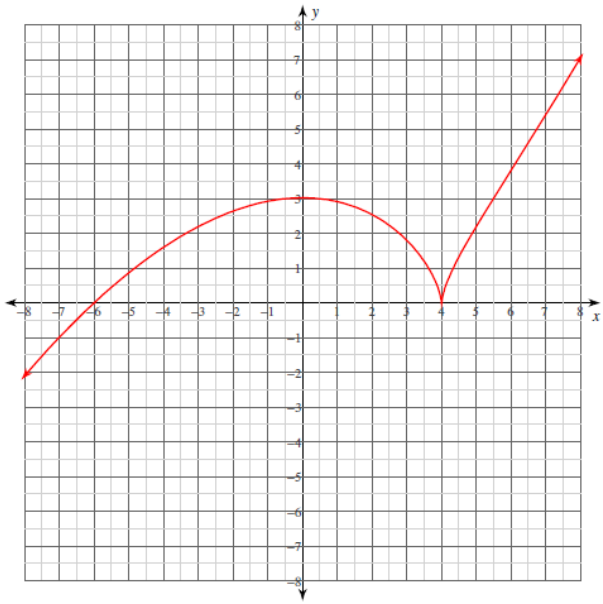
3) $y = \frac{1}{5}(x-4)^{\frac{5}{3}} + 2(x-4)^{\frac{2}{3}}$



Curve Sketching … Set 2

Answers

3) $y = \frac{1}{5}(x-4)^{\frac{5}{3}} + 2(x-4)^{\frac{2}{3}}$



x-intercepts at $x = -6, 4$

y-intercept at $y = \frac{12\sqrt[3]{2}}{5}$

Critical points at: $x = 0, 4$

Increasing: $(-\infty, 0), (4, \infty)$

Decreasing: $(0, 4)$

Inflection point at: $x = 6$

Concave up: $(6, \infty)$

Concave down: $(-\infty, 4), (4, 6)$

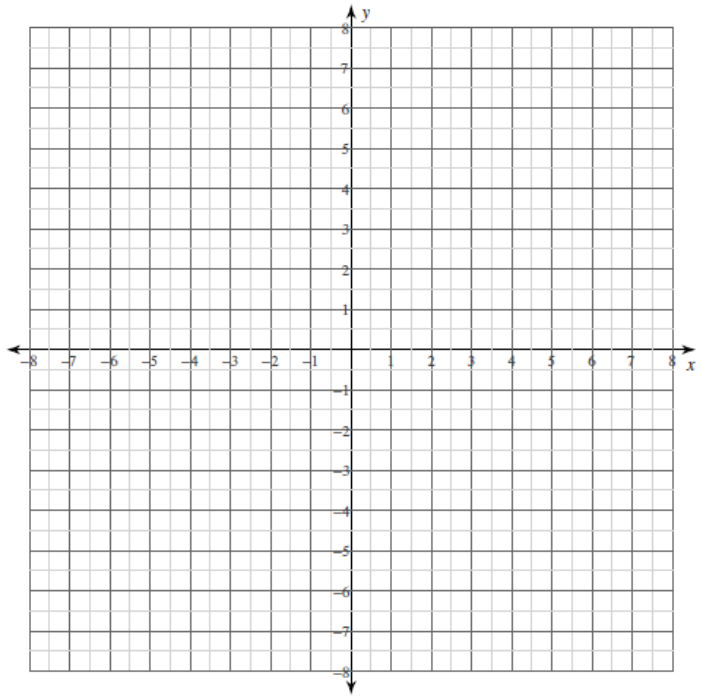
Relative minimum: $(4, 0)$

Relative maximum: $\left(0, \frac{12\sqrt[3]{2}}{5}\right)$

Curve Sketching … Set 2

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

4) $y = \dfrac{7x^2 - 7}{x^3}$



Curve Sketching … Set 2

Answers

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

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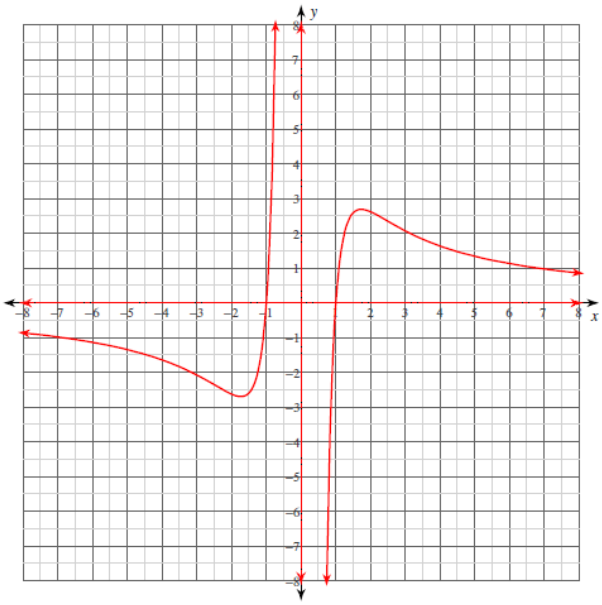

x-intercepts at $x = -1, 1$
No y-intercepts.
Vertical asymptote at: $x = 0$
Horizontal asymptote at: $y = 0$
Critical points at: $x = -\sqrt{3}, \sqrt{3}$
Increasing: $(-\sqrt{3}, 0), (0, \sqrt{3})$
Decreasing: $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Inflection points at: $x = -\sqrt{6}, \sqrt{6}$
Concave up: $(-\sqrt{6}, 0), (\sqrt{6}, \infty)$
Concave down: $(-\infty, -\sqrt{6}), (0, \sqrt{6})$
Relative minimum: $\left(-\sqrt{3}, -\dfrac{14\sqrt{3}}{9}\right)$
Relative maximum: $\left(\sqrt{3}, \dfrac{14\sqrt{3}}{9}\right)$