

AP Calculus AB ... 1973 MC

90 Minutes—No Calculator

Note: In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

1. $\int (x^3 - 3x) dx =$

(A) $3x^2 - 3 + C$

(B) $4x^4 - 6x^2 + C$

(C) $\frac{x^4}{3} - 3x^2 + C$

(D) $\frac{x^4}{4} - 3x + C$

(E) $\frac{x^4}{4} - \frac{3x^2}{2} + C$

2. If $f(x) = x^3 + 3x^2 + 4x + 5$ and $g(x) = 5$, then $g(f(x)) =$

(A) $5x^2 + 15x + 25$

(B) $5x^3 + 15x^2 + 20x + 25$

(C) 1125

(D) 225

(E) 5

3. The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

(A) $\frac{1}{e^2}$

(B) $\frac{2}{e^2}$

(C) $\frac{4}{e^2}$

(D) $\frac{1}{e^4}$

(E) $\frac{4}{e^4}$

4. If $f(x) = x + \sin x$, then $f'(x) =$

(A) $1 + \cos x$

(B) $1 - \cos x$

(C) $\cos x$

(D) $\sin x - x \cos x$

(E) $\sin x + x \cos x$

5. If $f(x) = e^x$, which of the following lines is an asymptote to the graph of f ?

(A) $y = 0$

(B) $x = 0$

(C) $y = x$

(D) $y = -x$

(E) $y = 1$

6. If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) -1

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

(E) 1

AP Calculus AB ... 1973 MC

Answers

1. E $\int (x^3 - 3x) dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 + C$
2. E $g(x) = 5 \Rightarrow g(f(x)) = 5$
3. B $y = \ln x^2$; $y' = \frac{2x}{x^2} = \frac{2}{x}$. At $x = e^2$, $y' = \frac{2}{e^2}$.
4. A $f(x) = x + \sin x$; $f'(x) = 1 + \cos x$
5. A $\lim_{x \rightarrow -\infty} e^x = 0 \Rightarrow y = 0$ is a horizontal asymptote
6. D $f'(x) = \frac{(1)(x+1) - (x-1)(1)}{(x+1)^2}$, $f'(1) = \frac{2}{4} = \frac{1}{2}$

AP Calculus AB ... 1973 MC

7. Which of the following equations has a graph that is symmetric with respect to the origin?

(A) $y = \frac{x+1}{x}$

(B) $y = -x^5 + 3x$

(C) $y = x^4 - 2x^2 + 6$

(D) $y = (x-1)^3 + 1$

(E) $y = (x^2 + 1)^2 - 1$

8. A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$

(B) $\frac{7}{3}$

(C) 3

(D) 7

(E) 8

9. If $y = \cos^2 3x$, then $\frac{dy}{dx} =$

(A) $-6 \sin 3x \cos 3x$

(B) $-2 \cos 3x$

(C) $2 \cos 3x$

(D) $6 \cos 3x$

(E) $2 \sin 3x \cos 3x$

10. The derivative of $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$ attains its maximum value at $x =$

(A) -1

(B) 0

(C) 1

(D) $\frac{4}{3}$

(E) $\frac{5}{3}$

11. If the line $3x - 4y = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then k is

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) 0

(D) $-\frac{1}{8}$

(E) $-\frac{1}{2}$

12. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6

(B) -3

(C) -1

(D) 2

(E) It cannot be determined from the information given.

