Related Rates … Set 2

Related Rates

Solve each related rate problem.

1) A spherical balloon is deflated so that its radius decreases at a rate of 4 cm/sec. At what rate is the volume of the balloon changing when the radius is 3 cm?

2) A spherical balloon is deflated at a rate of $\frac{256\pi}{3}$ cm³/sec. At what rate is the radius of the balloon changing when the radius is 8 cm?

3) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 9 cm/min. How fast is the area of the pool increasing when the radius is 12 cm?

4) A 7 ft tall person is walking towards a 17 ft tall lamppost at a rate of 4 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 12 ft from the lamppost?

5) A conical paper cup is 30 cm tall with a radius of 10 cm. The cup is being filled with water at a rate of $\frac{2\pi}{3}$ cm³/sec. How fast is the water level rising when the water level is 2 cm?

6) A 13 ft ladder is leaning against a wall and sliding towards the floor. The top of the ladder is sliding down the wall at a rate of 7 ft/sec. How fast is the base of the ladder sliding away from the wall when the base of the ladder is 12 ft from the wall?

7) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The radius of the spill increases at a rate of 2 m/min. How fast is the area of the spill increasing when the radius is 13 m?

Related Rates … Set 2

Answers

1) V = volume of sphere r = radius t = time

Equation: $V=\frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt}=-4$ Find: $\left.\frac{dV}{dt}\right|_{r=3}$

$\left.\frac{dV}{dt}\right|_{r=3}=4\pi r^2\cdot\frac{dr}{dt}=-144\pi$ cm³/sec

2) V = volume of sphere r = radius t = time

Equation: $V=\frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt}=-\frac{256\pi}{3}$ Find: $\left.\frac{dr}{dt}\right|_{r=8}$

$\left.\frac{dr}{dt}\right|_{r=8}=\frac{1}{4\pi r^2}\cdot\frac{dV}{dt}=-\frac{1}{3}$ cm/sec

3) A = area of circle r = radius t = time

Equation: $A=\pi r^2$ Given rate: $\frac{dr}{dt}=9$ Find: $\left.\frac{dA}{dt}\right|_{r=12}$

$\left.\frac{dA}{dt}\right|_{r=12}=2\pi r\cdot\frac{dr}{dt}=216\pi$ cm²/min

4) x = distance from person to lamppost y = length of shadow t = time

Equation: $\frac{x+y}{17}=\frac{y}{7}$ Given rate: $\frac{dx}{dt}=-4$ Find: $\left.\frac{dy}{dt}\right|_{x=12}$

$\left.\frac{dy}{dt}\right|_{x=12}=\frac{7}{10}\cdot\frac{dx}{dt}=-\frac{14}{5}$ ft/sec

5) V = volume of material in cone h = height t = time

Equation: $V=\frac{\pi h^3}{27}$ Given rate: $\frac{dV}{dt}=\frac{2\pi}{3}$ Find: $\left.\frac{dh}{dt}\right|_{h=2}$

$\left.\frac{dh}{dt}\right|_{h=2}=\frac{9}{\pi h^2}\cdot\frac{dV}{dt}=\frac{3}{2}$ cm/sec

6) x = horizontal distance from base of ladder to wall y = vertical distance from top of ladder to floor t = time

Equation: $x^2+y^2=13^2$ Given rate: $\frac{dy}{dt}=-7$ Find: $\left.\frac{dx}{dt}\right|_{x=12}$

$\left.\frac{dx}{dt}\right|_{x=12}=-\frac{y}{x}\cdot\frac{dy}{dt}=\frac{91}{12}$ ft/sec

7) A = area of circle r = radius t = time

Equation: $A=\pi r^2$ Given rate: $\frac{dr}{dt}=2$ Find: $\left.\frac{dA}{dt}\right|_{r=13}$

$\left.\frac{dA}{dt}\right|_{r=13}=2\pi r\cdot\frac{dr}{dt}=52\pi$ m²/min

Related Rates … Set 2

8) A hypothetical cube shrinks so that the length of its sides are decreasing at a rate of 2 m/min. At what rate is the volume of the cube changing when the sides are 2 m each?

9) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water level goes down at a rate of 2 cm/sec. At what rate is the volume of water in the cup changing when the water level is 9 cm?

10) An observer stands 500 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 700 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 1200 ft from the ground?

