

AP Calculus AB ... 1993 MC

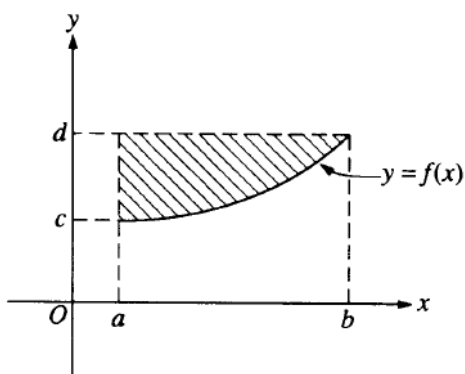
90 Minutes—Scientific Calculator

Notes: (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $f(x) = x^{\frac{3}{2}}$, then $f'(4) =$

- (A) -6 (B) -3 (C) 3 (D) 6 (E) 8
-



2. Which of the following represents the area of the shaded region in the figure above?

- (A) $\int_c^d f(y)dy$ (B) $\int_a^b (d - f(x))dx$ (C) $f'(b) - f'(a)$
(D) $(b - a)[f(b) - f(a)]$ (E) $(d - c)[f(b) - f(a)]$
-

3. $\lim_{n \rightarrow \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1}$ is

- (A) -5 (B) -2 (C) 1 (D) 3 (E) nonexistent

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Answers

1. C $f'(x) = \frac{3}{2}x^{\frac{1}{2}}; f'(4) = \frac{3}{2} \cdot 4^{\frac{1}{2}} = \frac{3}{2} \cdot 2 = 3$

2. B Summing pieces of the form: (vertical) · (small width), vertical = $(d - f(x))$, width = Δx
Area = $\int_a^b (d - f(x)) dx$

3. D Divide each term by n^3 . $\lim_{n \rightarrow \infty} \frac{3n^3 - 5n}{n^3 - 2n^2 + 1} = \lim_{n \rightarrow \infty} \frac{3 - \frac{5}{n^2}}{1 - \frac{2}{n} + \frac{1}{n^3}} = 3$

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4. If $x^3 + 3xy + 2y^3 = 17$, then in terms of x and y , $\frac{dy}{dx} =$

(A) $-\frac{x^2 + y}{x + 2y^2}$

(B) $-\frac{x^2 + y}{x + y^2}$

(C) $-\frac{x^2 + y}{x + 2y}$

(D) $-\frac{x^2 + y}{2y^2}$

(E) $\frac{-x^2}{1 + 2y^2}$

5. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) =$

(A) -4

(B) -2

(C) -1

(D) 0

(E) 2

6. The area of the region enclosed by the curve $y = \frac{1}{x-1}$, the x -axis, and the lines $x = 3$ and $x = 4$ is

(A) $\frac{5}{36}$

(B) $\ln \frac{2}{3}$

(C) $\ln \frac{4}{3}$

(D) $\ln \frac{3}{2}$

(E) $\ln 6$

7. An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1, 5)$ is

(A) $13x - y = 8$

(B) $13x + y = 18$

(C) $x - 13y = 64$

(D) $x + 13y = 66$

(E) $-2x + 3y = 13$

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Answers

4. A $3x^2 + 3(y + x \cdot y') + 6y^2 \cdot y' = 0$; $y'(3x + 6y^2) = -(3x^2 + 3y)$
$$y' = -\frac{3x^2 + 3y}{3x + 6y^2} = -\frac{x^2 + y}{x + 2y^2}$$
5. A $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2} = \lim_{x \rightarrow -2} \frac{(x+2)(x-2)}{x+2} = \lim_{x \rightarrow -2} (x-2) = -4$. For continuity $f(-2)$ must be -4 .
6. D $\text{Area} = \int_3^4 \frac{1}{x-1} dx = (\ln|x-1|) \Big|_3^4 = \ln 3 - \ln 2 = \ln \frac{3}{2}$
7. B $y' = \frac{2 \cdot (3x-2) - (2x+3) \cdot 3}{(3x-2)^2}$; $y'(1) = -13$. Tangent line: $y - 5 = -13(x-1) \Rightarrow 13x + y = 18$

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8. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

- (A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$
-

9. If h is the function given by $h(x) = f(g(x))$, where $f(x) = 3x^2 - 1$ and $g(x) = |x|$, then $h(x) =$

- (A) $3x^3 - |x|$ (B) $|3x^2 - 1|$ (C) $3x^2|x| - 1$ (D) $3|x| - 1$ (E) $3x^2 - 1$
-