AP Calculus AB ... 1973 MC

Answers

7. B Replace x with $(-x)$ and see if the result is the opposite of the original. This is true for B.
$$-(-x)^5 + 3(-x) = x^5 - 3x = -(-x^5 + 3x).$$
8. B Distance = $\int_1^2 |t^2| dx = \int_1^2 t^2 dt = \frac{1}{3} t^3 \Big|_1^2 = \frac{1}{3} (2^3 - 1^3) = \frac{7}{3}$
9. A $y' = 2 \cos 3x \cdot \frac{d}{dx}(\cos 3x) = 2 \cos 3x \cdot (-\sin 3x) \cdot \frac{d}{dx}(3x) = 2 \cos 3x \cdot (-\sin 3x) \cdot (3)$
$$y' = -6 \sin 3x \cos 3x$$
10. C $f(x) = \frac{x^4}{3} - \frac{x^5}{5}$; $f'(x) = \frac{4x^3}{3} - x^4$; $f''(x) = 4x^2 - 4x^3 = 4x^2(1-x)$
 $f'' > 0$ for $x < 1$ and $f'' < 0$ for $x > 1 \Rightarrow f'$ has its maximum at $x = 1$.
11. B Curve and line have the same slope when $3x^2 = \frac{3}{4} \Rightarrow x = \frac{1}{2}$. Using the line, the point of tangency is $\left(\frac{1}{2}, \frac{3}{8}\right)$. Since the point is also on the curve, $\frac{3}{8} = \left(\frac{1}{2}\right)^3 + k \Rightarrow k = \frac{1}{4}$.
12. C Substitute the points into the equation and solve the resulting linear system.
 $3 = 16 + 4A + 2B - 5$ and $-37 = -16 + 4A - 2B - 5$; $A = -3$, $B = 2 \Rightarrow A + B = -1$.

AP Calculus AB ... 1973 MC

13. The acceleration α of a body moving in a straight line is given in terms of time t by $\alpha = 8 - 6t$. If the velocity of the body is 25 at $t = 1$ and if $s(t)$ is the distance of the body from the origin at time t , what is $s(4) - s(2)$?

(A) 20 (B) 24 (C) 28 (D) 32 (E) 42

14. If $f(x) = x^{\frac{1}{3}}(x-2)^{\frac{2}{3}}$ for all x , then the domain of f' is

(A) $\{x \mid x \neq 0\}$ (B) $\{x \mid x > 0\}$ (C) $\{x \mid 0 \leq x \leq 2\}$
 (D) $\{x \mid x \neq 0 \text{ and } x \neq 2\}$ (E) $\{x \mid x \text{ is a real number}\}$

15. The area of the region bounded by the lines $x = 0$, $x = 2$, and $y = 0$ and the curve $y = e^{\frac{x}{2}}$ is

(A) $\frac{e-1}{2}$ (B) $e-1$ (C) $2(e-1)$ (D) $2e-1$ (E) $2e$

16. The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t . At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

17. What is the area of the region completely bounded by the curve $y = -x^2 + x + 6$ and the line $y = 4$?

(A) $\frac{3}{2}$ (B) $\frac{7}{3}$ (C) $\frac{9}{2}$ (D) $\frac{31}{6}$ (E) $\frac{33}{2}$

18. $\frac{d}{dx}(\arcsin 2x) =$

(A) $\frac{-1}{2\sqrt{1-4x^2}}$ (B) $\frac{-2}{\sqrt{4x^2-1}}$ (C) $\frac{1}{2\sqrt{1-4x^2}}$
 (D) $\frac{2}{\sqrt{1-4x^2}}$ (E) $\frac{2}{\sqrt{4x^2-1}}$

AP Calculus AB ... 1973 MC

Answers

13. D $v(t) = 8t - 3t^2 + C$ and $v(1) = 25 \Rightarrow C = 20$ so $v(t) = 8t - 3t^2 + 20$.
 $s(4) - s(2) = \int_2^4 v(t) dt = (4t^2 - t^3 + 20t) \Big|_2^4 = 32$
14. D $f(x) = x^{1/3} (x-2)^{2/3}$
 $f'(x) = x^{1/3} \cdot \frac{2}{3} (x-2)^{-1/3} + (x-2)^{2/3} \cdot \frac{1}{3} x^{-2/3} = \frac{1}{3} x^{-2/3} (x-2)^{-1/3} (3x-2)$
 f' is not defined at $x = 0$ and at $x = 2$.
15. C $\text{Area} = \int_0^2 e^{x/2} dx = 2e^{x/2} \Big|_0^2 = 2(e-1)$
16. C $\frac{dN}{dt} = 3000e^{2t}$, $N = 7500e^{2t} + C$ and $N(0) = 7500 \Rightarrow C = 0$
 $N = 7500e^{2t}$, $N(5) = 7500e^2$
17. C Determine where the curves intersect. $-x^2 + x + 6 = 4 \Rightarrow x^2 - x - 2 = 0$
 $(x-2)(x+1) = 0 \Rightarrow x = -1, x = 2$. Between these two x values the parabola lies above the line $y = 4$.
 $\text{Area} = \int_{-1}^2 ((-x^2 + x + 6) - 4) dx = \left(-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right) \Big|_{-1}^2 = \frac{9}{2}$
18. D $\frac{d}{dx}(\arcsin 2x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot \frac{d}{dx}(2x) = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$

