AP Calculus AB … 1988 MC

90 Minutes—No Calculator

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e).

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $y = x^2 e^x$, then $\dfrac{dy}{dx} =$

(A) $2xe^x$ (B) $x\left(x + 2e^x\right)$ (C) $xe^x(x+2)$

(D) $2x + e^x$ (E) $2x + e$

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2. What is the domain of the function f given by $f(x) = \dfrac{\sqrt{x^2 - 4}}{x - 3}$?

(A) $\{x:\ x \neq 3\}$ (B) $\{x:\ |x| \leq 2\}$ (C) $\{x:\ |x| \geq 2\}$

(D) $\{x:\ |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x:\ x \geq 2 \text{ and } x \neq 3\}$

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3. A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\dfrac{e^3}{3}$

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4. The graph of $y = \dfrac{-5}{x - 2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

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5. $\int \sec^2 x\, dx =$

(A) $\tan x + C$ (B) $\csc^2 x + C$ (C) $\cos^2 x + C$

(D) $\dfrac{\sec^3 x}{3} + C$ (E) $2\sec^2 x \tan x + C$

AP Calculus AB … 1988 MC

Answers

1. C $\frac{dy}{dx} = x^2 \cdot \frac{d}{dx}(e^x) + e^x \cdot \frac{d}{dx}(x^2) = x^2e^x + 2xe^x = xe^x(x+2)$

2. D $x^2 - 4 \geq 0$ and $x \neq 3 \Rightarrow |x| \geq 2$ and $x \neq 3$

3. A Distance $= \int_0^2 |v(t)|\, dt = \int_0^2 e^t\, dt = e^t \Big|_0^2 = e^2 - e^0 = e^2 - 1$

4. E Students should know what the graph looks like without a calculator and choose option E.
Or $y = -5(x-2)^{-1}$; $y' = 5(x-2)^{-2}$; $y'' = -10(x-2)^{-3}$. $y'' < 0$ for $x > 2$.

5. A $\int \sec^2 x\, dx = \int d(\tan x) = \tan x + C$

6. If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

(A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

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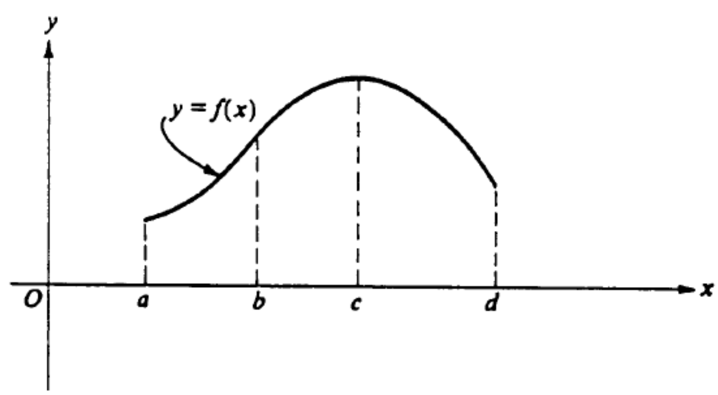
7. $\int \frac{x\,dx}{\sqrt{3x^2+5}} =$

(A) $\frac{1}{9}\left(3x^2+5\right)^{\frac{3}{2}} + C$ (B) $\frac{1}{4}\left(3x^2+5\right)^{\frac{3}{2}} + C$ (C) $\frac{1}{12}\left(3x^2+5\right)^{\frac{1}{2}} + C$

(D) $\frac{1}{3}\left(3x^2+5\right)^{\frac{1}{2}} + C$ (E) $\frac{3}{2}\left(3x^2+5\right)^{\frac{1}{2}} + C$



8. The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

Answers

6. D $\dfrac{dy}{dx}=\dfrac{x\cdot\dfrac{d}{dx}(\ln x)-\ln x\cdot\dfrac{d}{dx}(x)}{x^2}=\dfrac{x\cdot\left(\dfrac{1}{x}\right)-\ln x\cdot(1)}{x^2}=\dfrac{1-\ln x}{x^2}$

