

AP Calculus AB ... 1985 MC

90 Minutes—No Calculator

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_1^2 x^{-3} dx =$

- (A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$
-

2. If $f(x) = (2x+1)^4$, then the 4th derivative of $f(x)$ at $x=0$ is

- (A) 0 (B) 24 (C) 48 (D) 240 (E) 384
-

3. If $y = \frac{3}{4+x^2}$, then $\frac{dy}{dx} =$

- (A) $\frac{-6x}{(4+x^2)^2}$ (B) $\frac{3x}{(4+x^2)^2}$ (C) $\frac{6x}{(4+x^2)^2}$ (D) $\frac{-3}{(4+x^2)^2}$ (E) $\frac{3}{2x}$
-

4. If $\frac{dy}{dx} = \cos(2x)$, then $y =$

- (A) $-\frac{1}{2}\cos(2x) + C$ (B) $-\frac{1}{2}\cos^2(2x) + C$ (C) $\frac{1}{2}\sin(2x) + C$
(D) $\frac{1}{2}\sin^2(2x) + C$ (E) $-\frac{1}{2}\sin(2x) + C$
-

5. $\lim_{n \rightarrow \infty} \frac{4n^2}{n^2 + 10,000n}$ is

- (A) 0 (B) $\frac{1}{2,500}$ (C) 1 (D) 4 (E) nonexistent

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Answers

1. D $\int_1^2 x^{-3} dx = -\frac{1}{2}x^{-2} \Big|_1^2 = -\frac{1}{2}\left(\frac{1}{4} - 1\right) = \frac{3}{8}.$

2. E $f'(x) = 4(2x+1)^3 \cdot 2, f''(1) = 4 \cdot 3(2x+1)^2 \cdot 2^2, f'''(1) = 4 \cdot 3 \cdot 2(2x+1)^1 \cdot 2^3,$
 $f^{(4)}(1) = 4! \cdot 2^4 = 384$

3. A $y = 3(4+x^2)^{-1}$ so $y' = -3(4+x^2)^{-2}(2x) = \frac{-6x}{(4+x^2)^2}$

Or using the quotient rule directly gives $y' = \frac{(4+x^2)(0) - 3(2x)}{(4+x^2)^2} = \frac{-6x}{(4+x^2)^2}$

4. C $\int \cos(2x) dx = \frac{1}{2} \int \cos(2x)(2 dx) = \frac{1}{2} \sin(2x) + C$

5. D $\lim_{n \rightarrow \infty} \frac{4n^2}{n^2 + 10000n} = \lim_{n \rightarrow \infty} \frac{4}{1 + \frac{10000}{n}} = 4$

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6. If $f(x) = x$, then $f'(5) =$

- (A) 0 (B) $\frac{1}{5}$ (C) 1 (D) 5 (E) $\frac{25}{2}$
-

7. Which of the following is equal to $\ln 4$?

- (A) $\ln 3 + \ln 1$ (B) $\frac{\ln 8}{\ln 2}$ (C) $\int_1^4 e^t dt$ (D) $\int_1^4 \ln x dx$ (E) $\int_1^4 \frac{1}{t} dt$
-

8. The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

- (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4
-

9. If $\int_{-1}^1 e^{-x^2} dx = k$, then $\int_{-1}^0 e^{-x^2} dx =$

- (A) $-2k$ (B) $-k$ (C) $-\frac{k}{2}$ (D) $\frac{k}{2}$ (E) $2k$
-

10. If $y = 10^{(x^2-1)}$, then $\frac{dy}{dx} =$

- (A) $(\ln 10)10^{(x^2-1)}$ (B) $(2x)10^{(x^2-1)}$ (C) $(x^2-1)10^{(x^2-2)}$
 (D) $2x(\ln 10)10^{(x^2-1)}$ (E) $x^2(\ln 10)10^{(x^2-1)}$
-

11. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

- (A) 0 (B) 2 (C) 4 (D) 8 (E) 12
-

12. If $f(g(x)) = \ln(x^2 + 4)$, $f(x) = \ln(x^2)$, and $g(x) > 0$ for all real x , then $g(x) =$

- (A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

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Answers

6. C $f'(x) = 1 \Rightarrow f'(5) = 1$

7. E $\int_1^4 \frac{1}{t} dt = \ln t \Big|_1^4 = \ln 4 - \ln 1 = \ln 4$

8. B $y = \ln\left(\frac{x}{2}\right) = \ln x - \ln 2$, $y' = \frac{1}{x}$, $y'(4) = \frac{1}{4}$

9. D Since e^{-x^2} is even, $\int_{-1}^0 e^{-x^2} dx = \frac{1}{2} \int_{-1}^1 e^{-x^2} dx = \frac{1}{2}k$

10. D $y' = 10^{(x^2-1)} \cdot \ln(10) \cdot \frac{d}{dx}(x^2-1) = 2x \cdot 10^{(x^2-1)} \cdot \ln(10)$

11. B $v(t) = 2t + 4 \Rightarrow a(t) = 2 \therefore a(4) = 2$

12. C $f(g(x)) = \ln(g(x)^2) = \ln(x^2 + 4) \Rightarrow g(x) = \sqrt{x^2 + 4}$

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13. If $x^2 + xy + y^3 = 0$, then, in terms of x and y , $\frac{dy}{dx} =$
- (A) $-\frac{2x+y}{x+3y^2}$ (B) $-\frac{x+3y^2}{2x+y}$ (C) $\frac{-2x}{1+3y^2}$ (D) $\frac{-2x}{x+3y^2}$ (E) $-\frac{2x+y}{x+3y^2-1}$
-
14. The velocity of a particle moving on a line at time t is $v = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ meters per second. How many meters did the particle travel from $t = 0$ to $t = 4$?
- (A) 32 (B) 40 (C) 64 (D) 80 (E) 184
-
15. The domain of the function defined by $f(x) = \ln(x^2 - 4)$ is the set of all real numbers x such that
- (A) $|x| < 2$ (B) $|x| \leq 2$ (C) $|x| > 2$ (D) $|x| \geq 2$ (E) x is a real number
-
16. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$
- (A) -2 (B) 0 (C) 1 (D) 2 (E) 4
-
17. $\int_0^1 xe^{-x} dx =$
- (A) $1 - 2e$ (B) -1 (C) $1 - 2e^{-1}$ (D) 1 (E) $2e - 1$
-
18. If $y = \cos^2 x - \sin^2 x$, then $y' =$
- (A) -1 (B) 0 (C) $-2\sin(2x)$ (D) $-2(\cos x + \sin x)$ (E) $2(\cos x - \sin x)$
-
19. If $f(x_1) + f(x_2) = f(x_1 + x_2)$ for all real numbers x_1 and x_2 , which of the following could define f ?
- (A) $f(x) = x + 1$ (B) $f(x) = 2x$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = e^x$ (E) $f(x) = x^2$

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Answers

13. A $2x + x \cdot y' + y + 3y^2 \cdot y' = 0 \Rightarrow y' = -\frac{2x + y}{x + 3y^2}$
14. D Since $v(t) \geq 0$, distance $= \int_0^4 |v(t)| dt = \int_0^4 \left(3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}\right) dt = \left(2t^{\frac{3}{2}} + 2t^{\frac{5}{2}}\right) \Big|_0^4 = 80$
15. C $x^2 - 4 > 0 \Rightarrow |x| > 2$
16. B $f'(x) = 3x^2 - 6x = 3x(x - 2)$ changes sign from positive to negative only at $x = 0$.
17. C Use the technique of antiderivatives by parts:
 $u = x \quad dv = e^{-x} dx$
 $du = dx \quad v = -e^{-x}$
 $-xe^{-x} + \int e^{-x} dx = \left(-xe^{-x} - e^{-x}\right) \Big|_0^1 = 1 - 2e^{-1}$
18. C $y = \cos^2 x - \sin^2 x = \cos 2x, \quad y' = -2 \sin 2x$
19. B Quick solution: lines through the origin have this property.
Or, $f(x_1) + f(x_2) = 2x_1 + 2x_2 = 2(x_1 + x_2) = f(x_1 + x_2)$