AP Calculus AB ... 1973 MC

19. Suppose that f is a function that is defined for all real numbers. Which of the following conditions assures that f has an inverse function?

- (A) The function f is periodic.
 - (B) The graph of f is symmetric with respect to the y -axis.
 - (C) The graph of f is concave up.
 - (D) The function f is a strictly increasing function.
 - (E) The function f is continuous.
-

20. If F and f are continuous functions such that $F'(x) = f(x)$ for all x , then $\int_a^b f(x) dx$ is

- (A) $F'(a) - F'(b)$
 - (B) $F'(b) - F'(a)$
 - (C) $F(a) - F(b)$
 - (D) $F(b) - F(a)$
 - (E) none of the above
-

21. $\int_0^1 (x+1)e^{x^2+2x} dx =$

- (A) $\frac{e^3}{2}$ (B) $\frac{e^3-1}{2}$ (C) $\frac{e^4-e}{2}$ (D) e^3-1 (E) e^4-e
-

22. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

- (A) $x > 0$
- (B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
- (C) $-2 < x < 0$ or $x > 2$
- (D) $x > \sqrt{2}$
- (E) $-2 < x < 2$

AP Calculus AB ... 1973 MC

Answers

19. D If f is strictly increasing then it must be one to one and therefore have an inverse.
20. D By the Fundamental Theorem of Calculus, $\int_a^b f(x) dx = F(b) - F(a)$ where $F'(x) = f(x)$.
21. B
$$\int_0^1 (x+1)e^{x^2+2x} dx = \frac{1}{2} \int_0^1 e^{x^2+2x} ((2x+2) dx) = \frac{1}{2} \left(e^{x^2+2x} \right) \Big|_0^1 = \frac{1}{2} (e^3 - e^0) = \frac{e^3 - 1}{2}$$
22. B $f(x) = 3x^5 - 20x^3$; $f'(x) = 15x^4 - 60x^2$; $f''(x) = 60x^3 - 120x = 60x(x^2 - 2)$
The graph of f is concave up where $f'' > 0$: $f'' > 0$ for $x > \sqrt{2}$ and for $-\sqrt{2} < x < 0$.

AP Calculus AB ... 1973 MC

23. $\lim_{h \rightarrow 0} \frac{1}{h} \ln\left(\frac{2+h}{2}\right)$ is
- (A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) nonexistent
-

24. Let $f(x) = \cos(\arctan x)$. What is the range of f ?

- (A) $\left\{x \mid -\frac{\pi}{2} < x < \frac{\pi}{2}\right\}$ (B) $\{x \mid 0 < x \leq 1\}$ (C) $\{x \mid 0 \leq x \leq 1\}$
 (D) $\{x \mid -1 < x < 1\}$ (E) $\{x \mid -1 \leq x \leq 1\}$
-

25. $\int_0^{\pi/4} \tan^2 x \, dx =$

- (A) $\frac{\pi}{4} - 1$ (B) $1 - \frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2} - 1$ (E) $\frac{\pi}{4} + 1$
-

26. The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V ? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

- (A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π
-

27. $\int_0^{1/2} \frac{2x}{\sqrt{1-x^2}} \, dx =$

- (A) $1 - \frac{\sqrt{3}}{2}$ (B) $\frac{1}{2} \ln \frac{3}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{6} - 1$ (E) $2 - \sqrt{3}$
-

28. A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t - 3t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

- (A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

AP Calculus AB ... 1973 MC

Answers

23. C $\lim_{h \rightarrow 0} \frac{\ln(2+h) - \ln 2}{h} = f'(2)$ where $f(x) = \ln x$; $f'(x) = \frac{1}{x} \Rightarrow f'(2) = \frac{1}{2}$

24. B $f(x) = \cos(\arctan x)$; $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$ and the cosine in this domain takes on all values in the interval $(0, 1]$.