7. D $\int x(3x^2+5)^{-\frac{1}{2}}\,dx=\frac{1}{6}\int(3x^2+5)^{-\frac{1}{2}}(6x\,dx)=\frac{1}{6}\cdot 2(3x^2+5)^{\frac{1}{2}}+C=\frac{1}{3}(3x^2+5)^{\frac{1}{2}}+C$

8. B $\dfrac{dy}{dx}>0\Rightarrow y$ is increasing; $\dfrac{d^2y}{dx^2}<0\Rightarrow$ graph is concave down. This is only on $b<x<c$.

9. If $x+2xy-y^2=2$, then at the point $(1,1)$, $\frac{dy}{dx}$ is

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{3}{2}$ (E) nonexistent

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10. If $\int_0^k \left(2kx-x^2\right)dx=18$, then $k=$

(A) -9 (B) -3 (C) 3 (D) 9 (E) 18

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11. An equation of the line tangent to the graph of $f(x)=x(1-2x)^3$ at the point $(1,-1)$ is

(A) $y=-7x+6$ (B) $y=-6x+5$ (C) $y=-2x+1$

(D) $y=2x-3$ (E) $y=7x-8$

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12. If $f(x)=\sin x$, then $f'\left(\frac{\pi}{3}\right)=$

(A) $-\frac{1}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{3}}{2}$ (E) $\sqrt{3}$

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13. If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx=$

(A) $f(c)-f(0)$ (B) $|f(c)-f(0)|$ (C) $f(c)$ (D) $f(x)+c$ (E) $f''(c)-f''(0)$

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14. $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{1+\sin\theta}}\,d\theta=$

(A) $-2\left(\sqrt{2}-1\right)$ (B) $-2\sqrt{2}$ (C) $2\sqrt{2}$

(D) $2\left(\sqrt{2}-1\right)$ (E) $2\left(\sqrt{2}+1\right)$

Answers

9. E $1+(2x\cdot y'+2y)-2y\cdot y'=0;\ y'=\dfrac{1+2y}{2y-2x}$. This cannot be evaluated at $(1,1)$ and so y' does not exist at $(1,1)$.

10. C $18=\left(kx^2-\frac{1}{3}x^3\right)\Big|_0^k=\frac{2}{3}k^3\Rightarrow k^3=27,\ \text{so}\ k=3$

11. A $f'(x)=x\cdot 3(1-2x)^2(-2)+(1-2x)^3;\ f'(1)=-7$. Only option A has a slope of –7.

12. B $f'\left(\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$

13. A By the Fundamental Theorem of Calculus $\int_0^c f'(x)\,dx=f(x)\Big|_0^c=f(c)-f(0)$

14. D $\int_0^{\frac{\pi}{2}}(1+\sin\theta)^{-1/2}(\cos\theta\,d\theta)=2(1+\sin\theta)^{1/2}\Big|_0^{\frac{\pi}{2}}=2\left(\sqrt{2}-1\right)$

15. If $f(x)=\sqrt{2x}$, then $f'(2)=$

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 1 (E) $\sqrt{2}$

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16. A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t)=t^3-3t^2-9t+1$. For what values of t is the particle at rest?

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

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17. $\int_0^1 (3x-2)^2\,dx=$

(A) $-\frac{7}{3}$ (B) $-\frac{7}{9}$ (C) $\frac{1}{9}$ (D) 1 (E) 3

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18. If $y=2\cos\left(\frac{x}{2}\right)$, then $\frac{d^2y}{dx^2}=$

(A) $-8\cos\left(\frac{x}{2}\right)$ (B) $-2\cos\left(\frac{x}{2}\right)$ (C) $-\sin\left(\frac{x}{2}\right)$ (D) $-\cos\left(\frac{x}{2}\right)$ (E) $-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

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19. $\int_2^3 \frac{x}{x^2+1}\,dx=$

(A) $\frac{1}{2}\ln\frac{3}{2}$ (B) $\frac{1}{2}\ln 2$ (C) $\ln 2$ (D) $2\ln 2$ (E) $\frac{1}{2}\ln 5$

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20. Let f be a polynomial function with degree greater than 2. If $a \neq b$ and $f(a)=f(b)=1$, which of the following must be true for at least one value of x between a and b?