10. If $f(x) = (x-1)^2 \sin x$, then $f'(0) =$

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2
-

11. The acceleration of a particle moving along the x -axis at time t is given by $a(t) = 6t - 2$. If the velocity is 25 when $t = 3$ and the position is 10 when $t = 1$, then the position $x(t) =$

- (A) $9t^2 + 1$
 (B) $3t^2 - 2t + 4$
 (C) $t^3 - t^2 + 4t + 6$
 (D) $t^3 - t^2 + 9t - 20$
 (E) $36t^3 - 4t^2 - 77t + 55$
-

12. If f and g are continuous functions, and if $f(x) \geq 0$ for all real numbers x , which of the following must be true?

I. $\int_a^b f(x)g(x)dx = \left(\int_a^b f(x)dx\right)\left(\int_a^b g(x)dx\right)$

II. $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

III. $\int_a^b \sqrt{f(x)} dx = \sqrt{\int_a^b f(x)dx}$

- (A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

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Answers

8. E $y' = \sec^2 x + \csc^2 x$
9. E $h(x) = f(|x|) = 3|x|^2 - 1 = 3x^2 - 1$
10. D $f'(x) = 2(x-1) \cdot \sin x + (x-1)^2 \cos x$; $f'(0) = (-2) \cdot 0 + 1 \cdot 1 = 1$
11. C $a(t) = 6t - 2$; $v(t) = 3t^2 - 2t + C$ and $v(3) = 25 \Rightarrow 25 = 27 - 6 + C$; $v(t) = 3t^2 - 2t + 4$
 $x(t) = t^3 - t^2 + 4t + K$; Since $x(1) = 10$, $K = 6$; $x(t) = t^3 - t^2 + 4t + 6$.
12. B The only one that is true is II. The others can easily be seen as false by examples. For example, let $f(x) = 1$ and $g(x) = 1$ with $a = 0$ and $b = 2$. Then I gives $2 = 4$ and III gives $2 = \sqrt{2}$, both false statements.

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13. The fundamental period of $2 \cos(3x)$ is

- (A) $\frac{2\pi}{3}$ (B) 2π (C) 6π (D) 2 (E) 3
-

14. $\int \frac{3x^2}{\sqrt{x^3+1}} dx =$

- (A) $2\sqrt{x^3+1} + C$
(B) $\frac{3}{2}\sqrt{x^3+1} + C$
(C) $\sqrt{x^3+1} + C$
(D) $\ln\sqrt{x^3+1} + C$
(E) $\ln(x^3+1) + C$
-

15. For what value of x does the function $f(x) = (x-2)(x-3)^2$ have a relative maximum?

- (A) -3 (B) $-\frac{7}{3}$ (C) $-\frac{5}{2}$ (D) $\frac{7}{3}$ (E) $\frac{5}{2}$
-

16. The slope of the line normal to the graph of $y = 2 \ln(\sec x)$ at $x = \frac{\pi}{4}$ is

- (A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2
(E) nonexistent

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Answers

13. A $\text{period} = \frac{2\pi}{B} = \frac{2\pi}{3}$

14. A Let $u = x^3 + 1$. Then $\int \frac{3x^2}{\sqrt{x^3 + 1}} dx = \int u^{-1/2} du = 2u^{1/2} + C = 2\sqrt{x^3 + 1} + C$

15. D $f'(x) = (x-3)^2 + 2(x-2)(x-3) = (x-3)(3x-7)$; $f'(x)$ changes from positive to negative at $x = \frac{7}{3}$.

16. B $y' = 2 \frac{\sec x \tan x}{\sec x} = 2 \tan x$; $y'(\pi/4) = 2 \tan(\pi/4) = 2$. The slope of the normal line $-\frac{1}{y'(\pi/4)} = -\frac{1}{2}$

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17. $\int (x^2 + 1)^2 dx =$

(A) $\frac{(x^2 + 1)^3}{3} + C$

(B) $\frac{(x^2 + 1)^3}{6x} + C$

(C) $\left(\frac{x^3}{3} + x\right)^2 + C$

(D) $\frac{2x(x^2 + 1)^3}{3} + C$

(E) $\frac{x^5}{5} + \frac{2x^3}{3} + x + C$

18. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

(A) $\frac{2\pi}{3}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{6}$

(D) π

(E) $\frac{3\pi}{2}$

19. Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \leq 0, \\ x & \text{for } x > 0. \end{cases}$ Which of the following statements about f is true?

