

# Maximizing and Output ... Set 3

## Solving Problems Involving Cost, Revenue, Profit

The **cost function  $C(x)$**  is the total cost of making  $x$  items. If the cost per item is fixed, it is equal to the cost per item ( $c$ ) times the number of items produced ( $x$ ), or  $C(x) = c \cdot x$ .

The **price function  $p(x)$**  - also called the demand function - describes how price affects the number of items sold. Normally, when the price increases, customers will not demand as many items, and so  $x$  will decrease. To sell more items, the price usually has to decrease.

The **revenue function  $R(x)$**  is the income from sales. It is equal to the price times the number sold, or  $R(x) = x \cdot p(x)$ .

The **profit function  $P(x)$**  is the money that is left over from the revenue (income) after the costs (expenses) have been subtracted. In other words,  $P(x) = R(x) - C(x)$ .

The **break-even point** occurs when the total revenue equals the total cost - or, in other words, when the profit is zero. To solve for a break-even quantity, set  $P(x) = 0$  and solve for  $x$  using factored form or the quadratic formula.

## Max and Min Problems

Max and min problems can be solved using any of the forms of quadratic equation:

Vertex form	$y = a(x - h)^2 + k$	the vertex is $(h, k)$
Factored form	$y = a(x - p)(x - q)$	the vertex will lie on the axis of symmetry, which is found using the mean of the roots $p$ and $q$ .
Standard form	$y = ax^2 + bx + c$	find the vertex by completing the square (or by using the shortcut method related to the axis of symmetry)

A vertex is a **minimum** if the parabola opens up ( $a > 0$ )

A vertex is a **maximum** if the parabola opens down ( $a < 0$ )

Depending on the situation, the "best" solution (either max or min) is called the **optimal value**.

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### Warm up:

You are the sole owner of a denim store downtown, Toronto. Last week, you sold 200 pairs of jeans priced at \$36. You buy the pants from a local manufacturer located in Montreal, Quebec. Calculate your **total revenue** for the last week.

$$\begin{aligned}\text{Revenue} &= (\text{\#of jeans sold})(\text{unit price}) \\ &= (200)(36) \\ &= \$7200\end{aligned}$$

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## The Real Deal: How to maximize the Revenue

Ex.1 Ever since you took grade 11 math course in high school with Mr. Bulut, you have been wondering if you can apply **maximizing total revenue** concept in your business to make more money. When you set your price for \$36, you sell average of 200 pairs a week. After doing a mini survey, you find out that for each \$2 increase in price, the demand for jeans is less and 5 fewer pants are sold.

a) What price will **maximize** your total revenue?

$$\begin{aligned}
 \text{Revenue} &= (\# \text{ of items sold}) (\text{Unit Price}) \\
 &= (200 \times 36) \quad \begin{array}{l} \# \text{ of times of price} \\ \text{increase} \end{array} \\
 &\quad (200-5)(36+2) \quad \begin{array}{l} 1 \text{ time} \\ 2 \\ 3 \\ x \end{array} \\
 &\quad (200-10)(36+4) \\
 &\quad (200-15)(36+6) \\
 R(x) &= (200-5x)(36+2x) \quad x
 \end{aligned}
 \left. \vphantom{\begin{aligned} \text{Revenue} \\ = \\ \\ \\ \\ R(x) \end{aligned}} \right\} \begin{array}{l} R(x) = (200-5x)(36+2x) \\ \text{or} \\ R(x) = 7200 + 400x - 180x - 10x^2 \\ R(x) = -10x^2 + 220x + 7200 \end{array}$$

$R(x) = -10x^2 + 220x + 7200$  "x" represents the number of times of price increase

$\frac{-22}{-2} = 11$        $= -10(x^2 - 22x) + 7200$       By completing the square you can find the vertex which will give you the max revenue

$(-11)^2 = 121$        $= -10(x^2 - 22x + 121 - 121) + 7200$

$R(x) = -10(x-11)^2 + 8410$        $V(11, 8410)$

if you increase the price 11 times

your revenue will max at \$8410

$$\begin{aligned}
 \# \text{ of jeans sold} &= 200 - 5x \\
 &= 200 - 5 \cdot 11 \\
 &= 145 \text{ jeans sold}
 \end{aligned}$$

$$\begin{aligned}
 \text{Unit (Price) } p(x) &= 36 + 2x \\
 &= 36 + 2 \cdot 11 \\
 &= \underline{\underline{\$58}}
 \end{aligned}$$

∴ When the price is set to \$58, I will maximize my revenue!