I. $f(x)=0$
II. $f'(x)=0$
III. $f''(x)=0$

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

Answers

15. B $f(x)=\sqrt{2x}=\sqrt{2}\cdot\sqrt{x}$; $f'(x)=\sqrt{2}\cdot\frac{1}{2\sqrt{x}}$; $f'(2)=\sqrt{2}\cdot\frac{1}{2\sqrt{2}}=\frac{1}{2}$

16. C At rest when $0=v(t)=x'(t)=3t^2-6t-9=3\left(t^2-2t-3\right)=3(t-3)(t+1)$

$t=-1,\ 3$ and $t\geq 0 \Rightarrow t=3$

17. D $\int_0^1 (3x-2)^2\,dx=\frac{1}{3}\int_0^1 (3x-2)^2(3\,dx)=\frac{1}{3}\cdot\frac{1}{3}(3x-2)^3\Big|_0^1=\frac{1}{9}(1-(-8))=1$

18. E $y'=2\cdot\left(-\sin\left(\frac{x}{2}\right)\cdot\frac{1}{2}\right)=-\sin\left(\frac{x}{2}\right)$; $y''=-\left(\cos\left(\frac{x}{2}\right)\cdot\left(\frac{1}{2}\right)\right)=-\frac{1}{2}\cos\left(\frac{x}{2}\right)$

19. B $\int_2^3 \frac{x}{x^2+1}dx=\frac{1}{2}\int_2^3 \frac{2x\,dx}{x^2+1}=\frac{1}{2}\ln\left(x^2+1\right)\Big|_2^3=\frac{1}{2}(\ln 10-\ln 5)=\frac{1}{2}\ln 2$

20. C Consider the cases:

I. false if $f(x)=1$

II. This is true by the Mean Value Theorem

III. false if the graph of f is a parabola with vertex at $x=\frac{a+b}{2}$.

Only II must be true.

21. The area of the region enclosed by the graphs of $y = x$ and $y = x^2 - 3x + 3$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{4}{3}$ (D) 2 (E) $\frac{14}{3}$

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22. If $\ln x - \ln\left(\frac{1}{x}\right) = 2$, then $x =$

(A) $\frac{1}{e^2}$ (B) $\frac{1}{e}$ (C) e (D) $2e$ (E) e^2

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23. If $f'(x) = \cos x$ and $g'(x) = 1$ for all x, and if $f(0) = g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) 0 (D) -1 (E) nonexistent

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24. $\frac{d}{dx}\left(x^{\ln x}\right) =$

(A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)\left(x^{\ln x}\right)$ (D) $(\ln x)\left(x^{\ln x - 1}\right)$ (E) $2(\ln x)\left(x^{\ln x}\right)$

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25. For all $x > 1$, if $f(x) = \int_1^x \frac{1}{t}\,dt$, then $f'(x) =$

(A) 1 (B) $\frac{1}{x}$ (C) $\ln x - 1$ (D) $\ln x$ (E) e^x

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26. $\int_0^{\frac{\pi}{2}} x \cos x \, dx =$

(A) $-\frac{\pi}{2}$ (B) -1 (C) $1 - \frac{\pi}{2}$ (D) 1 (E) $\frac{\pi}{2} - 1$