25. B $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \, dx = (\tan x - x) \Big|_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4}$

26. E $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = S \cdot \frac{dr}{dt} = 100\pi(0.3) = 30\pi$

27. E $\int_0^{\frac{1}{2}} \frac{2x}{\sqrt{1-x^2}} \, dx = -\int_0^{\frac{1}{2}} (1-x^2)^{-\frac{1}{2}} (-2x \, dx) = -2(1-x^2)^{\frac{1}{2}} \Big|_0^{\frac{1}{2}} = 2 - \sqrt{3}$

28. C $v(t) = 8 - 6t$ changes sign at $t = \frac{4}{3}$. Distance = $\left| x(1) - x\left(\frac{4}{3}\right) \right| + \left| x(2) - x\left(\frac{4}{3}\right) \right| = \frac{5}{3}$.

Alternative Solution: Distance = $\int_1^2 |v(t)| \, dt = \int_1^2 |8 - 6t| \, dt = \frac{5}{3}$

AP Calculus AB ... 1973 MC

29. Let $f(x) = \left| \sin x - \frac{1}{2} \right|$. The maximum value attained by f is

- (A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{\pi}{2}$ (E) $\frac{3\pi}{2}$
-

30. $\int_1^2 \frac{x-4}{x^2} dx =$

- (A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$
-

31. If $\log_a(2^a) = \frac{a}{4}$, then $a =$

- (A) 2 (B) 4 (C) 8 (D) 16 (E) 32
-

32. $\int \frac{5}{1+x^2} dx =$

- (A) $\frac{-10x}{(1+x^2)^2} + C$ (B) $\frac{5}{2x} \ln(1+x^2) + C$ (C) $5x - \frac{5}{x} + C$
 (D) $5 \arctan x + C$ (E) $5 \ln(1+x^2) + C$
-

33. Suppose that f is an odd function; i.e., $f(-x) = -f(x)$ for all x . Suppose that $f'(x_0)$ exists. Which of the following must necessarily be equal to $f'(-x_0)$?

- (A) $f'(x_0)$
 (B) $-f'(x_0)$
 (C) $\frac{1}{f'(x_0)}$
 (D) $\frac{-1}{f'(x_0)}$
 (E) None of the above

AP Calculus AB ... 1973 MC

Answers

29. C $-1 \leq \sin x \leq 1 \Rightarrow -\frac{3}{2} \leq \sin x - \frac{1}{2} \leq \frac{1}{2}$; The maximum for $\left| \sin x - \frac{1}{2} \right|$ is $\frac{3}{2}$.
30. B $\int_1^2 \frac{x-4}{x^2} dx = \int_1^2 \left(\frac{1}{x} - 4x^{-2} \right) dx = \left(\ln x + \frac{4}{x} \right) \Big|_1^2 = (\ln 2 + 2) - (\ln 1 + 4) = \ln 2 - 2$
31. D $\log_a(2^a) = \frac{a}{4} \Rightarrow \log_a 2 = \frac{1}{4} \Rightarrow 2 = a^{\frac{1}{4}}; a = 16$
32. D $\int \frac{5}{1+x^2} dx = 5 \int \frac{1}{1+x^2} dx = 5 \tan^{-1}(x) + C$
33. A $f(-x) = -f(x) \Rightarrow f'(-x) \cdot (-1) = -f'(x) \Rightarrow f'(-x) = -f'(x)$ thus $f'(-x_0) = -f'(x_0)$.