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20. If $y = \arctan(\cos x)$, then $\frac{dy}{dx} =$
- (A) $\frac{-\sin x}{1 + \cos^2 x}$ (B) $-(\operatorname{arcsec}(\cos x))^2 \sin x$ (C) $(\operatorname{arcsec}(\cos x))^2$
- (D) $\frac{1}{(\arccos x)^2 + 1}$ (E) $\frac{1}{1 + \cos^2 x}$
-

21. If the domain of the function f given by $f(x) = \frac{1}{1-x^2}$ is $\{x: |x| > 1\}$, what is the range of f ?
- (A) $\{x: -\infty < x < -1\}$ (B) $\{x: -\infty < x < 0\}$ (C) $\{x: -\infty < x < 1\}$
- (D) $\{x: -1 < x < \infty\}$ (E) $\{x: 0 < x < \infty\}$
-

22. $\int_1^2 \frac{x^2 - 1}{x + 1} dx =$
- (A) $\frac{1}{2}$ (B) 1 (C) 2 (D) $\frac{5}{2}$ (E) $\ln 3$
-

23. $\frac{d}{dx} \left(\frac{1}{x^3} - \frac{1}{x} + x^2 \right)$ at $x = -1$ is
- (A) -6 (B) -4 (C) 0 (D) 2 (E) 6
-

24. If $\int_{-2}^2 (x^7 + k) dx = 16$, then $k =$
- (A) -12 (B) -4 (C) 0 (D) 4 (E) 12
-

25. If $f(x) = e^x$, which of the following is equal to $f'(e)$?
- (A) $\lim_{h \rightarrow 0} \frac{e^{x+h}}{h}$ (B) $\lim_{h \rightarrow 0} \frac{e^{x+h} - e^e}{h}$ (C) $\lim_{h \rightarrow 0} \frac{e^{e+h} - e}{h}$
- (D) $\lim_{h \rightarrow 0} \frac{e^{x+h} - 1}{h}$ (E) $\lim_{h \rightarrow 0} \frac{e^{e+h} - e^e}{h}$

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20. A $\frac{dy}{dx} = \frac{1}{1 + \cos^2 x} \cdot \frac{d}{dx}(\cos x) = \frac{-\sin x}{1 + \cos^2 x}$

21. B $|x| > 1 \Rightarrow x^2 > 1 \Rightarrow f(x) < 0$ for all x in the domain. $\lim_{|x| \rightarrow \infty} f(x) = 0$. $\lim_{|x| \rightarrow 1} f(x) = -\infty$. The only option that is consistent with these statements is (B).

22. A $\int_1^2 \frac{x^2 - 1}{x + 1} dx = \int_1^2 \frac{(x+1)(x-1)}{x+1} dx = \int_1^2 (x-1) dx = \frac{1}{2}(x-1)^2 \Big|_1^2 = \frac{1}{2}$

23. B $\frac{d}{dx}(x^{-3} - x^{-1} + x^2) \Big|_{x=-1} = (-3x^{-4} + x^{-2} + 2x) \Big|_{x=-1} = -3 + 1 - 2 = -4$

24. D $16 = \int_{-2}^2 (x^7 + k) dx = \int_{-2}^2 x^7 dx + \int_{-2}^2 k dx = 0 + (2 - (-2))k = 4k \Rightarrow k = 4$

25. E $f'(e) = \lim_{h \rightarrow 0} \frac{f(e+h) - f(e)}{h} = \lim_{h \rightarrow 0} \frac{e^{e+h} - e^e}{h}$

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26. The graph of $y^2 = x^2 + 9$ is symmetric to which of the following?