Answers

21. C $x = x^2 - 3x + 3$ at $x = 1$ and at $x = 3$.

Area $= \int_1^3 \left(x - \left(x^2 - 3x + 3\right)\right) dx = \int_1^3 \left(-x^2 + 4x - 3\right) dx = \left(-\frac{1}{3}x^3 + 2x^2 - 3x\right)\Big|_1^3 = \frac{4}{3}$

22. C $2 = \ln x - \ln\frac{1}{x} = \ln x + \ln x \Rightarrow \ln x = 1 \Rightarrow x = e$

23. B By L'Hôpital's rule (which is no longer part of the AB Course Description),

$\lim\limits_{x\to 0} \frac{f(x)}{g(x)} = \lim\limits_{x\to 0} \frac{f'(x)}{g'(x)} = \frac{f'(0)}{g'(0)} = \frac{\cos 0}{1} = \frac{1}{1} = 1$

Alternatively, $f'(x) = \cos x$ and $f(0) = 0 \Rightarrow f(x) = \sin x$. Also $g'(x) = 1$ and $g(0) = 0 \Rightarrow g(x) = x$. Hence $\lim\limits_{x\to 0} \frac{f(x)}{g(x)} = \lim\limits_{x\to 0} \frac{\sin x}{x} = 1$.

24. C Let $y = x^{\ln x}$ and take the ln of each side. $\ln y = \ln x^{\ln x} = \ln x \cdot \ln x$. Take the derivative of each side with respect to x. $\frac{y'}{y} = 2\ln x \cdot \frac{1}{x} \Rightarrow y' = 2\ln x \cdot \frac{1}{x} \cdot x^{\ln x}$

25. B Use the Fundamental Theorem of Calculus. $f'(x) = \frac{1}{x}$

26. E Use the technique of antiderivatives by parts: Let $u = x$ and $dv = \cos x\, dx$.

$\int_0^{\frac{\pi}{2}} x\cos x\, dx = \left(x\sin x - \int \sin x\, dx\right)\Big|_0^{\frac{\pi}{2}} = \left(x\sin x + \cos x\right)\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1$

27. At $x=3$, the function given by $f(x)=\begin{cases} x^2, & x<3 \\ 6x-9, & x\geq 3 \end{cases}$ is

(A) undefined.
(B) continuous but not differentiable.
(C) differentiable but not continuous.
(D) neither continuous nor differentiable.
(E) both continuous and differentiable.

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28. $\int_1^4 |x-3|\,dx =$

(A) $-\frac{3}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$ (E) 5

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29. The $\lim\limits_{h\to 0}\dfrac{\tan 3(x+h)-\tan 3x}{h}$ is

(A) 0 (B) $3\sec^2(3x)$ (C) $\sec^2(3x)$ (D) $3\cot(3x)$ (E) nonexistent

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30. A region in the first quadrant is enclosed by the graphs of $y=e^{2x}$, $x=1$, and the coordinate axes. If the region is rotated about the <u>y-axis</u>, the volume of the solid that is generated is represented by which of the following integrals?

(A) $2\pi\int_0^1 xe^{2x}\,dx$

(B) $2\pi\int_0^1 e^{2x}\,dx$

(C) $\pi\int_0^1 e^{4x}\,dx$

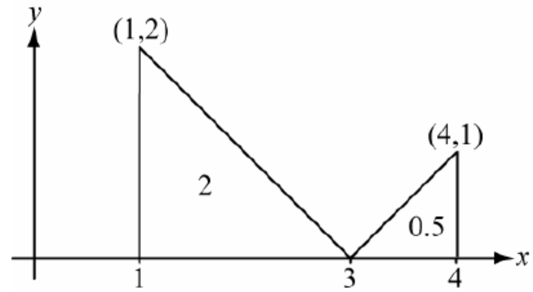
(D) $\pi\int_0^e y\ln y\,dy$

(E) $\frac{\pi}{4}\int_0^e \ln^2 y\,dy$

Answers

27. E The function is continuous at $x = 3$ since $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = 9 = f(3)$. Also, the derivative as you approach $x = 3$ from the left is 6 and the derivative as you approach $x = 3$ from the right is also 6. These two facts imply that f is differentiable at $x = 3$. The function is clearly continuous and differentiable at all other values of x.