AP Calculus AB ... 1973 MC

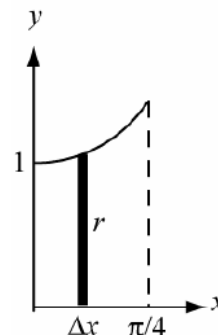
34. The average value of \sqrt{x} over the interval $0 \leq x \leq 2$ is
- (A) $\frac{1}{3}\sqrt{2}$ (B) $\frac{1}{2}\sqrt{2}$ (C) $\frac{2}{3}\sqrt{2}$ (D) 1 (E) $\frac{4}{3}\sqrt{2}$
-
35. The region in the first quadrant bounded by the graph of $y = \sec x$, $x = \frac{\pi}{4}$, and the axes is rotated about the x -axis. What is the volume of the solid generated?
- (A) $\frac{\pi^2}{4}$ (B) $\pi - 1$ (C) π (D) 2π (E) $\frac{8\pi}{3}$
-
36. If $y = e^{nx}$, then $\frac{d^n y}{dx^n} =$
- (A) $n^n e^{nx}$ (B) $n!e^{nx}$ (C) ne^{nx} (D) $n^n e^x$ (E) $n!e^x$
-
37. If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$
- (A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$
-
38. If $\int_1^2 f(x-c) dx = 5$ where c is a constant, then $\int_{1-c}^{2-c} f(x) dx =$
- (A) $5 + c$ (B) 5 (C) $5 - c$ (D) $c - 5$ (E) -5
-
39. The point on the curve $2y = x^2$ nearest to $(4, 1)$ is
- (A) $(0, 0)$ (B) $(2, 2)$ (C) $(\sqrt{2}, 1)$ (D) $(2\sqrt{2}, 4)$ (E) $(4, 8)$
-
40. If $\tan(xy) = x$, then $\frac{dy}{dx} =$
- (A) $\frac{1 - y \tan(xy) \sec(xy)}{x \tan(xy) \sec(xy)}$ (B) $\frac{\sec^2(xy) - y}{x}$ (C) $\cos^2(xy)$
- (D) $\frac{\cos^2(xy)}{x}$ (E) $\frac{\cos^2(xy) - y}{x}$

AP Calculus AB ... 1973 MC

Answers

34. C $\frac{1}{2} \int_0^2 \sqrt{x} dx = \frac{1}{2} \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^2 = \frac{1}{3} \cdot 2^{\frac{3}{2}} = \frac{2}{3} \sqrt{2}$

35. C Washers: $\sum \pi r^2 \Delta x$ where $r = y = \sec x$.
 Volume = $\pi \int_0^{\frac{\pi}{4}} \sec^2 x dx = \pi \tan x \Big|_0^{\pi/4} = \pi(\tan \frac{\pi}{4} - \tan 0) = \pi$



36. A $y = e^{nx}$, $y' = ne^{nx}$, $y'' = n^2 e^{nx}$, ..., $y^{(n)} = n^n e^{nx}$

37. A $\frac{dy}{dx} = 4y$, $y(0) = 4$. This is exponential growth. The general solution is $y = Ce^{4x}$. Since $y(0) = 4$, $C = 4$ and so the solution is $y = 4e^{4x}$.

38. B Let $z = x - c$. Then $5 = \int_1^2 f(x - c) dx = \int_{1-c}^{2-c} f(z) dz$

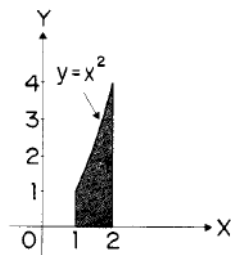
39. B Use the distance formula to determine the distance, L , from any point $(x, y) = (x, \frac{1}{2}x^2)$ on the curve to the point $(4, 1)$. The distance L satisfies the equation $L^2 = (x - 4)^2 + \left(\frac{1}{2}x^2 - 1\right)$. Determine where L is a maximum by examining critical points. Differentiating with respect to x , $2L \cdot \frac{dL}{dx} = 2(x - 4) + 2\left(\frac{1}{2}x^2 - 1\right)x = x^3 - 8$. $\frac{dL}{dx}$ changes sign from positive to negative at $x = 2$ only. The point on the curve has coordinates $(2, 2)$.

