Derivatives

Question 10: Find the derivative of the following function:

$$f(x) = 1963$$

(A) $+\infty$

(B) 1963

(C) $-\infty$

(D) 0

(E) None of the above

Answer: (D) The derivative of a constant function is always zero.

Question 11: Find the derivative of the following function:

$$f(x) = x^2 + 6x + 9$$

(A) $f'(x) = 2x + 6 + 9$

(B) $f'(x) = x^2 + 6$

(C) $f'(x) = 2x + 6$

(D) $f'(x) = 2x$

(E) None of the above

Answer: (C) Remember that 1) the derivative of a sum of functions is simply the sum of the derivatives of each of the functions, and 2) the power rule for derivatives says that if $f(x) = kx^n$, then $f'(x) = nkx^{n-1}$. Thus $f'(x) = 2x^{2-1} + 6x^{1-1} + 0 = 2x + 6$.

Question 12: Find the derivative of the following function:

$$f(x) = x^{\frac{1}{2}}$$

(A) $f'(x) = -\frac{1}{2\sqrt{x}}$

(B) $f'(x) = \frac{1}{\sqrt{x}}$

(C) $f'(x) = \frac{1}{2\sqrt{x}}$

(D) $f'(x) = \sqrt{x}$

(E) None of the above

Answer: (C) Remember that the power rule for derivatives works with fractional exponents as well! Thus $f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$.

Question 13: Find the derivative of the following function:

$$f(x) = 5x^2(x + 47)$$

(A) $f'(x) = 15x^2 + 470x$

(B) $f'(x) = 5x^2 + 470x$

(C) $f'(x) = 10x$

(D) $f'(x) = 15x^2 - 470x$

(E) None of the above

Answer: (A) Ideally, you would solve this problem by applying the product rule. Set $g(x) = 5x^2$ and $h(x) = (x + 47)$, then $f(x) = g(x)h(x)$. Apply the product rule:

$$\begin{aligned} f'(x) &= g'(x)h(x) + g(x)h'(x) \\ &= 10x(x + 47) + 5x^2(1) \\ &= 10x^2 + 470x + 5x^2 \\ &= 15x^2 + 470x \end{aligned}$$

Question 14: Find the derivative of the following function:

$$f(x) = \frac{5x^2}{x + 47}$$

(A) $f'(x) = \frac{5x^2 - 470x}{(x+47)^2}$

(B) $f'(x) = \frac{10x^2 + 470x}{(x+47)}$

(C) $f'(x) = 10x$

(D) $f'(x) = \frac{5x^2 + 470}{(x+47)^2}$

(E) None of the above

Answer: (E) Ideally, you would solve this problem by applying the quotient rule. Set $g(x) = 5x^2$ and $h(x) = (x + 47)$, then $f(x) = \frac{g(x)}{h(x)}$. Apply the quotient rule:

$$\begin{aligned} f'(x) &= \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2} \\ &= \frac{10x(x + 47) - 5x^2(1)}{(x + 47)^2} \\ &= \frac{10x^2 + 470x - 5x^2}{(x + 47)^2} \\ &= \frac{5x^2 + 470x}{(x + 47)^2} \end{aligned}$$

Question 15: Find the derivative of the following function:

$$f(x) = 5(x + 47)^2$$

(A) $f'(x) = 15x^2 + 470x$

(B) $f'(x) = 10x - 470$

(C) $f'(x) = 10x + 470$

(D) $f'(x) = 15x^2 - 470x$

(E) None of the above

Answer: (C) Ideally, you would solve this problem by applying the chain rule. Set $g(h) = 5h^2$ and $h(x) = (x+47)$, then $f(x) = g(h(x))$. Apply the chain rule:

$$\begin{aligned} f'(x) &= g'(h)h'(x) \\ &= 10h \\ &= 10(x + 47) \\ &= 10x + 470 \end{aligned}$$

Question 16: Find the derivative of the following function:

$$f(x) = (7x - 4)(3x + 8)^4$$

Answer: Combine the product rule and the chain rule:

$$\begin{aligned} f'(x) &= 7(3x + 8)^4 + (7x - 4)(4)(3)(3x + 8)^3 \\ &= 7(3x + 8)^4 + 12(7x - 4)(3x + 8)^3 \\ &= 7(3x + 8)^4 + (84x - 48)(3x + 8)^3 \end{aligned}$$

Question 17: Find the derivative of the following function:

$$f(x) = (122x^3 - 49)^{-4}$$

Answer: Use the chain rule:

$$\begin{aligned} f'(x) &= -4 * (122)(3)x^2(122x^3 - 49)^{-5} \\ &= -\frac{1464x^2}{(122x^3 - 49)^5} \end{aligned}$$