28. C The graph is a V with vertex at $x = 3$. The integral gives the sum of the areas of the two triangles that the V forms with the horizontal axis for x from 1 to 4. These triangles have areas of 2 and 0.5 respectively.

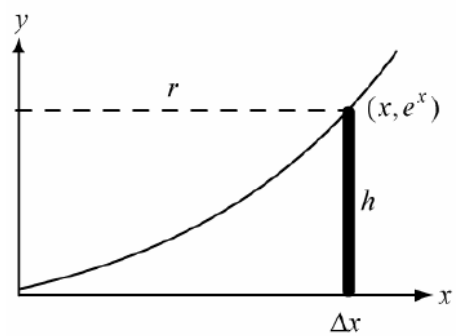


29. B This limit gives the derivative of the function $f(x) = \tan(3x)$. $f'(x) = 3\sec^2(3x)$

30. A Shells (which is no longer part of the AB Course Description)

$\sum 2\pi rh\Delta x$, where $r = x, h = e^{2x}$

Volume = $2\pi\int_0^1 xe^{2x}dx$



31. If $f(x)=\dfrac{x}{x+1}$, then the inverse function, f^{-1}, is given by $f^{-1}(x)=$

(A) $\dfrac{x-1}{x}$ (B) $\dfrac{x+1}{x}$ (C) $\dfrac{x}{1-x}$ (D) $\dfrac{x}{x+1}$ (E) x

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32. Which of the following does NOT have a period of π ?

(A) $f(x)=\sin\left(\dfrac{1}{2}x\right)$ (B) $f(x)=\left|\sin x\right|$ (C) $f(x)=\sin^2 x$

(D) $f(x)=\tan x$ (E) $f(x)=\tan^2 x$

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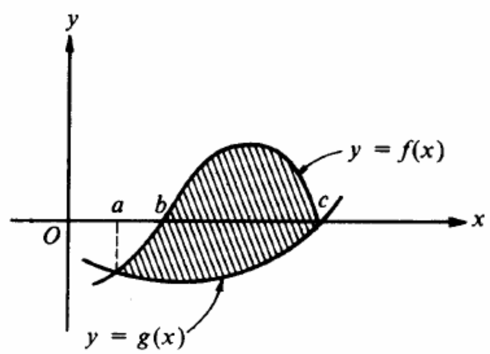
33. The absolute maximum value of $f(x)=x^3-3x^2+12$ on the closed interval $[-2,4]$ occurs at $x=$

(A) 4 (B) 2 (C) 1 (D) 0 (E) -2



34. The area of the shaded region in the figure above is represented by which of the following integrals?

(A) $\int_a^c \left(|f(x)|-|g(x)|\right)dx$

(B) $\int_b^c f(x)\,dx-\int_a^c g(x)\,dx$

(C) $\int_a^c \left(g(x)-f(x)\right)dx$

(D) $\int_a^c \left(f(x)-g(x)\right)dx$

(E) $\int_a^b \left(g(x)-f(x)\right)dx+\int_b^c \left(f(x)-g(x)\right)dx$

Answers

31. C Let $y = f(x)$ and solve for x.

$y = \frac{x}{x+1}; xy + y = x;\ x(y-1) = -y;\ x = \frac{y}{1-y} \Rightarrow f^{-1}(x) = \frac{x}{1-x}$

32. A The period for $\sin\left(\frac{x}{2}\right)$ is $\frac{2\pi}{\frac{1}{2}} = 4\pi$.

33. A Check the critical points and the endpoints.

$f'(x) = 3x^2 - 6x = 3x(x-2)$ so the critical points are 0 and 2.

x	-2	0	2	4
$f(x)$	-8	12	8	28

Absolute maximum is at $x = 4$.

