Derivative as a Rate of Change … Set 1

Average rates of change (Word Problems)

[1]. A train travels from A to B to C. The distance from A to B is 10 miles and the distance from B to C is 40 miles. The average velocity from A to B was 20 miles per hour and the average velocity from B to C was 40 miles per hour. What was the average velocity from A to C in miles per hour?

(a) 180/5 (b) 90/3 (c) 100/3 (d) 180/3 (e) 100/5

[2]. A train travels from city A to city B. It leaves city A at 10:30 am and arrives at city B at 1:30 pm. The distance between the cities is 150 miles. What was the average velocity of the train in miles per hour?

(a) 60 (b) 150 (c) 50 (d) 75 (e) 130

[3]. A train travels from city A to city B to city C. The distance from A to B is 20 miles. The distance from B to C is 45 miles. The train took 1 hour for the trip from A to B, stopped at city B for 30 minutes, and then went from B to C at an average velocity of 30 miles per hour. What was the average velocity of the train for the entire trip (in miles per hour)?

(a) 65 (b) 25 (c) $\frac{65}{2}$ (d) 50 (e) $\frac{65}{3}$

[4]. A train travels from A to B to C. The distance from A to B is 30 miles and the distance from B to C is 80 miles. The train leaves A at 10:00 AM and arrives at C at 3:00 PM. The average speed from A to B was 30 miles per hour. What was the average speed from B to C in miles per hour?

(a) 20 (b) 25 (c) 30 (d) 35 (e) 40

[5]. A train travels from city A to city B. The cities are 600 miles apart. The distance from city A at t hours after the train leaves A is given by

$$d(t) = 50t + t^2.$$

What is the average velocity of the train in miles per hour during the trip from A to B?
(**Hint:** First find how long it takes for the train to get from A to B.)

(a) 50 (b) 55 (c) 60 (d) 65 (e) 70

[6]. John leaves at 9:00 am and drives from Lexington to Ashland arriving at 11:00 am. He stops for two hours since his girlfriend Mary is not yet ready. Then they drive together from Ashland to Columbus arriving at Columbus after a three-hour drive. The distance from Lexington to Ashland is 110 miles and the distance from Ashland to Columbus is 130 miles. Find the average velocity of John's car in miles per hour for the entire trip (including the two hour stop) correct to two decimal places.

(a) 33.81 (b) 33.42 (c) 35.00 (d) 34.29 (e) 34.47

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Average rates of change

[7]. If $g(x) = (x-1)^2$ what is the average rate of change of $g(x)$ with respect to x as x changes from -3 to 3?

(a) -4 (b) -2 (c) 0 (d) 2 (e) 4

[8]. Suppose that $h(t) = \frac{2}{t}$. Find the average rate of change of $h(t)$ from $t = 5$ to $t = 10$.

(a) $-.05$ (b) $-.04$ (c) .05 (d) .04 (e) .02

[9]. Find the average rate of change of the function $R(t) = \sqrt{2t+7}$ as t changes from 1 to 9.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 4 (e) 2

[10]. If $g(x) = |x-7|$ what is the average rate of change of $g(x)$ with respect to x as x changes from -3 to 3?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

[11]. Find the average rate of change of the function $G(t) = |t^2 - 1|$ as t changes from -1 to 2.

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

[12]. Let $g(s) = s^2 - 3s + 1$. Find a value $A \geq 0$ such that the average rate of change of $g(s)$ from 0 to A equals 8.

(a) 0 (b) 8 (c) 11 (d) 15 (e) 22

[13]. Suppose $f(t) = t^3 + 1$. Find a value A greater than 0 such that the average rate of change of $f(t)$ from 0 to A equals 2.

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2 (e) $\sqrt{5}$

Derivative as a Rate of Change … Set 1

Difference quotients

[14]. Compute $\dfrac{f(2+h)-f(2)}{h}$ where $f(x)=3x^2+1$.

(a) 12 **(b)** $12+h$ **(c)** $12+2h$ **(d)** $12+3h$ **(e)** None of the above

[15]. What is the average rate of change of $g(s)=s^2-4$ as s changes from 1 to $1+h$?

(a) $6+3h$ **(b)** $2+h$ **(c)** $4+2h$ **(d)** 2 **(e)** h

[16]. Let $f(x)=2x^2-3x$. Find the average rate of change of $f(x)$ from $x=3$ to $x=3+h$.

(a) $9-h$ **(b)** $9+h$ **(c)** 9 **(d)** $9-2h$ **(e)** $9+2h$

[17]. Let $g(t)=(t-5)^2+1$. What is the average rate of change of $g(t)$ as t changes from 4 to $4+h$?

(a) h^2-2h **(b)** $h+2$ **(c)** h^2+2h **(d)** $h-2$ **(e)** 1

[18]. If $f(t)=3t^2+4$ then

$$\frac{f(1+h)-f(1)}{h}=$$

(a) $4+3h$ **(b)** $3+4h$ **(c)** $6+3h$ **(d)** $8+3h$ **(e)** $8+4h$

[19]. If $f(t)=1/t$ then

$$\frac{f(t+h)-f(t)}{h}=$$

(a) $1/(h^2)$ **(b)** $1/(t(t+h))$ **(c)** $-1/(t(t+h))$

(d) $1/(t(t-h))$ **(e)** $-1/(t(t-h))$

Derivative as a Rate of Change … Set 1

Instantaneous rates of change

[20]. Consider a triangle with base x and height $2x$. Find the instantaneous rate of change of the area of the triangle with respect to x when $x = 5$.