- I. The x -axis
- II. The y -axis
- III. The origin

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

27. $\int_0^3 |x-1| dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) $\frac{5}{2}$ (E) 6

28. If the position of a particle on the x -axis at time t is $-5t^2$, then the average velocity of the particle for $0 \leq t \leq 3$ is

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

29. Which of the following functions are continuous for all real numbers x ?

- I. $y = x^{\frac{2}{3}}$
- II. $y = e^x$
- III. $y = \tan x$

(A) None (B) I only (C) II only (D) I and II (E) I and III

30. $\int \tan(2x) dx =$

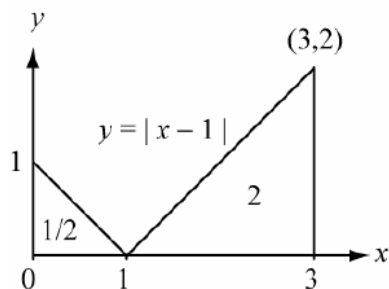
(A) $-2 \ln |\cos(2x)| + C$ (B) $-\frac{1}{2} \ln |\cos(2x)| + C$ (C) $\frac{1}{2} \ln |\cos(2x)| + C$
(D) $2 \ln |\cos(2x)| + C$ (E) $\frac{1}{2} \sec(2x) \tan(2x) + C$

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Answers

26. E I: Replace y with $(-y)$: $(-y)^2 = x^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
 II: Replace x with $(-x)$: $y^2 = (-x)^2 + 9 \Rightarrow y^2 = x^2 + 9$, no change, so yes.
 III: Since there is symmetry with respect to both axes there is origin symmetry.

27. D The graph is a V with vertex at $x = 1$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for x from 0 to 3. These triangles have areas of $1/2$ and 2 respectively.



28. C Let $x(t) = -5t^2$ be the position at time t . Average velocity $= \frac{x(3) - x(0)}{3 - 0} = \frac{-45 - 0}{3} = -15$
29. D The tangent function is not defined at $x = \pi/2$ so it cannot be continuous for all real numbers. Option E is the only one that includes item III. In fact, the functions in I and II are a power and an exponential function that are known to be continuous for all real numbers x .
30. B
$$\int \tan(2x) dx = -\frac{1}{2} \int \frac{-2 \sin(2x)}{\cos(2x)} dx = -\frac{1}{2} \ln |\cos(2x)| + C$$

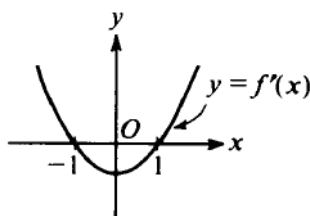
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31. The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

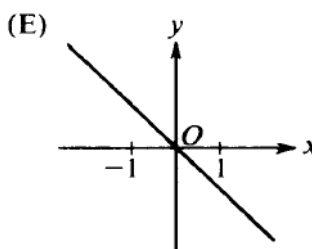
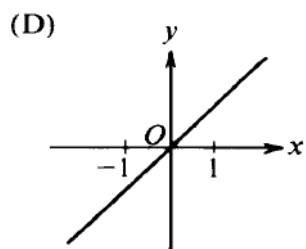
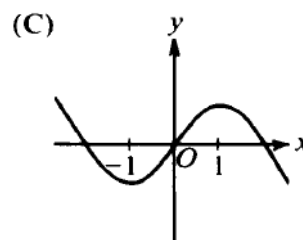
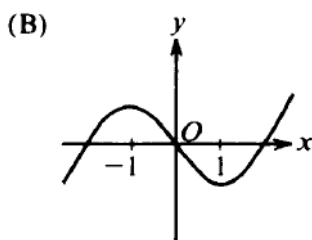
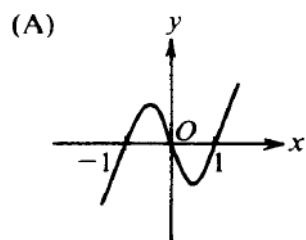
(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

32. $\int_0^{\frac{\pi}{3}} \sin(3x) dx =$

(A) -2 (B) $-\frac{2}{3}$ (C) 0 (D) $\frac{2}{3}$ (E) 2



33. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f ?



