Related Rates … Set 4

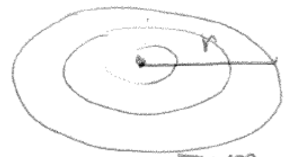
Related Rates

Solve each related rate problem.

1) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm? $\frac{\Delta r}{\Delta t} = 4$ cm/m

A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dr}{dt} = 4$ Find: $\left.\frac{dA}{dt}\right|_{r=5}$



$$\left.\frac{dA}{dt}\right|_{r=5} = 2\pi r \cdot \frac{dr}{dt} = 40\pi \text{ cm}^2/\text{min}$$

$\frac{dA}{dt} = \left[\pi r^2\right]' = \boxed{\pi \cdot 2r\frac{dr}{dt}}$

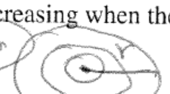
when $r = 5$ cm,

$\frac{dA}{dt} = \pi \cdot 2 \cdot 5 \cdot 4 = \boxed{40\pi \text{ cm}^2/\text{min or} \approx 125.66 \text{ cm}^2/\text{min}}$

when the radius is 5cm, the area of the pool increases at a rate of 40π or ≈125.66 cm²/min

2) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 9π m²/min. How fast is the radius of the spill increasing when the radius is 10 m?

Given: $\frac{dA}{dt} = 9\pi$ m²/min



A = area of circle r = radius t = time

Equation: $A = \pi r^2$ Given rate: $\frac{dA}{dt} = 9\pi$ Find: $\left.\frac{dr}{dt}\right|_{r=10}$

Find: $\left.\frac{dr}{dt}\right|_{r=10\text{m}}$ $A = \pi r^2$

$$\left.\frac{dr}{dt}\right|_{r=10} = \frac{1}{2\pi r} \cdot \frac{dA}{dt} = \frac{9}{20} \text{ m/min}$$

$\frac{dA}{dt} = \pi 2r\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{\pi \cdot 2r} \cdot \frac{dA}{dt}$

$\frac{dr}{dt} = \frac{1}{\cancel{\pi} \cdot 2 \cdot 10} \cdot 9\cancel{\pi} = \frac{9}{20}$ m/min or ≈ .45 m/min

when the radius is 10m, it is increasing at a rate of ≈ .45m/min

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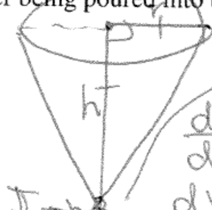
3) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?



$h = 10\text{cm}$

$r = 10\text{cm}$

$\frac{dh}{dt} = 2\text{cm/sec}$

$\left.\frac{dV}{dt} = ?\right|_{h=8\text{cm}}$

V = volume of material in cone h = height t = time

Equation: $V = \frac{\pi h^3}{3}$ Given rate: $\frac{dh}{dt} = 2$ Find: $\left.\frac{dV}{dt}\right|_{h=8}$ $h^2 \cdot h$

$\left.\frac{dV}{dt}\right|_{h=8} = \pi h^2 \cdot \frac{dh}{dt} = 128\pi$ cm³/sec

$V = \frac{1}{3}\pi r^2 \cdot h = \frac{\pi}{3} h^3 \Rightarrow$

$\frac{dV}{dt} = \frac{\pi}{3} 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{\cancel{3}} \cdot \cancel{3} \cdot 8^2 \cdot 2 = \boxed{128\pi} \approx \boxed{402.12\text{cm}^3/\text{sec}}$

answer:
when the water level is 8 cm, it's being poured into the cup at a rate of $\boxed{\approx 402.12\text{cm}^3/\text{sec}}$

4) A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{2}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 4 cm?

$\frac{dr}{dt} = \frac{2}{r}$ cm/sec

$\left.\frac{dV}{dt}\right|_{r=4} = ?$

V = volume of sphere r = radius t = time

Equation: $V = \frac{4}{3}\pi r^3$ Given rate: $\frac{dr}{dt} = \frac{2}{r}$ Find: $\left.\frac{dV}{dt}\right|_{r=4}$

$\left.\frac{dV}{dt}\right|_{r=4} = 4\pi r^2 \cdot \frac{dr}{dt} = 32\pi$ cm³/sec

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3} \cdot 4^2 \cdot \frac{2}{4} = 32\pi \approx \boxed{100.53\text{cm}^3/\text{sec}}$

When the radius of the balloon is 4cm, its volume increases at a rate of $\approx \boxed{100.53\text{cm}^3/\text{sec}}$

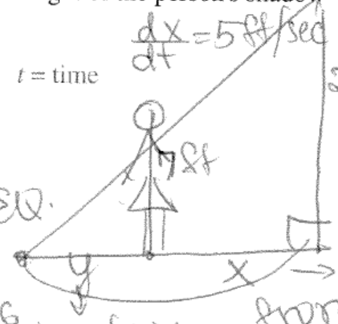
Related Rates … Set 4

5) A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?

x = distance from person to lamppost $\quad y$ = length of shadow $\quad t$ = time

Equation: $\dfrac{x+y}{20} = \dfrac{y}{7}$ Given rate: $\dfrac{dx}{dt} = 5$ Find: $\left.\dfrac{dy}{dt}\right|_{x=16}$

$\left.\dfrac{dy}{dt}\right|_{x=16} = \dfrac{7}{13} \cdot \dfrac{dx}{dt} = \boxed{\dfrac{35}{13}\ \text{ft/sec}}$



$\dfrac{y+x}{y} = \dfrac{20}{7} \to$ EQ.

$\left.\dfrac{dy}{dt} = ?\ \right|_{x=16}$

$7(y+x) = 20y$

$7\left(\dfrac{dy}{dt} + \dfrac{dx}{dt}\right) = 20\dfrac{dy}{dt} \Rightarrow 7\dfrac{dy}{dt} - 20\dfrac{dy}{dt} = -\dfrac{dx}{dt} \Rightarrow 7\dfrac{dx}{dt} = 13\dfrac{dy}{dt}$

$\dfrac{dy}{dt} = \dfrac{7}{13}\dfrac{dx}{dt} = \dfrac{35}{13}$ ft/sec ≈ 2.69 ft/sec

6) An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?

a = altitute of rocket $\quad z$ = distance from observer to rocket $\quad t$ = time

Equation: $a^2 + 490000 = z^2$ Given rate: $\dfrac{da}{dt} = 900$ Find: $\left.\dfrac{dz}{dt}\right|_{a=2400}$

$\left.\dfrac{dz}{dt}\right|_{a=2400} = \dfrac{a}{z} \cdot \dfrac{da}{dt} = 864$ ft/sec