34. D The interval is $x = a$ to $x = c$. The height of a rectangular slice is the top curve, $f(x)$, minus the bottom curve, $g(x)$. The area of the rectangular slice is therefore $(f(x) - g(x))\Delta x$. Set up a Riemann sum and take the limit as Δx goes to 0 to get a definite integral.

35. $4\cos\left(x+\frac{\pi}{3}\right)=$

(A) $2\sqrt{3}\cos x-2\sin x$ (B) $2\cos x-2\sqrt{3}\sin x$ (C) $2\cos x+2\sqrt{3}\sin x$

(D) $2\sqrt{3}\cos x+2\sin x$ (E) $4\cos x+2$

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36. What is the average value of y for the part of the curve $y=3x-x^2$ which is in the first quadrant?

(A) -6 (B) -2 (C) $\frac{3}{2}$ (D) $\frac{9}{4}$ (E) $\frac{9}{2}$

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37. If $f(x)=e^x\sin x$, then the number of zeros of f on the closed interval $[0,2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

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38. For $x>0$, $\int\left(\frac{1}{x}\int_1^x\frac{du}{u}\right)dx=$

(A) $\frac{1}{x^3}+C$ (B) $\frac{8}{x^4}-\frac{2}{x^2}+C$ (C) $\ln(\ln x)+C$

(D) $\frac{\ln\left(x^2\right)}{2}+C$ (E) $\frac{(\ln x)^2}{2}+C$

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39. If $\int_1^{10}f(x)\,dx=4$ and $\int_{10}^{3}f(x)\,dx=7$, then $\int_1^{3}f(x)\,dx=$

(A) -3 (B) 0 (C) 3 (D) 10 (E) 11

Answers

35. B $4\cos\left(x+\frac{\pi}{3}\right)=4\left(\cos x\cdot\cos\left(\frac{\pi}{3}\right)-\sin x\cdot\sin\left(\frac{\pi}{3}\right)\right)$

$$=4\left(\cos x\cdot\frac{1}{2}-\sin x\cdot\frac{\sqrt{3}}{2}\right)=2\cos x-2\sqrt{3}\sin x$$

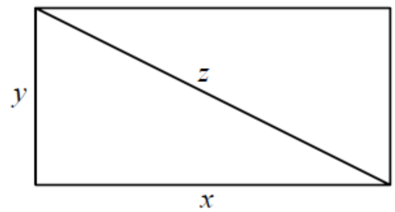
36. C $3x-x^2=x(3-x)>0$ for $0<x<3$

Average value $=\frac{1}{3}\int_0^3\left(3x-x^2\right)dx=\frac{1}{3}\left(\frac{3}{2}x^2-\frac{1}{3}x^3\right)\Big|_0^3=\frac{3}{2}$

37. D Since $e^x>0$ for all x, the zeros of $f(x)$ are the zeros of $\sin x$, so $x=0,\pi,2\pi$.

38. E $\int\left(\frac{1}{x}\int_1^x\frac{du}{u}\right)dx=\int\frac{1}{x}\ln x\,dx=\int\ln x\left(\frac{dx}{x}\right)$. This is $\int u\,du$ with $u=\ln x$, so the value is $\frac{(\ln x)^2}{2}+C$

39. E $\int_3^{10}f(x)\,dx=-\int_{10}^3 f(x)\,dx$; $\int_1^3 f(x)\,dx=\int_1^{10}f(x)\,dx-\int_3^{10}f(x)\,dx=4-(-7)=11$



40. The sides of the rectangle above increase in such a way that $\frac{dz}{dt}=1$ and $\frac{dx}{dt}=3\frac{dy}{dt}$. At the instant when $x=4$ and $y=3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

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41. If $\lim_{x\to 3} f(x)=7$, which of the following must be true?

