Prerequisites

Before starting this Section you should ...

- have a good knowledge of the exponential function
- have knowledge of odd and even functions
- have familiarity with the definitions of $\tan,\ \sec,\ \text{cosec}, \cot$ and of trigonometric identities

Key Point 3

Hyperbolic Functions

$$\cosh x \equiv \frac{1}{2}(\mathrm{e}^{x} + \mathrm{e}^{-x})$$

$$\sinh x \equiv \frac{1}{2}(\mathrm{e}^{x} - \mathrm{e}^{-x})$$

These two functions, when added and subtracted, give

$$\cosh x + \sinh x \equiv \mathrm{e}^{x} \qquad \text{and} \qquad \cosh x - \sinh x \equiv \mathrm{e}^{-x}$$

The graphs of $\cosh x$ and $\sinh x$ are shown in Figure 4.

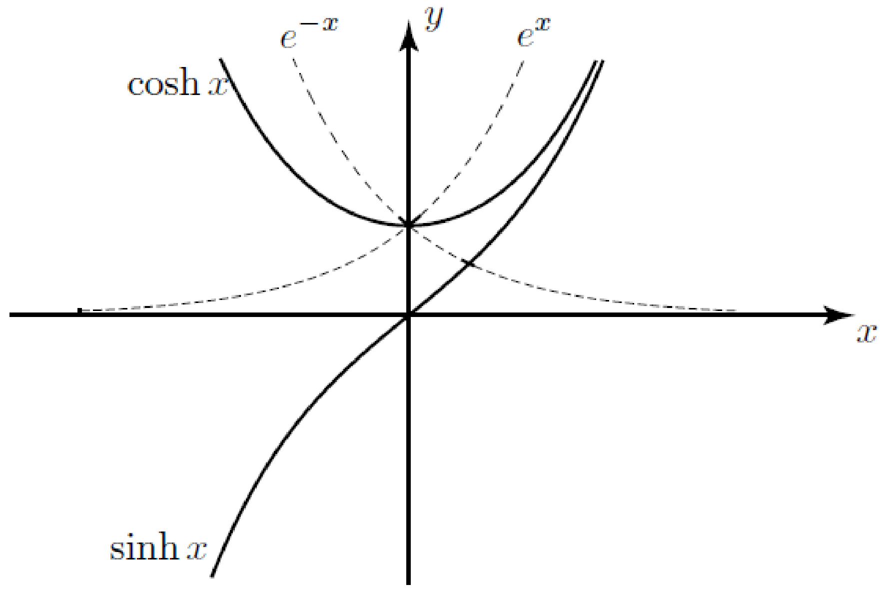

Figure 4: $\sinh x$ and $\cosh x$

Note that $\cosh x > 0$ for all values of x and that $\sinh x$ is zero only when $x = 0$.

Key Point 4

The fundamental identity relating hyperbolic functions is:

$$\cosh^2 x - \sinh^2 x \equiv 1$$

This is the hyperbolic function equivalent of the trigonometric identity: $\cos^2 x + \sin^2 x \equiv 1$

Key Point 5

Hyperbolic Identities

- $\cosh^2 - \sinh^2 \equiv 1$
- $\cosh(x + y) \equiv \cosh x \cosh y + \sinh x \sinh y$
- $\sinh(x + y) \equiv \sinh x \cosh y + \cosh x \sinh y$
- $\sinh 2x \equiv 2 \sinh x \cosh y$
- $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$ or $\cosh 2x \equiv 2\cosh^2 - 1$ or $\cosh 2x \equiv 1 + 2\sinh^2 x$

3. Related hyperbolic functions

Given the trigonometric functions $\cos x$, $\sin x$ related functions can be defined; $\tan x$, $\sec x$, $\text{cosec } x$ through the relations:

$$\tan x \equiv \frac{\sin x}{\cos x} \qquad \sec x \equiv \frac{1}{\cos x} \qquad \text{cosec } x \equiv \frac{1}{\sin x} \qquad \cot x \equiv \frac{\cos x}{\sin x}$$

In an analogous way, given $\cosh x$ and $\sinh x$ we can introduce hyperbolic functions $\tanh x$, $\text{sec h } x$, $\text{cosech } x$ and $\coth x$. These functions are defined in the following Key Point:

Key Point 6

Further Hyperbolic Functions

$$\begin{aligned} \tanh x &\equiv \frac{\sinh x}{\cosh x} \\ \text{sech } x &\equiv \frac{1}{\cosh x} \\ \text{cosech } x &\equiv \frac{1}{\sinh x} \\ \coth x &\equiv \frac{\cosh x}{\sinh x} \end{aligned}$$

Learning Outcomes

On completion you should be able to ...

- explain how hyperbolic functions are defined in terms of exponential functions
- obtain and use hyperbolic function identities
- manipulate expressions involving hyperbolic functions