Calculus
Ch. 5.6 Inverse Trig Derivatives
Classwork Worksheet

THEOREM 5.16 Derivatives of Inverse Trigonometric Functions

Let u be a differentiable function of x.

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Evaluating an Expression In Exercises 21–24, evaluate each expression without using a calculator. (*Hint:* See Example 3.)

21. (a) $\sin\left(\arctan\frac{3}{4}\right)$

22. (a) $\tan\left(\arccos\frac{\sqrt{2}}{2}\right)$

23. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$

24. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$

Simplifying an Expression Using a Right Triangle In Exercises 25–32, write the expression in algebraic form. (*Hint:* Sketch a right triangle, as demonstrated in Example 3.)

25. $\cos(\arcsin 2x)$

26. $\sec(\arctan 4x)$

29. $\tan\left(\text{arcsec}\,\frac{x}{3}\right)$

31. $\csc\left(\arctan\frac{x}{\sqrt{2}}\right)$

Answers

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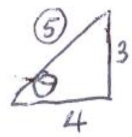
Evaluating an Expression In Exercises 21–24, evaluate each expression without using a calculator. (*Hint:* See Example 3.)

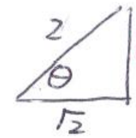
21. (a) $\sin\left(\arctan \frac{3}{4}\right)$



$\tan\theta = 3/4$

$= \boxed{\frac{3}{5}}$

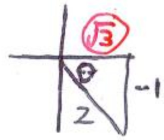
22. (a) $\tan\left(\arccos \frac{\sqrt{2}}{2}\right)$



$\cos\theta = \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{\sqrt{2}} = \boxed{1}$

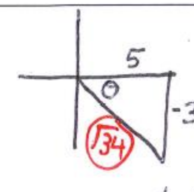
23. (a) $\cot\left[\arcsin\left(-\frac{1}{2}\right)\right]$



$= \frac{\sqrt{3}}{-1} = \boxed{-\sqrt{3}}$

$[-\pi/2, \pi/2]$

24. (a) $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$



$\tan\theta = \frac{-3}{5}$

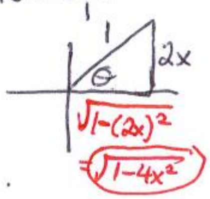
$= \boxed{\frac{\sqrt{34}}{5}}$

$[-\pi/2, \pi/2]$

Simplifying an Expression Using a Right Triangle In Exercises 25–32, write the expression in algebraic form. (*Hint:* Sketch a right triangle, as demonstrated in Example 3.)

25. $\cos(\arcsin 2x)$

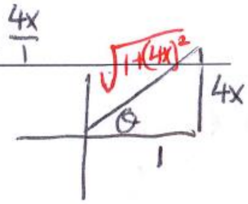
$\sin\theta = \frac{2x}{1}$



$= \frac{\sqrt{1-4x^2}}{1} = \boxed{\sqrt{1-4x^2}}$

26. $\sec(\arctan 4x)$

$\tan\theta = \frac{4x}{1}$

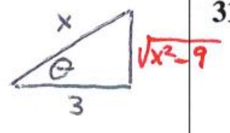


$= \frac{\sqrt{1+16x^2}}{1}$

$= \boxed{\sqrt{1+16x^2}}$

29. $\tan\left(\text{arcsec} \frac{x}{3}\right)$

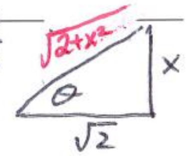
$\sec\theta = \frac{x}{3}$



$= \boxed{\frac{\sqrt{x^2-9}}{3}}$

31. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$

$\tan\theta = \frac{x}{\sqrt{2}}$



$= \boxed{\frac{\sqrt{2+x^2}}{x}}$

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Finding a Derivative **In Exercises 39–58, find the derivative of the function.**

39. $f(x) = 2\arcsin(x-1)$

44. $f(x) = \arctan\sqrt{x}$

46. $h(x) = x^2 \arctan 5x$

47. $h(t) = \sin(\arccos t)$

50. $y = \ln(t^2+4) - \frac{1}{2}\arctan\frac{t}{2}$

Answers

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Finding a Derivative **In Exercises 39–58, find the derivative of the function.**

39. $f(x) = 2\arcsin(x-1)$

$$f'(x) = 2\cdot\left(\frac{1}{\sqrt{1-(x-1)^2}}\right) = \boxed{\frac{2}{\sqrt{1-(x-1)^2}}}$$

44. $f(x) = \arctan\sqrt{x}$ $\quad \arctan\left(x^{1/2}\right)$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}}{1+(\sqrt{x})^2} = \frac{\frac{1}{2\sqrt{x}}}{1+x}$$

$$= \boxed{\frac{1}{2\sqrt{x}(1+x)}}$$

46. $h(x) = \underbrace{x^2}_{f}\,\underbrace{\arctan 5x}_{g}$

*product rule

$$h'(x) = \underbrace{2x}_{f'}\cdot\underbrace{\arctan(5x)}_{g} + \underbrace{x^2}_{f}\cdot\underbrace{\frac{5}{1+(5x)^2}}_{g'}$$

$$\boxed{h'(x) = 2x\arctan(5x) + \frac{5x^2}{1+25x^2}}$$

47. $h(t) = \sin(\arccos t)$

*chain rule

$$h'(t) = \underbrace{\cos(\arccos t)}_{t}\cdot\frac{-1}{\sqrt{1-t^2}}$$

out: sin( )

in: arccost

$$h'(t) = t\cdot\left(\frac{-1}{\sqrt{1-t^2}}\right)$$

$$\boxed{h'(t) = \frac{-t}{\sqrt{1-t^2}}}$$

50. $y = \ln(t^2+4) - \frac{1}{2}\arctan\frac{t}{2}$ $\quad \leftarrow \frac{1}{2}\arctan\left(\frac{1}{2}t\right)$

*apply $\frac{d}{dx}\ln u = \frac{u'}{u}$

$$y' = \frac{2t}{t^2+4} - \frac{1}{2}\cdot\frac{\frac{1}{2}}{1+\left(\frac{t}{2}\right)^2}$$

$$y' = \frac{2t}{t^2+4} - \frac{1}{4\left(1+\frac{t^2}{4}\right)}$$

$$y' = \frac{2t}{t^2+4} - \frac{1}{4+t^2} = \boxed{\frac{2t-1}{t^2+4}}$$