I. f is continuous at $x=3$.
II. f is differentiable at $x=3$.
III. $f(3)=7$

(A) None (B) II only (C) III only
(D) I and III only (E) I, II, and III

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42. The graph of which of the following equations has $y=1$ as an asymptote?

(A) $y=\ln x$ (B) $y=\sin x$ (C) $y=\frac{x}{x+1}$ (D) $y=\frac{x^2}{x-1}$ (E) $y=e^{-x}$

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43. The volume of the solid obtained by revolving the region enclosed by the ellipse $x^2+9y^2=9$ about the x-axis is

(A) 2π (B) 4π (C) 6π (D) 9π (E) 12π

Answers

40. B $x^2 + y^2 = z^2$, take the derivative of both sides with respect to t. $2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 2z \cdot \frac{dz}{dt}$

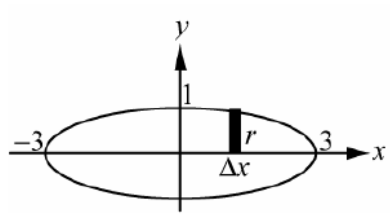
Divide by 2 and substitute: $4 \cdot \frac{dx}{dt} + 3 \cdot \frac{1}{3}\frac{dx}{dt} = 5 \cdot 1 \Rightarrow \frac{dx}{dt} = 1$

41. A The statement makes no claim as to the behavior of f at $x = 3$, only the value of f for input arbitrarily close to $x = 3$. None of the statements are true.

42. C $\lim_{x\to\infty} \frac{x}{x+1} = \lim_{x\to\infty} \frac{\frac{x}{x}}{\frac{x}{x} + \frac{1}{x}} = \lim_{x\to\infty} \frac{1}{1 + \frac{1}{x}} = 1$.

None of the other functions have a limit of 1 as $x \to \infty$

43. B The cross-sections are disks with radius $r = y$ where $y = \frac{1}{3}\sqrt{9 - x^2}$.



Volume = $\pi\int_{-3}^{3} y^2\,dx = 2\pi\int_0^3 \frac{1}{9}\left(9 - x^2\right)dx = \frac{2\pi}{9}\left(9x - \frac{1}{3}x^3\right)\Big|_0^3 = 4\pi$

44. Let f and g be odd functions. If p, r, and s are nonzero functions defined as follows, which must be odd?

I. $p(x) = f(g(x))$
II. $r(x) = f(x) + g(x)$
III. $s(x) = f(x)g(x)$

(A) I only (B) II only (C) I and II only
(D) II and III only (E) I, II, and III

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45. The volume of a cylindrical tin can with a top and a bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must be the height, in inches, of the can?

(A) $2\sqrt[3]{2}$ (B) $2\sqrt{2}$ (C) $2\sqrt[3]{4}$ (D) 4 (E) 8

Answers

44. C For I: $p(-x) = f\left(g(-x)\right) = f\left(-g(x)\right) = -f\left(g(x)\right) = -p(x) \Rightarrow$ p is odd.
For II: $r(-x) = f(-x) + g(-x) = -f(x) - g(x) = -\left(f(x) + g(x)\right) = -r(x) \Rightarrow$ r is odd.
For III: $s(-x) = f(-x) \cdot g(-x) = \left(-f(x)\right) \cdot \left(-g(x)\right) = f(x) \cdot g(x) = s(x) \Rightarrow$ s is not odd.

45. D Volume $= \pi r^2 h = 16\pi \Rightarrow h = 16r^{-2}$. $A = 2\pi rh + 2\pi r^2 = 2\pi\left(16r^{-1} + r^2\right)$

$\frac{dA}{dr} = 2\pi\left(-16r^{-2} + 2r\right) = 4\pi r^{-2}\left(-8 + r^3\right)$; $\frac{dA}{dr} < 0$ for $0 < r < 2$ and $\frac{dA}{dr} > 0$ for $r > 2$

The minimum surface area of the can is when $r = 2 \Rightarrow h = 4$.