Question 18: Find the derivative of the following function:

$$f(x) = \frac{8x^2 + 3x - 9}{7x^2 - 4}$$

Answer: The easiest way is to solve this is to get rid of the fraction, and then combine the product rule with the chain rule:

$$\begin{aligned} f(x) &= (8x^2 + 3x - 9)(7x^2 - 4)^{-1} \\ f'(x) &= (8(2)x + 3)(7x^2 - 4)^{-1} + (8x^2 + 3x - 9)(-1)(7x^2 - 4)^{-2} \\ &= \frac{16x + 3}{7x^2 - 4} - \frac{8x^2 + 3x - 9}{(7x^2 - 4)^2} \end{aligned}$$

Question 19: Find the derivative of the following function:

$$f(x) = (22 - 9x^6)^{\frac{1}{2}}$$

Answer: Use the chain rule:

$$\begin{aligned} f'(x) &= \frac{1}{2}(22 - 9x^6)^{-\frac{1}{2}}(9)(6)x^5 \\ &= 7(3x + 8)^4 + 12(7x - 4)(3x + 8)^3 \\ &= \frac{27x^5}{2(22 - 9x^6)^{\frac{1}{2}}} \end{aligned}$$

Question 20: Find the derivative of the following function:

$$f(x) = (18x^2 + 23)^{\frac{1}{3}}$$

Answer: Use the chain rule:

$$\begin{aligned} f'(x) &= \frac{1}{3}(2)(18)x(18x^2 + 23)^{-\frac{1}{3}} \\ &= \frac{12x}{(18x^2 + 23)^{\frac{1}{3}}} \end{aligned}$$

Question 21: Find the derivative of the following function:

$$f(x) = 5x^2(4x-9)^3$$

Answer: Combine the product rule and the chain rule:

$$\begin{aligned} f'(x) &= 5(2)x(4x-9)^3 + 5x^2(3)(4)(4x-9)^2 \\ &= 10x(4x-9)^3 + 60x^2(4x-9)^2 \end{aligned}$$

Higher Order Derivatives

Question 22: Find the second derivative of the following function:

$$f(x) = 5x^2(x+47)$$

(A) $f''(x) = 30x - 470$

(B) $f''(x) = 30x + 470$

(C) $f''(x) = 15x^2 + 235$

(D) $f''(x) = 15x^2 + 470x$

(E) None of the above

Answer: (B) The second derivative is just the derivative of the first derivative. Simplest solution would be to multiply to re-write the function as $f(x) = 5x^2(x+47) = 5x^3+235x^2$. Now take the derivative: $f'(x) = 15x^2+470x$. Taking the derivative again yields the second derivative: $f''(x) = 30x + 470$.

Question 23: Find the third derivative of the following function:

$$f(x) = 5x^2(x+47)$$

(A) 15

(B) $15 + x$

(C) $30x$

(D) $30x + 470$

(E) None of the above

Answer: (E) Just take the derivative of your answer to Question 12 to get the third derivative of $f(x) = 5x^2(x+47)$. Answer: $f'''(x) = 30$.

Question 24: Suppose that you have the following utility function:

$$u(x) = \sqrt{x}$$

Find $-\frac{u''(x)}{u'(x)}$.

(A) $\frac{1}{2x}$

(B) $-\frac{1}{2x}$

(C) $2x$

(D) $-2x$

(E) None of the above

Answer: (A) The ratio $-\frac{u''(x)}{u'(x)}$ is called the Arrow-Pratt measure of relative risk aversion and you will encounter it in core microeconomics. The first derivative of the utility function (otherwise known as marginal utility) is $u'(x) = \frac{1}{2\sqrt{x}}$ (see Question 9 above). The second derivative is $u''(x) = -\frac{1}{4}x^{-\frac{3}{2}} = -\frac{1}{4\sqrt{x^3}}$. Thus the Arrow-Pratt measure of relative risk aversion is:

$$-\frac{u''(x)}{u'(x)} = -\frac{-\frac{1}{4\sqrt{x^3}}}{\frac{1}{2\sqrt{x}}} = \frac{2\sqrt{x}}{4\sqrt{x^3}} = \frac{1}{2x}$$

Question 25: Find the first, second and third derivatives of the following function:

$$f(x) = 3x^4 - 5x^3 + 8x^2 - 7x - 13$$

Answer:

$$f'(x) = 12x^3 - 15x^2 + 16x - 7$$
$$f''(x) = 36x^2 - 30x + 16$$
$$f'''(x) = 72x^2 - 30$$

Question 26: Find the first, second and third derivatives of the following function:

$$f(x) = (5 - 2x)^4$$

Answer:

$$f'(x) = 4(-2)(5 - 2x)^3 = -8(5 - 2x)^3$$
$$f''(x) = -8(3)(-2)(5 - 2x)^2 = 48(5 - 2x)^2$$
$$f'''(x) = 48(2)(-2)(5 - 2x) = -192(5 - 2x) = 384x - 960$$