Maximization, Minimization … Set 4

Calculus Practice: Optimization

Solve each optimization problem.

1) Which points on the graph of $y = 3 - x^2$ are closest to the point $(0, 2)$?

2) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(4, 0)$?

3) A geometry student wants to draw a rectangle inscribed in the ellipse $x^2 + 4y^2 = 36$. What is the area of the largest rectangle that the student can draw?

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Answers

Calculus Practice: Optimization

Solve each optimization problem.

1) Which points on the graph of $y = 3 - x^2$ are closest to the point $(0, 2)$?

d = the distance from point $(0, 2)$ to a point on the parabola x = the x-coord. of a point on the parabola

Function to minimize: $d = \sqrt{x^2 + (3 - x^2 - 2)^2}$ where $-\infty < x < \infty$

Points on the parabola that are closest to the point $(0, 2)$: $\left(-\frac{\sqrt{2}}{2}, \frac{5}{2}\right), \left(\frac{\sqrt{2}}{2}, \frac{5}{2}\right)$

2) Which point on the graph of $y = \sqrt{x}$ is closest to the point $(4, 0)$?

d = the distance from point $(4, 0)$ to a point on the curve x = the x-coordinate of a point on the curve

Function to minimize: $d = \sqrt{(x - 4)^2 + (\sqrt{x})^2}$ where $-\infty < x < \infty$

Point on the curve that is closest to the point $(4, 0)$: $\left(\frac{7}{2}, \frac{\sqrt{14}}{2}\right)$

3) A geometry student wants to draw a rectangle inscribed in the ellipse $x^2 + 4y^2 = 36$.
What is the area of the largest rectangle that the student can draw?

A = the area of the rectangle x = half the base of the rectangle

Function to maximize: $A = 2x \cdot 2 \cdot \frac{\sqrt{36 - x^2}}{2}$ where $0 < x < 6$

Area of largest rectangle: 36

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4) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 8. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

5) A graphic designer is asked to create a movie poster with a 50 in^2 photo surrounded by a 2 in border at the top and bottom and a 1 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?

6) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 1372 ft^3 of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

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Answers

4) A geometry student wants to draw a rectangle inscribed in a semicircle of radius 8. If one side must be on the semicircle's diameter, what is the area of the largest rectangle that the student can draw?

A = the area of the rectangle x = half the base of the rectangle

Function to maximize: $A = 2x\sqrt{8^2 - x^2}$ where $0 < x < 8$

Area of largest rectangle: 64

5) A graphic designer is asked to create a movie poster with a 50 in² photo surrounded by a 2 in border at the top and bottom and a 1 in border on each side. What overall dimensions for the poster should the designer choose to use the least amount of paper?

A = the area of the poster x = the width of the photo

Function to minimize: $A = (x + 2 \cdot 1)\left(\frac{50}{x} + 2 \cdot 2\right)$ where $0 < x < \infty$

Dimensions of the entire poster: 7 in wide by 14 in tall

6) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 1372 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

A = the area of the glass x = the length of the sides of the square bottom

Function to minimize: $A = x^2 + 4x \cdot \frac{1372}{x^2}$ where $0 < x < \infty$

Dimensions of the aquarium: 14 ft by 14 ft by 7 ft tall