(A) f is an odd function.

(B) f is discontinuous at $x = 0$.

(C) f has a relative maximum.

(D) $f'(0) = 0$

(E) $f'(x) > 0$ for $x \neq 0$

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17. E Expand the integrand. $\int (x^2 + 1)^2 dx = \int (x^4 + 2x^2 + 1) dx = \frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$

18. D Want c so that $f'(c) = \frac{f\left(\frac{3\pi}{2}\right) - f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \frac{\sin\left(\frac{3\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right)}{\pi} = \frac{0}{\pi}$.

$$f'(c) = \frac{1}{2} \cos\left(\frac{c}{2}\right) = 0 \Rightarrow c = \pi$$

19. E The only one that is true is E. A consideration of the graph of $y = f(x)$, which is a standard cubic to the left of 0 and a line with slope 1 to the right of 0, shows the other options to be false.

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20. Let R be the region in the first quadrant enclosed by the graph of $y = (x+1)^{\frac{1}{3}}$, the line $x = 7$, the x -axis, and the y -axis. The volume of the solid generated when R is revolved about the y -axis is given by

(A) $\pi \int_0^7 (x+1)^{\frac{2}{3}} dx$ (B) $2\pi \int_0^7 x(x+1)^{\frac{1}{3}} dx$ (C) $\pi \int_0^2 (x+1)^{\frac{2}{3}} dx$
 (D) $2\pi \int_0^2 x(x+1)^{\frac{1}{3}} dx$ (E) $\pi \int_0^7 (y^3 - 1)^2 dy$

21. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x

22. An antiderivative for $\frac{1}{x^2 - 2x + 2}$ is

(A) $-(x^2 - 2x + 2)^{-2}$
 (B) $\ln(x^2 - 2x + 2)$
 (C) $\ln \left| \frac{x-2}{x+1} \right|$
 (D) $\operatorname{arcsec}(x-1)$
 (E) $\arctan(x-1)$

23. How many critical points does the function $f(x) = (x+2)^5(x-3)^4$ have?

(A) One (B) Two (C) Three (D) Five (E) Nine

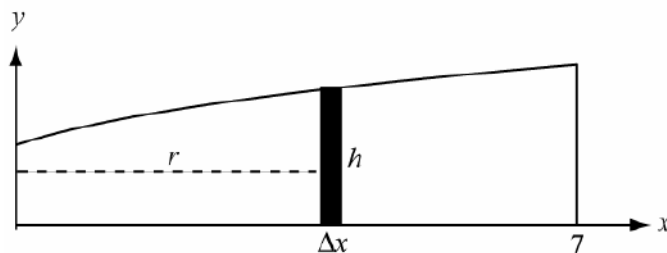
24. If $f(x) = (x^2 - 2x - 1)^{\frac{2}{3}}$, then $f'(0)$ is

(A) $\frac{4}{3}$ (B) 0 (C) $-\frac{2}{3}$ (D) $-\frac{4}{3}$ (E) -2

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Answers

20. B Use Cylindrical Shells which is no part of the AP Course Description. The volume of each shell is of the form $(2\pi rh)\Delta x$ with $r = x$ and $h = y$. Volume = $2\pi \int_0^7 x(x+1)^{\frac{1}{3}} dx$.



21. C $y = x^{-2} - x^{-3}$; $y' = -2x^{-3} + 3x^{-4}$; $y'' = 6x^{-4} - 12x^{-5} = 6x^{-5}(x - 2)$. The only domain value at which there is a sign change in y'' is $x = 2$. Inflection point at $x = 2$.
22. E $\int \frac{1}{x^2 - 2x + 2} dx = \int \frac{1}{(x^2 - 2x + 1) + 1} dx = \int \frac{1}{(x-1)^2 + 1} dx = \tan^{-1}(x-1) + C$
23. C A quick way to do this problem is to use the effect of the multiplicity of the zeros of f on the graph of $y = f(x)$. There is point of inflection and a horizontal tangent at $x = -2$. There is a horizontal tangent and turning point at $x = 3$. There is a horizontal tangent on the interval $(-2, 3)$. Thus, there must be 3 critical points. Also, $f'(x) = (x-3)^3(x+2)^4(9x-7)$.
24. A $f'(x) = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{1}{3}}(2x - 2)$, $f'(0) = \frac{2}{3} \cdot (-1) \cdot (-2) = \frac{4}{3}$