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Answers

31. C $V = \frac{1}{3}\pi r^2 h, \frac{dV}{dt} = \frac{1}{3}\pi \left(2rh \frac{dr}{dt} + r^2 \frac{dh}{dt} \right) = \frac{1}{3}\pi \left(2(6)(9) \left(\frac{1}{2} \right) + 6^2 \left(\frac{1}{2} \right) \right) = 24\pi$

32. D $\int_0^{\pi/3} \sin(3x) dx = -\frac{1}{3} \cos(3x) \Big|_0^{\pi/3} = -\frac{1}{3} (\cos \pi - \cos 0) = \frac{2}{3}$

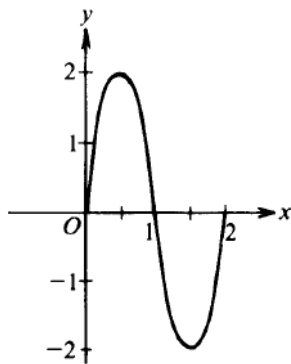
33. B f' changes sign from positive to negative at $x = -1$ and therefore f changes from increasing to decreasing at $x = -1$.

Or f' changes sign from positive to negative at $x = -1$ and from negative to positive at $x = 1$. Therefore f has a local maximum at $x = -1$ and a local minimum at $x = 1$.

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34. The area of the region in the first quadrant that is enclosed by the graphs of $y = x^3 + 8$ and $y = x + 8$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1 (E) $\frac{65}{4}$



35. The figure above shows the graph of a sine function for one complete period. Which of the following is an equation for the graph?

(A) $y = 2 \sin\left(\frac{\pi}{2}x\right)$ (B) $y = \sin(\pi x)$ (C) $y = 2 \sin(2x)$
 (D) $y = 2 \sin(\pi x)$ (E) $y = \sin(2x)$

36. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7 , then which of the following must be true?

- I. The maximum value of $f(|x|)$ is 5.
- II. The maximum value of $|f(x)|$ is 7.
- III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

37. $\lim_{x \rightarrow 0} (x \csc x)$ is

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

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Answers

34. A $\int_0^1 ((x+8) - (x^3+8)) dx = \int_0^1 (x-x^3) dx = \left(\frac{1}{2}x^2 - \frac{1}{4}x^4 \right) \Big|_0^1 = \frac{1}{4}$

35. D The amplitude is 2 and the period is 2.

$$y = A \sin Bx \text{ where } |A| = \text{amplitude} = 2 \text{ and } B = \frac{2\pi}{\text{period}} = \frac{2\pi}{2} = \pi$$

36. B II is true since $|-7| = 7$ will be the maximum value of $|f(x)|$. To see why I and III do not

have to be true, consider the following: $f(x) = \begin{cases} 5 & \text{if } x \leq -5 \\ -x & \text{if } -5 < x < 7 \\ -7 & \text{if } x \geq 7 \end{cases}$

For $f(|x|)$, the maximum is 0 and the minimum is -7 .

37. D $\lim_{x \rightarrow 0} x \csc x = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$

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38. Let f and g have continuous first and second derivatives everywhere. If $f(x) \leq g(x)$ for all real x , which of the following must be true?

- I. $f'(x) \leq g'(x)$ for all real x
- II. $f''(x) \leq g''(x)$ for all real x
- III. $\int_0^1 f(x) dx \leq \int_0^1 g(x) dx$

(A) None (B) I only (C) III only (D) I and II only (E) I, II, and III

39. If $f(x) = \frac{\ln x}{x}$, for all $x > 0$, which of the following is true?

- (A) f is increasing for all x greater than 0.
 - (B) f is increasing for all x greater than 1.
 - (C) f is decreasing for all x between 0 and 1.
 - (D) f is decreasing for all x between 1 and e .
 - (E) f is decreasing for all x greater than e .
-

