2017fa 140 Test3 Ch04 10Q Applications

Short Answer

1. Locate the absolute extrema of the function $y = x^2 - 2\cos x$ on the closed interval $[-3, 5]$. Round your answers to four decimal places.

2. Identify the open intervals on which the function $y = 9x - 18\cos x,\ 0 < x < 2\pi$ is increasing or decreasing.

3. Determine the open intervals on which the graph of $y = -6x^3 + 8x^2 + 6x - 5$ is concave downward or concave upward.

4. Determine the x-coordinate(s) of any relative extrema and inflection points of the function $y = x^5 \ln \frac{x}{9}$.

SHORT ANSWER

1. ANS:

   left endpoint: $(-3,\ 10.9800)$

   right endpoint: $(5,\ 24.4327)$ absolute maximum

   critical points: $(0,\ -2.0000)$ absolute minimum

   PTS: 1 DIF: Difficult REF: 4.1.35
   OBJ: Locate the absolute extrema of a function on a given closed interval
   MSC: Skill NOT: Section 4.1

2. ANS:

   increasing on $\left(0, \frac{7\pi}{6}\right)$ and $\left(\frac{11\pi}{6}, 2\pi\right)$; decreasing on $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$

   PTS: 1 DIF: Medium REF: 4.3.15
   OBJ: Identify the intervals on which a function is increasing or decreasing
   MSC: Skill NOT: Section 4.3

3. ANS:

   concave upward on $\left(-\infty, \frac{4}{9}\right)$; concave downward on $\left(\frac{4}{9}, \infty\right)$

   PTS: 1 DIF: Medium REF: 4.4.9
   OBJ: Identify the intervals on which a function is concave up or concave down
   MSC: Skill NOT: Section 4.4

4. ANS:

   relative minimum: $x = 9e^{\frac{-1}{5}}$; inflection point: $x = 9e^{\frac{-9}{20}}$

   PTS: 1 DIF: Medium REF: 4.4.62
   OBJ: Identify the relative extrema and inflection points of a function
   MSC: Skill NOT: Section 4.4

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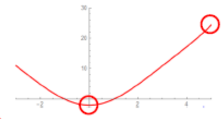
Short Answer

1. Locate the absolute extrema of the function $y = x^2 - 2\cos x$ on the closed interval $[-3,\ 5]$. Round your answers to four decimal places.



- $y'(x) = 0 \Rightarrow x = 0$
- Abs. min. at $x = 0$
- Check endpoints
- Abs. max. at $x = 5$

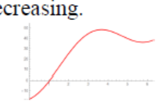
$y'(x) = 2x + 2\sin x = 0 \Rightarrow x = 0$

$y(-3) = 9 - 2\cos 3 \approx 10.9800$

$y(0) = 0 - 2\cos 0 = -2.0000$. Absolute minimum at $(0, -2.0000)$

$y(5) = 5 - 2\cos 5 \approx 24.4327$. Absolute maximum at $(5, 24.4327)$

2. Identify the open intervals on which the function $y = 9x - 18\cos x,\ 0 < x < 2\pi$ is increasing or decreasing.



- $y'(x) = 0$, two roots
- Increasing $(0, 7\pi/6)$
- Decreasing $(7\pi/6, 11\pi/6)$
- Increasing $(11\pi/6, 2\pi)$

$y'(x) = 9 + 18\sin x = 0 \Rightarrow x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

$y'\left(\frac{\pi}{2}\right) = 27 > 0$ or increasing on $\left(0, \frac{7\pi}{6}\right)$

$y'\left(\frac{3\pi}{2}\right) = -9$ or decreasing on $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$

$y'\left(\frac{23\pi}{12}\right) \approx 4.3413 > 0$ or increasing on $\left(\frac{11\pi}{6}, 2\pi\right)$

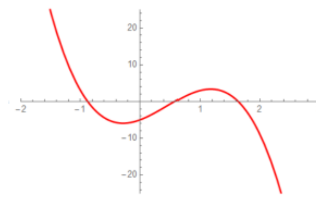
3. Determine the open intervals on which the graph of $y = -6x^3 + 8x^2 + 6x - 5$ is concave downward or concave upward.

- $y''(x) = 0$
- Point of inflection, $x = 4/9$
- Concave up $x < 4/9$
- Concave down $x > 4/9$

$y''(x) = 16 - 36x = 0 \Rightarrow x = \frac{4}{9}$

$y''(0) = 16$ or concave up on $\left(-\infty, \frac{4}{9}\right)$

$y''(1) = -20$ or concave down on $\left(\frac{4}{9}, \infty\right)$



4. Determine the x-coordinate(s) of any relative extrema and inflection points of the function $y = x^5 \ln \frac{x}{9}$.