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25. $\frac{d}{dx}(2^x) =$

- (A) 2^{x-1} (B) $(2^{x-1})x$ (C) $(2^x)\ln 2$ (D) $(2^{x-1})\ln 2$ (E) $\frac{2x}{\ln 2}$
-

26. A particle moves along a line so that at time t , where $0 \leq t \leq \pi$, its position is given by

$$s(t) = -4 \cos t - \frac{t^2}{2} + 10.$$

What is the velocity of the particle when its acceleration is zero?

- (A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13
-

27. The function f given by $f(x) = x^3 + 12x - 24$ is

- (A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
 (B) decreasing for $x < 0$, increasing for $x > 0$
 (C) increasing for all x
 (D) decreasing for all x
 (E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$
-

28. $\int_1^{500} (13^x - 11^x) dx + \int_2^{500} (11^x - 13^x) dx =$

- (A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364
-

29. $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2 \sin^2 \theta}$ is

- (A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) 1 (E) nonexistent
-

30. The region enclosed by the x -axis, the line $x = 3$, and the curve $y = \sqrt{x}$ is rotated about the x -axis. What is the volume of the solid generated?

- (A) 3π (B) $2\sqrt{3}\pi$ (C) $\frac{9}{2}\pi$ (D) 9π (E) $\frac{36\sqrt{3}}{5}\pi$

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25. C $\frac{d}{dx}(2^x) = 2^x \cdot \ln 2$

26. D $v(t) = 4 \sin t - t$; $a(t) = 4 \cos t - 1 = 0$ at $t = \cos^{-1}(1/4) = 1.31812$; $v(1.31812) = 2.55487$

27. C $f'(x) = 3x^2 + 12 > 0$. Thus f is increasing for all x .

28. B $\int_1^{500} (13^x - 11^x) dx + \int_2^{500} (11^x - 13^x) dx = \int_1^{500} (13^x - 11^x) dx - \int_2^{500} (13^x - 11^x) dx$
 $= \int_1^2 (13^x - 11^x) dx = \left(\frac{13^x}{\ln 13} - \frac{11^x}{\ln 11} \right) \Big|_1^2 = \frac{13^2 - 13}{\ln 13} - \frac{11^2 - 11}{\ln 11} = 14.946$

29. C Use L'Hôpital's Rule (which is no longer part of the AB Course Description).

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2 \sin^2 \theta} = \lim_{\theta \rightarrow 0} \frac{\sin \theta}{4 \sin \theta \cos \theta} = \lim_{\theta \rightarrow 0} \frac{1}{4 \cos \theta} = \frac{1}{4}$$

A way to do this without L'Hôpital's rule is the following

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2 \sin^2 \theta} = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2(1 - \cos^2 \theta)} = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{2(1 - \cos \theta)(1 + \cos \theta)} = \lim_{\theta \rightarrow 0} \frac{1}{2(1 + \cos \theta)} = \frac{1}{4}$$

30. C Each slice is a disk whose volume is given by $\pi r^2 \Delta x$, where $r = \sqrt{x}$.

$$\text{Volume} = \pi \int_0^3 (\sqrt{x})^2 dx = \pi \int_0^3 x dx = \frac{\pi}{2} x^2 \Big|_0^3 = \frac{9}{2} \pi.$$

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31. If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

- (A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$
-

32. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

- (A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) $-\ln 2$
-

33. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

- (A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$
-

34. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

- (A) $-\frac{7}{8}$ feet per minute
 (B) $-\frac{7}{24}$ feet per minute
 (C) $\frac{7}{24}$ feet per minute
 (D) $\frac{7}{8}$ feet per minute
 (E) $\frac{21}{25}$ feet per minute
-

35. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote $y = 2$ and a vertical asymptote $x = -3$, then $a + c =$