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Ex.2 When priced at \$40 each, a toy company sells 5000 toys. The manufacturer estimate that each \$1 increase in price will decrease sales by 100 units. Find the unit price of a toy that will maximize the total revenue.

$$\text{Revenue} = (\text{\# of toys sold}) (\text{Unit price})$$

$$R(x) = (5000 - 100x)(40 + x) \quad \text{expand by FOIL}$$

$$= 20000 + 5000x - 4000x - 100x^2$$

$$R(x) = -100x^2 + 1000x + 20000 \quad \text{complete the square}$$

$$= -100(x^2 - 10x) + 20000 \rightarrow \frac{-10}{2} = -5 \quad (-5)^2 = 25$$

$$= -100(x^2 - 10x + 25 - 25) + 20000$$

$$= -100(x^2 - 10x + 25) + 2500 + 20000$$

$$= -100(x - 5)^2 + 202500$$

$$V(5, 202500)$$

$$\begin{aligned} \text{\# of toys sold} &= 5000 - 100x \\ &= 5000 - 100(5) \\ &= \underline{4500} \end{aligned}$$

$$\begin{aligned} \text{Unit Price } p(x) &= 40 + x \\ &= 40 + 5 \\ &= \underline{\underline{\$45}} \end{aligned}$$

∴ When the price is set to \$45, the toy company will reach its max revenue. (\$202,500).

## Maximizing and Output ... Set 3

1. The city transit system carries 24800 bus riders per day for a fare of \$1.85. The city hopes to reduce car pollution by getting more people to ride the bus, while maximizing the transit system's revenue at the same time. A survey indicates that the number of riders will increase by 800 for every \$0.05 decrease in the fare. What fare will produce the **greatest revenue**?

$$\text{Revenue} = (\# \text{ of bus riders}) (\text{bus fare})$$

$$R(x) = (24800 + 800x)(1.85 - 0.05x)$$

$$= 45880 - 1240x + 1480x - 40x^2$$

$$R(x) = -40x^2 + 240x + 45880 \quad \text{complete the square}$$

$$= -40(x^2 - 6x) + 45880 \quad \frac{-b}{2} = -3 \quad (-3)^2 = 9$$

$$= -40(x^2 - 6x + 9 - 9) + 45880$$

$$= -40(x^2 - 6x + 9) + 360 + 45880$$

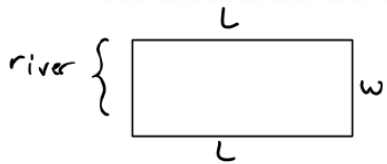
$$= -40(x-3)^2 + 46240$$

$$\leftarrow V(3, 46240)$$

$$\begin{aligned} \text{Unit Price } \cdot p(x) &= 1.85 - 0.05x \\ &= 1.85 - 0.05(3) \\ &= 1.85 - 0.15 \\ &= \$1.70 \end{aligned}$$

$\therefore$  When the bus fare is \$1.70, the max revenue is \$46240

- 7). A rectangular lot is bordered on one side by a stream and on the other three sides by 600 metres of fencing. Determine the dimensions of the lot if its area is a maximum.