11) A spherical snowball melts at a rate of 36π in^3/sec. At what rate is the radius of the snowball changing when the radius is 5 in?

12) A hypothetical cube grows at a rate of 8 m^3/min. How fast are the sides of the cube increasing when the sides are 2 m each?

13) A conical paper cup is 10 cm tall with a radius of 30 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 9 cm?

14) Water slowly evaporates from a circular shaped puddle. The radius of the puddle decreases at a rate of 8 in/hr. Assuming the puddle retains its circular shape, at what rate is the area of the puddle changing when the radius is 3 in?

Related Rates … Set 2

Answers

8) V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\frac{ds}{dt} = -2$ Find: $\left.\frac{dV}{dt}\right|_{s=2}$

$$\left.\frac{dV}{dt}\right|_{s=2} = 3s^2 \cdot \frac{ds}{dt} = -24 \text{ m}^3\text{/min}$$

9) V = volume of material in cone h = height t = time

Equation: $V = \frac{\pi h^3}{3}$ Given rate: $\frac{dh}{dt} = -2$ Find: $\left.\frac{dV}{dt}\right|_{h=9}$

$$\left.\frac{dV}{dt}\right|_{h=9} = \pi h^2 \cdot \frac{dh}{dt} = -162\pi \text{ cm}^3\text{/sec}$$

10) a = altitute of rocket z = distance from observer to rocket t = time

Equation: $a^2 + 250000 = z^2$ Given rate: $\frac{da}{dt} = 700$ Find: $\left.\frac{dz}{dt}\right|_{a=1200}$

$$\left.\frac{dz}{dt}\right|_{a=1200} = \frac{a}{z} \cdot \frac{da}{dt} = \frac{8400}{13} \text{ ft/sec}$$

11) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dV}{dt} = -36\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=5}$

$$\left.\frac{dr}{dt}\right|_{r=5} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt} = -\frac{9}{25} \text{ in/s}$$

12) V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\frac{dV}{dt} = 8$ Find: $\left.\frac{ds}{dt}\right|_{s=2}$

$$\left.\frac{ds}{dt}\right|_{s=2} = \frac{1}{3s^2} \cdot \frac{dV}{dt} = \frac{2}{3} \text{ m/min}$$

13) V = volume of material in cone h = height t = time

Equation: $V = 3\pi h^3$ Given rate: $\frac{dh}{dt} = 2$ Find: $\left.\frac{dV}{dt}\right|_{h=9}$

$$\left.\frac{dV}{dt}\right|_{h=9} = 9\pi h^2 \cdot \frac{dh}{dt} = 1458\pi \text{ cm}^3\text{/sec}$$

14) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = -8$ Find: $\left.\frac{dA}{dt}\right|_{r=3}$

$$\left.\frac{dA}{dt}\right|_{r=3} = 2\pi r \cdot \frac{dr}{dt} = -48\pi \text{ in}^2\text{/hr}$$

Related Rates … Set 2

15) A hypothetical square grows so that the length of its diagonals are increasing at a rate of 4 m/min. How fast is the area of the square increasing when the diagonals are 14 m each?

16) Water slowly evaporates from a circular shaped puddle. The area of the puddle decreases at a rate of 16π in^2/hr. Assuming the puddle retains its circular shape, at what rate is the radius of the puddle changing when the radius is 12 in?

17) A hypothetical cube grows so that the length of its sides are increasing at a rate of 4 m/min. How fast is the volume of the cube increasing when the sides are 7 m each?

18) A hypothetical square grows at a rate of 16 m^2/min. How fast are the sides of the square increasing when the sides are 15 m each?

19) A hypothetical cube shrinks at a rate of 8 m^3/min. At what rate are the sides of the cube changing when the sides are 3 m each?

20) A spherical snowball melts so that its radius decreases at a rate of 4 in/sec. At what rate is the volume of the snowball changing when the radius is 8 in?

21) A perfect cube shaped ice cube melts so that the length of its sides are decreasing at a rate of 2 mm/sec. Assume that the block retains its cube shape as it melts. At what rate is the volume of the ice cube changing when the sides are 2 mm each?