- (A) -5 (B) -1 (C) 0 (D) 1 (E) 5

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Answers

31. E $f(x) = e^{3\ln(x^2)} = e^{\ln(x^6)} = x^6$; $f'(x) = 6x^5$
32. A $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, a > 0$
 $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} = \sin^{-1}\left(\frac{x}{2}\right) \Big|_0^{\sqrt{3}} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}(0) = \frac{\pi}{3}$
33. B Separate the variables. $y^{-2}dy = 2dx$; $-\frac{1}{y} = 2x + C$; $y = \frac{-1}{2x+C}$. Substitute the point $(1, -1)$ to find the value of C . Then $-1 = \frac{-1}{2+C} \Rightarrow C = -1$, so $y = \frac{1}{1-2x}$. When $x = 2$, $y = -\frac{1}{3}$.
34. D Let x and y represent the horizontal and vertical sides of the triangle formed by the ladder, the wall, and the ground.
 $x^2 + y^2 = 25$; $2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$; $2(24) \frac{dx}{dt} + 2(7)(-3) = 0$; $\frac{dx}{dt} = \frac{7}{8}$.
35. E For there to be a vertical asymptote at $x = -3$, the value of c must be 3. For $y = 2$ to be a horizontal asymptote, the value of a must be 2. Thus $a + c = 5$.

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36. If the definite integral $\int_0^2 e^{x^2} dx$ is first approximated by using two inscribed rectangles of equal width and then approximated by using the trapezoidal rule with $n = 2$, the difference between the two approximations is

- (A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

37. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

II. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

III. $\lim_{x \rightarrow a} \frac{f(x+h) - f(x)}{h}$

- (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

38. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

(A) $\frac{x^3}{3} + \cos x - x + 1$

(B) $\frac{x^3}{3} - \cos x - x + 1$

(C) $x^3 + \cos x - x + 1$

(D) $x^2 - \sin x + 1$

(E) $x^2 + \sin x + 1$

39. The radius of a circle is increasing at a nonzero rate, and at a certain instant, the rate of increase in the area of the circle is numerically equal to the rate of increase in its circumference. At this instant, the radius of the circle is

(A) $\frac{1}{\pi}$

(B) $\frac{1}{2}$

(C) $\frac{2}{\pi}$

(D) 1

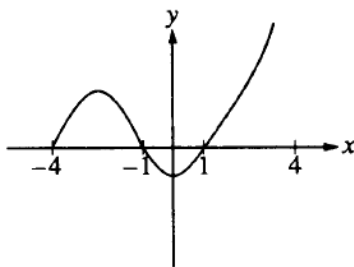
(E) 2

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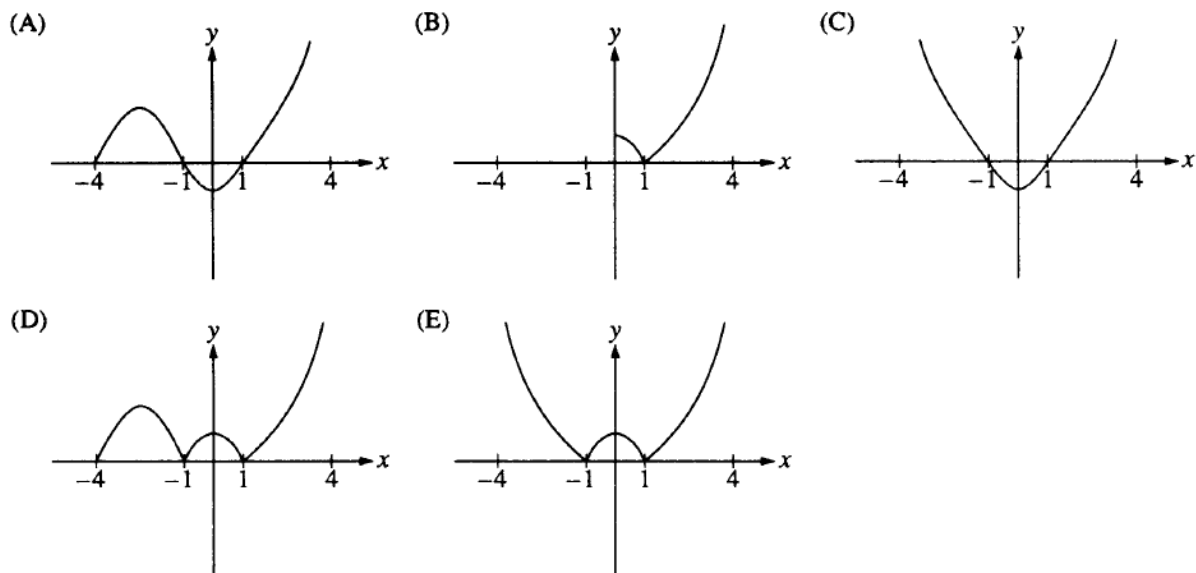
Answers