$$L + L + w = 600$$

$$2L + w = 600 \quad \text{if } 2L + w = 600, \text{ then } w = 600 - 2L$$

$$\text{Area} = L \times w$$

$$= L(600 - 2L)$$

$$= 600L - 2L^2$$

$$= -2L^2 + 600L$$

$$= -2(L^2 - 300L) \quad -\frac{300}{2} = -150 \quad (-150)^2 = 22500$$

$$= -2(L^2 - 300L + 22500 - 22500)$$

$$= -2(L^2 - 300L + 22500) + 45000$$

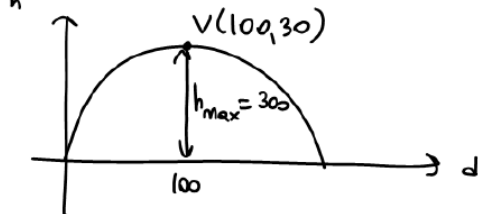
$$= -2(L - 150)^2 + 45000 \quad \therefore L = 150$$

$$V(150, 45000) \quad w = 300$$

## Maximizing and Output ... Set 3

4. The path of a golf ball can be modelled by the function  $h = -0.003d^2 + 0.6d$ , where  $h$  is the height of the golf ball, in metres, and  $d$  is the horizontal distance travelled, in metres. What is the maximum height of the golf ball? At what horizontal distance does the golf ball reach its maximum height?

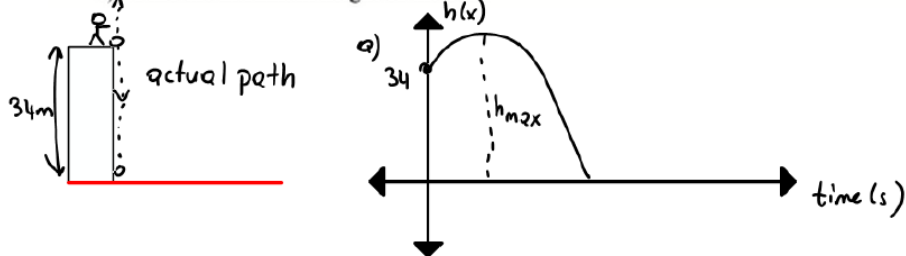
$$\begin{aligned}
 h &= -0.003d^2 + 0.6d \\
 &= -0.003(d^2 - 200d) \quad \rightarrow \quad -\frac{200}{2} = -100 \quad (-100)^2 = 10000 \\
 &= -0.003(d^2 - 200d + 10000 - 10000) \\
 &= -0.003(d^2 - 200d + 10000) + 30 \\
 &= -0.003(d - 100)^2 + 30 \quad V(100, 30)
 \end{aligned}$$



When the ball is at 100m horizontal distance, it reaches its max height of 30m.

5. A ball is thrown vertically upward off the roof of a 34 m tall building. The height of the ball  $h$  in metres, can be approximated by the function  $h = -5t^2 + 10t + 34$  where  $t$  is the time in seconds, after the ball is thrown.

- Sketch the graph.
- How high is the ball after 2 s?
- Find the maximum height of the ball.

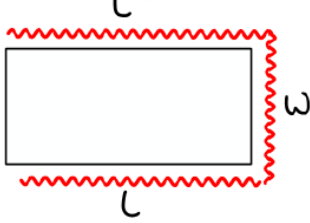


$$\begin{aligned}
 \text{b) } h(t) &= -5t^2 + 10t + 34 \\
 h(2) &= -5(2)^2 + 10(2) + 34 \\
 &= -20 + 20 + 34 \\
 h(2) &= 34\text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } h(t) &= -5t^2 + 10t + 34 \\
 &= -5(t^2 - 2t) + 34 \quad \rightarrow \quad -\frac{2}{2} = -1 \quad (-1)^2 = 1 \\
 &= -5(t^2 - 2t + 1 - 1) + 34 \\
 &= -5(t^2 - 2t + 1) + 5 + 34 \\
 &= -5(t - 1)^2 + 39 \quad V(1, 39) \\
 &\quad \underline{h_{\max} = 39\text{m}}
 \end{aligned}$$

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- 8 A lifeguard marks off a rectangular swimming area at a beach with 200 m of rope. What is the greatest area of water she can enclose if the rope only makes 3 sides of the rectangle? (No rope is needed along the shore side of the swimming area.)



