Maximization, Minimization … Set 1

Optimization

Solve each optimization problem.

1) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 500 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

2) A graphic designer is asked to create a movie poster with a 72 in^2 photo surrounded by a 2 in border at the top and bottom and a 1 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?

3) Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 1)$?

4) A geometry student wants to draw a rectangle inscribed in the ellipse $x^2 + 4y^2 = 25$. What is the area of the largest rectangle that the student can draw?

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Answers

Solve each optimization problem.

1) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 500 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

A = the area of the glass x = the length of the sides of the square bottom

Function to minimize: $A = x^2 + 4x \cdot \dfrac{500}{x^2}$ where $0 < x < \infty$

Dimensions of the aquarium: 10 ft by 10 ft by 5 ft tall

2) A graphic designer is asked to create a movie poster with a 72 in² photo surrounded by a 2 in border at the top and bottom and a 1 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?

A = the area of the poster x = the width of the photo

Function to minimize: $A = (x + 2 \cdot 1)\left(\dfrac{72}{x} + 2 \cdot 2\right)$ where $0 < x < \infty$

Dimensions of the entire poster: 8 in wide by 16 in tall

3) Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 1)$?

d = the distance from point $(0, 1)$ to a point on the parabola x = the x-coord. of a point on the parabola

Function to minimize: $d = \sqrt{x^2 + (4 - x^2 - 1)^2}$ where $-\infty < x < \infty$

Points on the parabola that are closest to the point $(0, 1)$: $\left(-\dfrac{\sqrt{10}}{2}, \dfrac{3}{2}\right), \left(\dfrac{\sqrt{10}}{2}, \dfrac{3}{2}\right)$

4) A geometry student wants to draw a rectangle inscribed in the ellipse $x^2 + 4y^2 = 25$. What is the area of the largest rectangle that the student can draw?

A = the area of the rectangle x = half the base of the rectangle

Function to maximize: $A = 2x \cdot 2 \cdot \dfrac{\sqrt{25 - x^2}}{2}$ where $0 < x < 5$

Area of largest rectangle: 25

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5) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 7. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

6) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(7, 0)$?

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Answers

5) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 7. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

A = the area of the rectangle x = half the base of the rectangle

Function to maximize: $A = 2x\sqrt{7^2 - x^2}$ where $0 < x < 7$

Area of largest rectangle: 49

6) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(7, 0)$?

d = the distance from point $(7, 0)$ to a point on the curve x = the x-coordinate of a point on the curve

Function to minimize: $d = \sqrt{(x-7)^2 + (\sqrt{x})^2}$ where $-\infty < x < \infty$

Point on the curve that is closest to the point $(7, 0)$: $\left(\frac{13}{2}, \frac{\sqrt{26}}{2}\right)$