36. D Rectangle approximation = $e^0 + e^1 = 1 + e$
Trapezoid approximation. = $(1 + 2e + e^4)/2$.
Difference = $(e^4 - 1)/2 = 26.799$.
37. C I and II both give the derivative at a . In III the denominator is fixed. This is not the derivative of f at $x = a$. This gives the slope of the secant line from $(a, f(a))$ to $(a+h, f(a+h))$.
38. A $f'(x) = x^2 - \sin x + C$, $f(x) = \frac{1}{3}x^3 + \cos x + Cx + K$. Option A is the only one with this form.
39. D $A = \pi r^2$ and $C = 2\pi r$; $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ and $\frac{dC}{dt} = 2\pi \frac{dr}{dt}$. For $\frac{dA}{dt} = \frac{dC}{dt}$, $r = 1$.

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40. The graph of $y = f(x)$ is shown in the figure above. Which of the following could be the graph of $y = f(|x|)$?



41. $\frac{d}{dx} \int_0^x \cos(2\pi u) du$ is

- (A) 0 (B) $\frac{1}{2\pi} \sin x$ (C) $\frac{1}{2\pi} \cos(2\pi x)$ (D) $\cos(2\pi x)$ (E) $2\pi \cos(2\pi x)$

-
42. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

- (A) 4.2 pounds (B) 4.6 pounds (C) 4.8 pounds (D) 5.6 pounds (E) 6.5 pounds

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40. C The graph of $y = f(|x|)$ is symmetric to the y -axis. This leaves only options C and E. For $x > 0$, x and $|x|$ are the same, so the graphs of $f(x)$ and $f(|x|)$ must be the same. This is option C.
41. D Answer follows from the Fundamental Theorem of Calculus.
42. B This is an example of exponential growth. We know from pre-calculus that $w = 2\left(\frac{3.5}{2}\right)^{\frac{t}{2}}$ is an exponential function that meets the two given conditions. When $t = 3$, $w = 4.630$. Using calculus the student may translate the statement “increasing at a rate proportional to its weight” to mean exponential growth and write the equation $w = 2e^{kt}$. Using the given conditions, $3.5 = 2e^{2k}$; $\ln(1.75) = 2k$; $k = \frac{\ln(1.75)}{2}$; $w = 2e^{t \frac{\ln(1.75)}{2}}$. When $t = 3$, $w = 4.630$.

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43. $\int x f(x) dx =$

(A) $x f(x) - \int x f'(x) dx$

(B) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x) dx$

(C) $x f(x) - \frac{x^2}{2} f(x) + C$

(D) $x f(x) - \int f'(x) dx$

(E) $\frac{x^2}{2} \int f(x) dx$

44. What is the minimum value of $f(x) = x \ln x$?

- (A) $-e$ (B) -1 (C) $-\frac{1}{e}$ (D) 0 (E) $f(x)$ has no minimum value.

45. If Newton's method is used to approximate the real root of $x^3 + x - 1 = 0$, then a first approximation $x_1 = 1$ would lead to a third approximation of $x_3 =$

- (A) 0.682 (B) 0.686 (C) 0.694 (D) 0.750 (E) 1.637

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Answers

43. B Use the technique of antiderivative by parts, which is no longer in the AB Course Description. The formula is $\int u dv = uv - \int v du$. Let $u = f(x)$ and $dv = x dx$. This leads to

$$\int x f(x) dx = \frac{1}{2} x^2 f(x) - \frac{1}{2} \int x^2 f'(x) dx.$$

44. C $f'(x) = \ln x + x \cdot \frac{1}{x}$; $f'(x)$ changes sign from negative to positive only at $x = e^{-1}$.

$$f(e^{-1}) = -e^{-1} = -\frac{1}{e}.$$

45. B Let $f(x) = x^3 + x - 1$. Then Newton's method (which is no longer part of the AP Course Description) gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 + x_n - 1}{3x_n^2 + 1}$$

$$x_2 = 1 - \frac{1+1-1}{3+1} = \frac{3}{4}$$

$$x_3 = \frac{3}{4} - \frac{\left(\frac{3}{4}\right)^3 + \frac{3}{4} - 1}{3\left(\frac{3}{4}\right)^2 + 1} = \frac{59}{86} = 0.686$$