40. Let f be a continuous function on the closed interval $[0, 2]$. If $2 \leq f(x) \leq 4$, then the greatest possible value of $\int_0^2 f(x) dx$ is

- (A) 0 (B) 2 (C) 4 (D) 8 (E) 16
-

41. If $\lim_{x \rightarrow a} f(x) = L$, where L is a real number, which of the following must be true?

- (A) $f'(a)$ exists.
- (B) $f(x)$ is continuous at $x = a$.
- (C) $f(x)$ is defined at $x = a$.
- (D) $f(a) = L$
- (E) None of the above

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38. C To see why I and II do not have to be true consider $f(x) = \sin x$ and $g(x) = 1 + e^x$. Then $f(x) \leq g(x)$ but neither $f'(x) \leq g'(x)$ nor $f''(x) < g''(x)$ is true for all real values of x .

III is true, since

$$f(x) \leq g(x) \Rightarrow g(x) - f(x) \geq 0 \Rightarrow \int_0^1 (g(x) - f(x)) dx \geq 0 \Rightarrow \int_0^1 f(x) dx \leq \int_0^1 g(x) dx$$

39. E $f'(x) = \frac{1}{x} \cdot \frac{1}{x} - \frac{1}{x^2} \ln x = \frac{1}{x^2} (1 - \ln x) < 0$ for $x > e$. Hence f is decreasing. for $x > e$.

40. D $\int_0^2 f(x) dx \leq \int_0^2 4 dx = 8$

41. E Consider the function whose graph is the horizontal line $y = 2$ with a hole at $x = a$. For this function $\lim_{x \rightarrow a} f(x) = 2$ and none of the given statements are true.

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42. $\frac{d}{dx} \int_2^x \sqrt{1+t^2} dt =$

(A) $\frac{x}{\sqrt{1+x^2}}$

(B) $\sqrt{1+x^2} - 5$

(C) $\sqrt{1+x^2}$

(D) $\frac{x}{\sqrt{1+x^2}} - \frac{1}{\sqrt{5}}$

(E) $\frac{1}{2\sqrt{1+x^2}} - \frac{1}{2\sqrt{5}}$

43. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$

(B) $y = -3x + 1$

(C) $y = 2x + 10$

(D) $y = 3x - 1$

(E) $y = 4x + 1$

44. The average value of $f(x) = x^2\sqrt{x^3+1}$ on the closed interval $[0, 2]$ is

(A) $\frac{26}{9}$

(B) $\frac{13}{3}$

(C) $\frac{26}{3}$

(D) 13

(E) 26

45. The region enclosed by the graph of $y = x^2$, the line $x = 2$, and the x -axis is revolved about the y -axis. The volume of the solid generated is

(A) 8π

(B) $\frac{32}{5}\pi$

(C) $\frac{16}{3}\pi$

(D) 4π

(E) $\frac{8}{3}\pi$

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42. C This is a direct application of the Fundamental Theorem of Calculus: $f'(x) = \sqrt{1+x^2}$

43. B $y' = 3x^2 + 6x$, $y'' = 6x + 6 = 0$ for $x = -1$. $y'(-1) = -3$. Only option B has a slope of -3 .

44. A $\frac{1}{2} \int_0^2 x^2 (x^3 + 1)^{1/2} dx = \frac{1}{2} \cdot \frac{1}{3} \int_0^2 (x^3 + 1)^{1/2} (3x^2 dx) = \frac{1}{6} (x^3 + 1)^{3/2} \cdot \frac{2}{3} \Big|_0^2 = \frac{26}{9}$

45. A Washers: $\sum \pi(R^2 - r^2) \Delta y$ where $R = 2$, $r = x$

$$\text{Volume} = \pi \int_0^4 (2^2 - x^2) dy = \pi \int_0^4 (4 - y) dy = \pi \left(4y - \frac{1}{2} y^2 \right) \Big|_0^4 = 8\pi$$