Related Rates … Set 2

Answers

15) A = area of square x = length of diagonals t = time

Equation: $A = \frac{x^2}{2}$ Given rate: $\frac{dx}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{x=14}$

$$\left.\frac{dA}{dt}\right|_{x=14} = x \cdot \frac{dx}{dt} = 56 \text{ m}^2\text{/min}$$

16) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = -16\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=12}$

$$\left.\frac{dr}{dt}\right|_{r=12} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = -\frac{2}{3} \text{ in/hr}$$

17) V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\frac{ds}{dt} = 4$ Find: $\left.\frac{dV}{dt}\right|_{s=7}$

$$\left.\frac{dV}{dt}\right|_{s=7} = 3s^2 \cdot \frac{ds}{dt} = 588 \text{ m}^3\text{/min}$$

18) A = area of square s = length of sides t = time

Equation: $A = s^2$ Given rate: $\frac{dA}{dt} = 16$ Find: $\left.\frac{ds}{dt}\right|_{s=15}$

$$\left.\frac{ds}{dt}\right|_{s=15} = \frac{1}{2s} \cdot \frac{dA}{dt} = \frac{8}{15} \text{ m/min}$$

19) V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\frac{dV}{dt} = -8$ Find: $\left.\frac{ds}{dt}\right|_{s=3}$

$$\left.\frac{ds}{dt}\right|_{s=3} = \frac{1}{3s^2} \cdot \frac{dV}{dt} = -\frac{8}{27} \text{ m/min}$$

20) V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt} = -4$ Find: $\left.\frac{dV}{dt}\right|_{r=8}$

$$\left.\frac{dV}{dt}\right|_{r=8} = 4\pi r^2 \cdot \frac{dr}{dt} = -1024\pi \text{ in}^3\text{/sec}$$

21) V = volume of cube s = length of sides t = time

Equation: $V = s^3$ Given rate: $\frac{ds}{dt} = -2$ Find: $\left.\frac{dV}{dt}\right|_{s=2}$

$$\left.\frac{dV}{dt}\right|_{s=2} = 3s^2 \cdot \frac{ds}{dt} = -24 \text{ mm}^3\text{/sec}$$

Related Rates … Set 2

22) A conical paper cup is 10 cm tall with a radius of 10 cm. The bottom of the cup is punctured so that the water leaks out at a rate of $\frac{9\pi}{4}$ cm³/sec. At what rate is the water level changing when the water level is 6 cm?

23) A hypothetical square shrinks so that the length of its diagonals are changing at a rate of -8 m/min. At what rate is the area of the square changing when the diagonals are 5 m each?

24) A hypothetical square shrinks at a rate of 2 m²/min. At what rate are the diagonals of the square changing when the diagonals are 7 m each?

25) Water leaking onto a floor forms a circular pool. The area of the pool increases at a rate of 25π cm²/min. How fast is the radius of the pool increasing when the radius is 6 cm?

Related Rates … Set 2

Answers

22) V = volume of material in cone h = height t = time

Equation: $V = \dfrac{\pi h^3}{3}$ Given rate: $\dfrac{dV}{dt} = -\dfrac{9\pi}{4}$ Find: $\left.\dfrac{dh}{dt}\right|_{h=6}$

$$\left.\frac{dh}{dt}\right|_{h=6} = \frac{1}{\pi h^2} \cdot \frac{dV}{dt} = -\frac{1}{16} \text{ cm/sec}$$

23) A = area of square x = length of diagonals t = time

Equation: $A = \dfrac{x^2}{2}$ Given rate: $\dfrac{dx}{dt} = -8$ Find: $\left.\dfrac{dA}{dt}\right|_{x=5}$

$$\left.\frac{dA}{dt}\right|_{x=5} = x \cdot \frac{dx}{dt} = -40 \text{ m}^2\text{/min}$$

24) A = area of square x = length of diagonals t = time

Equation: $A = \dfrac{x^2}{2}$ Given rate: $\dfrac{dA}{dt} = -2$ Find: $\left.\dfrac{dx}{dt}\right|_{x=7}$

$$\left.\frac{dx}{dt}\right|_{x=7} = \frac{1}{x} \cdot \frac{dA}{dt} = -\frac{2}{7} \text{ m/min}$$

25) A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\dfrac{dA}{dt} = 25\pi$ Find: $\left.\dfrac{dr}{dt}\right|_{r=6}$

$$\left.\frac{dr}{dt}\right|_{r=6} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = \frac{25}{12} \text{ cm/min}$$