(a) 1 (b) 2 (c) 5 (d) 10 (e) 20

[21]. Find the instantaneous rate of change of the function $H(t) = t^3$ at $t = 2$.

(a) 2 (b) 3 (c) 8 (d) 12 (e) 27

In what follows, you may use the following formula for the derivative of a quadratic function.

If $p(x) = Ax^2 + Bx + C$, then $p'(x) = 2Ax + B$.

[22]. If $g(s) = 3s^2 + s - 2$ what is the value of $g(s)$ when the instantaneous rate of change of $g(s)$ with respect to s equals 1?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

[23]. If $g(s) = 3s^2 + 2s - 2$ what is the value of s for which the instantaneous rate of change of $g(s)$ with respect to s equals 8?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

[24]. Suppose the price of a good is given by the quadratic function $P(t) = 2.58 + .14t + .01t^2$. What is the instantaneous rate of change in the price when $t = 3$?

(a) .18 (b) .20 (c) .22 (d) .24 (e) .26

Derivative as a Rate of Change … Set 1

[25]. Let $g(x) = x^2 + 4x + 5$. Find a value of c between 1 and 10 such that the average rate of change of $g(x)$ from $x = 1$ to $x = 10$ is equal to the instantaneous rate of $g(x)$ at $x = c$.

(a) 4.75 **(b)** 5.0 **(c)** 5.25 **(d)** 5.5 **(e)** 5.75

[26]. Find a nonnegative number A such that the average rate of change of $F(t) = t^2 - 2t + 1$ from $t = 1$ to $t = A$ equals the instantaneous rate of change of $F(t)$ at $t = 2$.

(a) $A = 0$ **(b)** $A = 2$ **(c)** $A = 3$ **(d)** $A = 4$ **(e)** $A = 5$

[27]. Suppose the cost $C(q)$ (in dollars) of producing a quantity q of a product equals

$$C(q) = 500 + 2q + \frac{1}{5}q^2.$$

The marginal cost $MC(q)$ equals the instantaneous rate of change of the total cost. Find the marginal cost when a quantity of 10 items are being produced.

(a) 2 **(b)** 6 **(c)** 10 **(d)** 20 **(e)** 500

Derivative as a Rate of Change … Set 1

Tangent lines

[28]. Find the slope of the tangent line to the graph of $f(x) = 3x^2 - 7x + 4$ at $x = 2$.

(a) 5 **(b)** 6 **(c)** 7 **(d)** 8 **(e)** 9

[29]. Find the equation of a line tangent to the curve $y = 2x^2 + x + 1$ at $x = 2$.

(a) $y = 9 + 11(x - 2)$ **(b)** $y = 11 + 9(x - 2)$ **(c)** $y = 22 + 13(x - 3)$

(d) $y = 13 + 22(x - 3)$ **(e)** $y = 7 + (4x + 1)$

[30]. Suppose $G(x) = x^2 + x - 2$. For what value of x is the tangent line to the graph of $y = G(x)$ parallel to the x-axis?

(a) $x = -1$ **(b)** $x = 0$ **(c)** $x = 2$ **(d)** $x = 1/2$ **(e)** $x = -1/2$

[31]. Suppose $g(s) = s^2 + 4s + 1$. Find a point of the graph of $t = g(s)$ such that the tangent line to the graph is parallel to the s-axis.

(a) $(2, 9)$ **(b)** $(-1, -2)$ **(c)** $(-2, -3)$ **(d)** $(-4, 8)$ **(e)** $(-4, 1)$

[32]. What is the value of x such that the slope of the tangent line to the graph of $f(x) = x^2 - 10x + 14$ is 6?

(a) 6 **(b)** 7 **(c)** 8 **(d)** 9 **(e)** There is no such x

[33]. Suppose $g(s) = s^2 + 1$. Find a point on the graph of $t = g(s)$ such that the tangent line to the graph is parallel to the line with equation $t = s$.

(a) $(0, 1)$ **(b)** $(1/2, 5/4)$ **(c)** $(1, 2)$ **(d)** $(3/2, 13/4)$ **(e)** $(2, 5)$

Derivative as a Rate of Change … Set 1

Velocity

[34]. Suppose $h(t)$ represents the height of an object above the ground at time t, where the height is measured in feet and the time t is measured in seconds. If

$$h(t) = -16t^2 + 48t + 144,$$

what is the velocity of the object at time $t = 0$?

(a) 48 feet per second **(b)** 144 miles per hour **(c)** 32 furlongs per fortnight

(d) 64 feet per second **(e)** 96 feet per second

[35]. If $h(t)$ represents the height of an object above ground level at time t and $h(t)$ is given by

$$h(t) = -16t^2 + 96t + 1,$$

find the height of the object at the time when the velocity is zero.

(a) 144 **(b)** 145 **(c)** 148 **(d)** 150 **(e)** 160

[36]. Suppose the position $P(t)$ of an object at time t is given by $t^2 + 1$. Find a value of t at which the instantaneous velocity of the object equals the average velocity on the interval $[0, 1]$.

(a) 1/2 **(b)** 1 **(c)** 3/2 **(d)** 2 **(e)** 5/2