$$2L + w = 200 \quad \text{then } w = 200 - 2L$$

$$\begin{aligned} \text{Area} &= L \cdot w \\ &= L(200 - 2L) \\ &= 200L - 2L^2 \end{aligned}$$

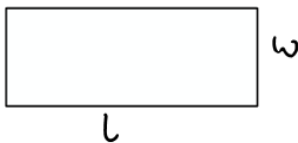
$$\begin{aligned} A(L) &= -2L^2 + 200L \\ &= -2(L^2 - 100L) \quad \frac{-(-100)}{2} = -50 \quad (-50)^2 = 2500 \\ &= -2(L^2 - 100L + 2500 - 2500) \\ &= -2(L - 50)^2 + 5000 \end{aligned}$$

$$\begin{array}{cc} \vee(50, 5000) \\ \downarrow \quad \downarrow \\ \text{length} \quad \text{area} \end{array}$$

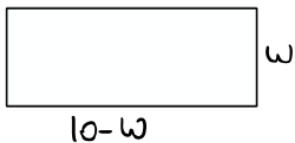
$$\begin{aligned} \text{width} &= 200 - 2L \\ &= 200 - 2(50) \\ \text{width} &= 100 \end{aligned}$$

$$\therefore \begin{aligned} \text{length} &= 50 \\ \text{width} &= 100 \end{aligned}$$

- 10 Determine the maximum possible area for a rectangle with perimeter 20.



$$\begin{aligned} P &= 2(L + w) \\ 20 &= 2(L + w) \\ 10 &= L + w, \quad \text{then } L = 10 - w \end{aligned}$$



$$\begin{aligned} A &= (10 - w)w \\ &= 10w - w^2 \\ &= -w^2 + 10w \\ &= -(w^2 - 10w) \quad \frac{-(-10)}{2} = -5 \quad (-5)^2 = 25 \\ &= -(w^2 - 10w + 25 - 25) \\ &= -(w - 5)^2 + 25 \quad \vee(5, 25) \end{aligned}$$

$$\left\{ \begin{array}{l} \text{length} = 5 \\ \text{width} = 5 \end{array} \right.$$

The max area occurs when length is 5m and width is 5m.

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2. The Thunderbirds professional indoor soccer team has 900 season ticket holders. The management of the team wants to increase the current price of \$400. A survey indicated that for every \$20 increase in price, the team will lose 15 season ticket holders. What price would maximize revenue from season ticket holders? What is the **maximum revenue** the team could receive?

$$R(x) = (900 - 15x)(400 + 20x) \quad \text{Let "x" be the \# of increase in price.}$$

$$= 360000 + 18000x - 6000x - 300x^2$$

$$= -300x^2 + 12000x + 360000$$

$$= -300(x^2 - 40x) + 360000 \quad \rightarrow \quad -\frac{40}{2} = -20 \quad (-20)^2 = 400$$

$$= -300(x^2 - 40x + 400 - 400) + 360000$$

$$= -300(x - 20)^2 + 120000 + 360000$$

$$= -300(x - 20)^2 + 480000$$

$$V(20, 480000)$$

$$\therefore \$480,000$$



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3. The school council sells sweatshirts to raise funds. The students sell 500 sweatshirts a year at \$45 each. They are planning to decrease the price to generate more sales. An informal survey was taken showing that for every \$1 decrease in price they can expect to sell an additional 20 sweatshirts. If the survey results are correct, what price would maximize revenue from sweatshirt sales? How many sweatshirts must be sold? What would be the **maximum revenue** generated?

$$\begin{aligned}R(x) &= (500 + 20x)(45 - x) \\&= 22500 - 500x + 900x - 20x^2 \\&= -20x^2 + 400x + 22500 \\&= -20(x^2 - 20x) + 22500 \rightarrow -\frac{20}{2} = -10 \quad (-10)^2 = 100 \\&= -20(x^2 - 20x + 100 - 100) + 22500 \\&= -20(x - 10)^2 + 2000 + 22500\end{aligned}$$

$$\begin{aligned}R(x) &= -20(x - 10)^2 + 24500 \\&\quad \vee(10, 24500)\end{aligned}$$