40. E $\sec^2(xy) \cdot (xy' + y) = 1$, $xy' \sec^2(xy) + y \sec^2(xy) = 1$, $y' = \frac{1 - y \sec^2(xy)}{x \sec^2(xy)} = \frac{\cos^2(xy) - y}{x}$

AP Calculus AB ... 1973 MC

41. Given $f(x) = \begin{cases} x+1 & \text{for } x < 0, \\ \cos \pi x & \text{for } x \geq 0, \end{cases}$ $\int_{-1}^1 f(x) dx =$

- (A) $\frac{1}{2} + \frac{1}{\pi}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2} - \frac{1}{\pi}$ (D) $\frac{1}{2}$ (E) $-\frac{1}{2} + \pi$
-



42. Calculate the approximate area of the shaded region in the figure by the trapezoidal rule, using divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$.

- (A) $\frac{50}{27}$ (B) $\frac{251}{108}$ (C) $\frac{7}{3}$ (D) $\frac{127}{54}$ (E) $\frac{77}{27}$
-

43. If the solutions of $f(x) = 0$ are -1 and 2 , then the solutions of $f\left(\frac{x}{2}\right) = 0$ are

- (A) -1 and 2 (B) $-\frac{1}{2}$ and $\frac{5}{2}$ (C) $-\frac{3}{2}$ and $\frac{3}{2}$
 (D) $-\frac{1}{2}$ and 1 (E) -2 and 4
-

44. For small values of h , the function $\sqrt[4]{16+h}$ is best approximated by which of the following?

- (A) $4 + \frac{h}{32}$ (B) $2 + \frac{h}{32}$ (C) $\frac{h}{32}$
 (D) $4 - \frac{h}{32}$ (E) $2 - \frac{h}{32}$

AP Calculus AB ... 1973 MC

Answers

$$\begin{aligned} 41. \quad D \quad \int_{-1}^1 f(x) dx &= \int_{-1}^0 (x+1) dx + \int_0^1 \cos(\pi x) dx = \frac{1}{2}(x+1)^2 \Big|_{-1}^0 + \frac{1}{\pi} \sin(\pi x) \Big|_0^1 \\ &= \frac{1}{2} + \frac{1}{\pi} (\sin \pi - \sin 0) = \frac{1}{2} \end{aligned}$$

$$42. \quad D \quad \Delta x = \frac{1}{3}; \quad T = \frac{1}{2} \cdot \frac{1}{3} \left(1^2 + 2 \left(\frac{4}{3} \right)^2 + 2 \left(\frac{5}{3} \right)^2 + 2^2 \right) = \frac{127}{54}$$

$$43. \quad E \quad \text{Solve } \frac{x}{2} = -1 \text{ and } \frac{x}{2} = 2; \quad x = -2, 4$$

$$\begin{aligned} 44. \quad B \quad \text{Use the linearization of } f(x) = \sqrt[4]{x} \text{ at } x = 16. \quad f'(x) = \frac{1}{4} x^{-\frac{3}{4}}, \quad f'(16) = \frac{1}{32} \\ L(x) = 2 + \frac{1}{32}(x-16); \quad f(16+h) \approx L(16+h) = 2 + \frac{h}{32} \end{aligned}$$

AP Calculus AB ... 1973 MC

45. If f is a continuous function on $[a, b]$, which of the following is necessarily true?

- (A) f' exists on (a, b) .
- (B) If $f(x_0)$ is a maximum of f , then $f'(x_0) = 0$.
- (C) $\lim_{x \rightarrow x_0} f(x) = f\left(\lim_{x \rightarrow x_0} x\right)$ for $x_0 \in (a, b)$
- (D) $f'(x) = 0$ for some $x \in [a, b]$
- (E) The graph of f' is a straight line.

AP Calculus AB ... 1973 MC

Answers

45. C This uses the definition of continuity of f at $x = x_0$.