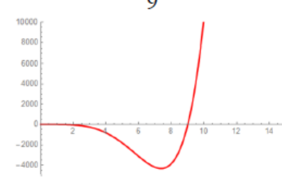
- $y' = 0$
- Solve for x.
- $y'' = 0$
- Solve for x.

$y' = x^4 + 5x^4 \ln \frac{x}{9}$

$y' = 0 \Rightarrow x = 9e^{-1/5} \approx 7.36858$

$y'' = 9x^3 + 20x^3 \ln \frac{x}{9}$

$y'' = 0 \Rightarrow x = 9e^{-9/20} \approx 5.73865$



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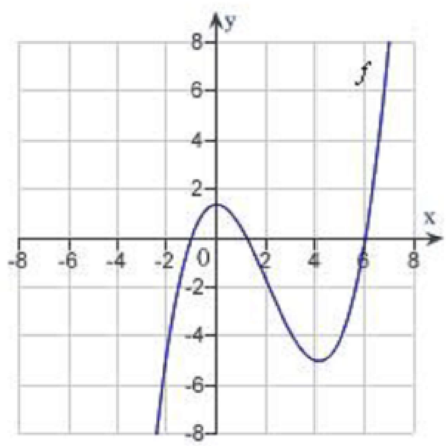
Short Answer

5. Find the limit.

$$\lim_{x \to \infty} \left(\frac{3}{4}x + \frac{2}{x^4} \right)$$

6. The graph of f is shown below. For which values of x is $f'(x)$ zero?



7. Find the point on the graph of the function $f(x) = \sqrt{x}$ that is closest to the point $(18, 0)$.

SHORT ANSWER

5. ANS:
∞

PTS: 1 DIF: Medium REF: 4.5.20
OBJ: Evaluate the limit of a function at infinity MSC: Skill
NOT: Section 4.5

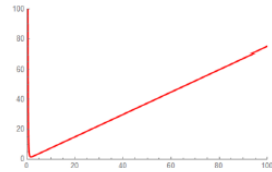
6. ANS:
$x = 0; x = 4$

PTS: 1 DIF: Easy REF: 4.6.89a
OBJ: Identify properties of the derivative of a function given the graph of the function
MSC: Skill NOT: Section 4.6

7. ANS:
$\left(\frac{35}{2}, \sqrt{\frac{35}{2}}\right)$

PTS: 1 DIF: Medium REF: 4.7.15
OBJ: Apply calculus techniques to solve a minimum/maximum problem involving the distance between points
MSC: Application NOT: Section 4.7

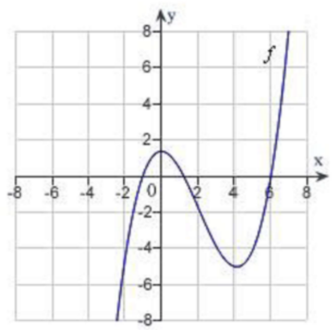
5. Find the limit.

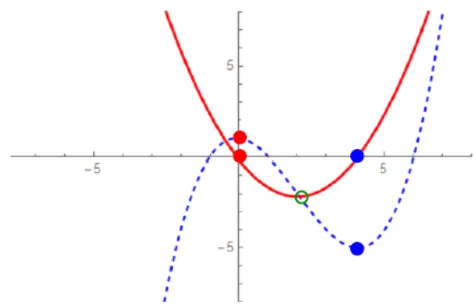


$$\lim_{x\to\infty}\left(\frac{3}{4}x+\frac{2}{x^4}\right) = \lim_{n\to\infty}\left(\frac{3x^5+2}{x^4}\right) = \infty$$

- lim 3x/4
- → ∞
- lim $2/x^4$
- → 0

6. The graph of f is shown below. For which values of x is $f'(x)$ zero?





- Max. (0,1)
- f'(x) = 0 at x = 0
- Min. (4,-5)
- f'(x) = 0 at x = 4

Possibility: $y=\frac{1}{6}(x+1)(x-1)(x-6)=\frac{1}{6}x^3-x^2-\frac{1}{6}x+1.$

$\therefore y'=\frac{1}{2}x^2-2x-\frac{1}{6}$

7. Find the point on the graph of the function $f(x)=\sqrt{x}$ that is closest to the point $(18,0)$.

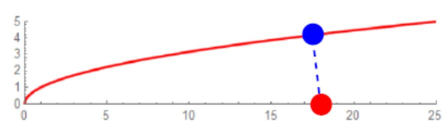
- $D = (x - 18)^2 + x$
- $D' = 2x - 35 = 0$
- $x = 35/2$
- $y = \sqrt{35/2}$