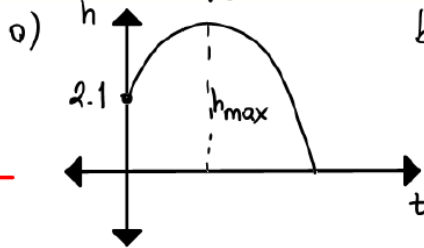
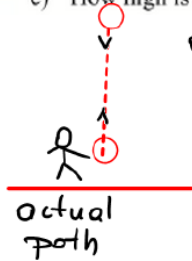
$$\begin{aligned}\text{Price} &= 45 - x \\&= 45 - 10 \\&= \$35\end{aligned}$$

∴ When the ticket price is set to \$35, the max revenue will be \$24,500.

## Maximizing and Output ... Set 3

- 6 A tennis ball is thrown up into the air. Its height  $h$  in metres after  $t$  seconds, is given by the function  
 $h = -4.9t^2 + 19.6t + 2.1$

- ✓ a) Sketch the graph.  
 ✓ b) Determine the maximum height of the ball and the time it takes to reach it.  
 c) How high is the ball after 3 s?  $V($



b) 
$$h = -4.9t^2 + 19.6t + 2.1$$

$$= -4.9(t^2 - 4t) + 2.1 \quad \left(\frac{-4}{2}\right) = -2$$

$$= -4.9(t^2 - 4t + 4 - 4) + 2.1 \quad (-2)^2 = 4$$

$$= -4.9(t-2)^2 + 19.6 + 2.1$$

$$h = -4.9(t-2)^2 + 21.7$$

$$V(2, 21.7)$$

c) 
$$h(3) = -4.9(3-2)^2 + 21.7$$

$$= -4.9(1)^2 + 21.7$$

$$= -4.9 + 21.7$$

$h(3) = 16.8$

∴ It takes 2 seconds to reach max height of 21.7 m.

## Maximizing and Output ... Set 3

14. An amusement park charges \$8 admission and averages 2000 visitors per day. A survey shows that for each \$1 increase in the admission price, 100 fewer people would visit the park.

- a) Determine what price the amusement park should charge to maximize revenue.
- b) At what price(s) will the revenue be equal to \$0?
- c) Find the price(s) that would generate revenue of \$11500.

Revenue = (# of visitors)(park fee)

$$R(x) = (2000 - 100x)(8 + x) \quad \text{let "x" be the number of times price gets increased. Expand.}$$

$$R(x) = 16000 + 2000x - 800x - 100x^2$$

$$= -100x^2 + 1200x + 16000 \quad \text{complete the square}$$

$$= -100(x^2 - 12x) + 16000 \quad \rightarrow \quad -\frac{12}{2} = -6 \quad (-b)^2 = 36$$

$$= -100(x^2 - 12x + 36 - 36) + 16000$$

$$= -100(x - 6)^2 + 3600 + 16000$$

$$R(x) = -100(x - 6)^2 + 19600$$

$$V(6, 19,600)$$

$$\begin{aligned} \text{Price} = P(x) &= 8 + x \\ &= 8 + 6 \\ &= \$14 \end{aligned}$$

∴ The fee should be \$14 to max the revenue.

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$$b) R(x) = 0$$
$$-100x^2 + 1200x + 16000 = 0$$

$$\frac{-100(x^2 - 12x - 160)}{-100} = \frac{0}{-100}$$

$$x^2 - 12x - 160 = 0$$

$$(x + 8)(x - 20) = 0$$

$$\downarrow$$
$$x + 8 = 0$$

$$x = -8$$

$$\downarrow$$
$$x - 20 = 0$$

$$x = 20$$

$$p(x) = 8 + x$$

$$p(-8) = 8 + (-8)$$

$$p(-8) = 0$$

$$\$0$$

$$p(x) = 8 + x$$

$$p(20) = 8 + 20$$

$$p(20) = 28$$

$$\$28$$

1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160

$\therefore$  When the price is set for \$0 and \$28, the revenue is \$0.