$d=\sqrt{(x-18)^2+(\sqrt{x}-0)^2}=\sqrt{(x-18)^2+x}$

$D=(x-18)^2+x$

$D'=2x-35=0\Rightarrow x=\frac{35}{2}$ = 17½

$y=\sqrt{\frac{35}{2}}$ ≈ 4.1833



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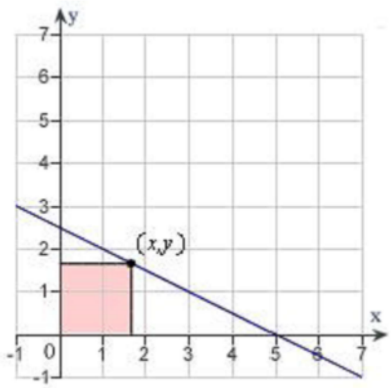
Short Answer

8. A rectangle is bounded by the x- and y-axes and the graph of $y = \dfrac{(5-x)}{2}$ (see figure). What length and width should the rectangle have so that its area is a maximum?



9. Find the differential dy of the function $y = x\cos(7x)$.

10. The range R of a projectile is $R = \dfrac{v_0^2}{32}(\sin 2\theta)$ where v_0 is the initial velocity in feet per second and θ is the angle of elevation. If $v_0 = 2300$ feet per second and θ is changed from 13° to 14° use differentials to approximate the change in the range. Round your answer to the nearest integer.

SHORT ANSWER

8. ANS:
$x = 2.5$; $y = 1.25$

PTS: 1 DIF: Difficult REF: 4.7.26
OBJ: Apply calculus techniques to solve a minimum/maximum problem involving the area of a rectangle bounded beneath a line MSC: Application NOT: Section 4.7

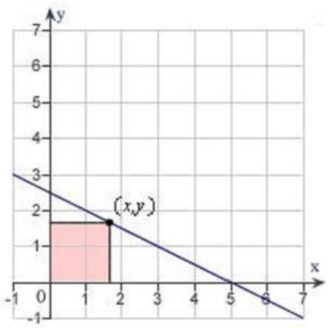
9. ANS:
$\cos(7x) - 7x\sin(7x)\,dx$

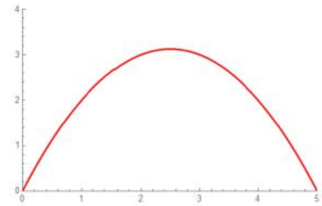
PTS: 1 DIF: Medium REF: 4.8.20
OBJ: Calculate the differential of y for a given function MSC: Skill
NOT: Section 4.8

10. ANS:
5,186 ft

PTS: 1 DIF: Medium REF: 4.8.45
OBJ: Estimate the total change of a function using differentials MSC: Application
NOT: Section 4.8

8. A rectangle is bounded by the x- and y-axes and the graph of $y = \frac{(5-x)}{2}$ (see figure). What length and width should the rectangle have so that its area is a maximum?





$$A(x) = x\left(\frac{5-x}{2}\right) = \frac{5}{2}x - \frac{1}{2}x^2$$

$$A'(x) = -x + \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$$

$$y\left(\frac{5}{2}\right) = \frac{5}{4}$$

- **Primary equation $A(x,y) = xy$**
- **$A'(x)=0$**
- **$x = 5/2$**
- **$y = 5/4$**

9. Find the differential dy of the function $y = x\cos(7x)$.

- **dy**
- **Product Rule**
- **Expand**
- **Simplify**

$$dy = \left[\cos(7x) - 7x\sin(7x)\right]dx$$

10. The range R of a projectile is $R = \frac{v_0^2}{32}(\sin 2\theta)$ where v_0 is the initial velocity in feet per second and θ is the angle of elevation. If $v_0 = 2300$ feet per second and θ is changed from 13° to 14° use differentials to approximate the change in the range. Round your answer to the nearest integer.

- **Tangent line slope**
- **Tangent line b**
- **Tangent line at 14°**
- **ΔR**

$$13^\circ = \frac{13\pi}{180} \approx 0.226893$$

$$14^\circ = \frac{7\pi}{90} \approx 0.244346$$

$$\frac{\Delta R}{\Delta\theta} = \frac{\frac{2300^2}{32}\sin(2\cdot 14^\circ) - \frac{2300^2}{32}\sin(2\cdot 13^\circ)}{14^\circ - 13^\circ} \approx \frac{77{,}609.5 - 72{,}468.2}{14^\circ - 13^\circ} = 5{,}141.3$$

$$\left.\frac{dR}{d\theta}\right|_{\theta=\frac{13\pi}{180}} = 330{,}625\cos 2\theta\Big|_{\theta=\frac{13\pi}{180}} \approx 297{,}164$$

$$R\left(\frac{13\pi}{180}\right) \approx 72{,}468.2$$

$$b \approx 72{,}468.2 - 297{,}164\cdot\frac{13\pi}{180} \approx 5043.9$$

$$\Delta R \approx \left[297{,}164\left(\frac{7\pi}{90}\right) + 5043.9\right] - 72{,}468.2 \approx 5186.49$$

Free points on #10