Derivative Graphs … Set 2

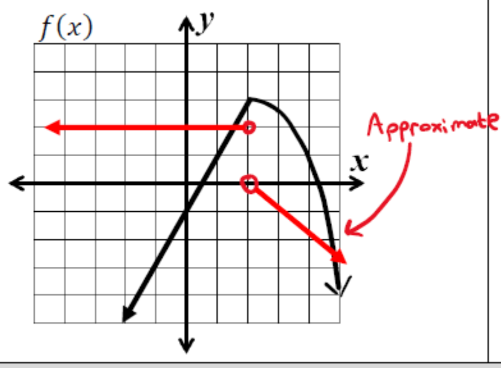
Sketching Graphs of Derivatives

Calculus

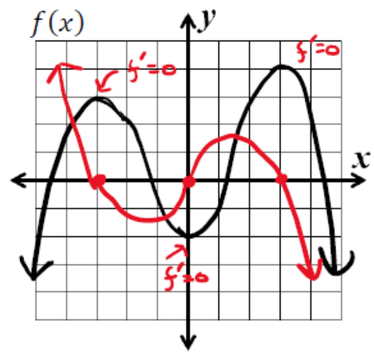
Practice

The graph of a function f is shown. On the same coordinate plane, sketch a graph of f', the derivative of f.

1.

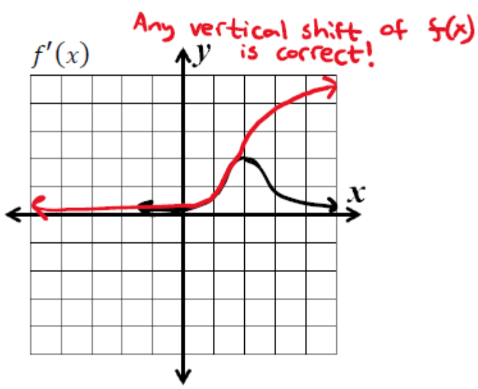


2.

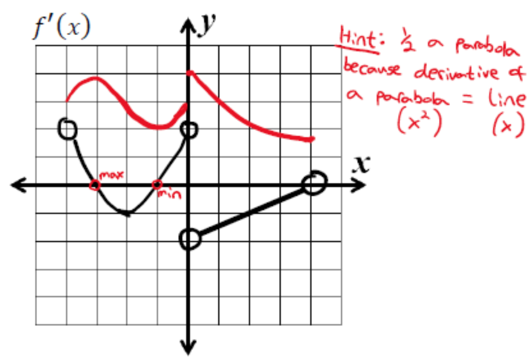


The graph of f', the derivative of f, is shown. On the same coordinate plane, sketch a possible graph of f.

3.



4.



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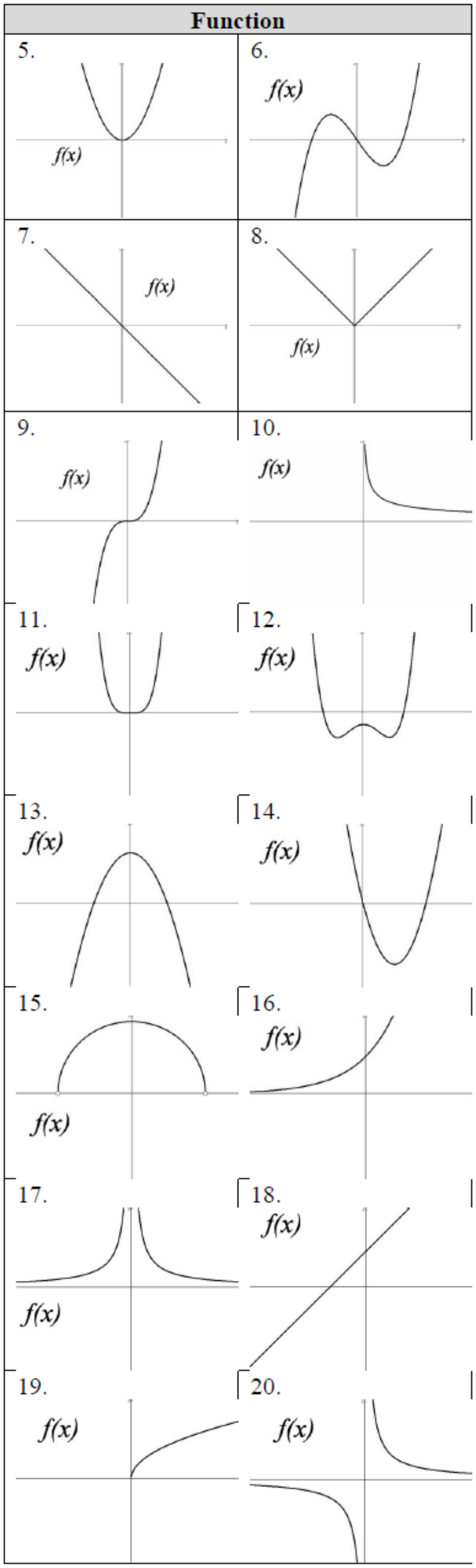
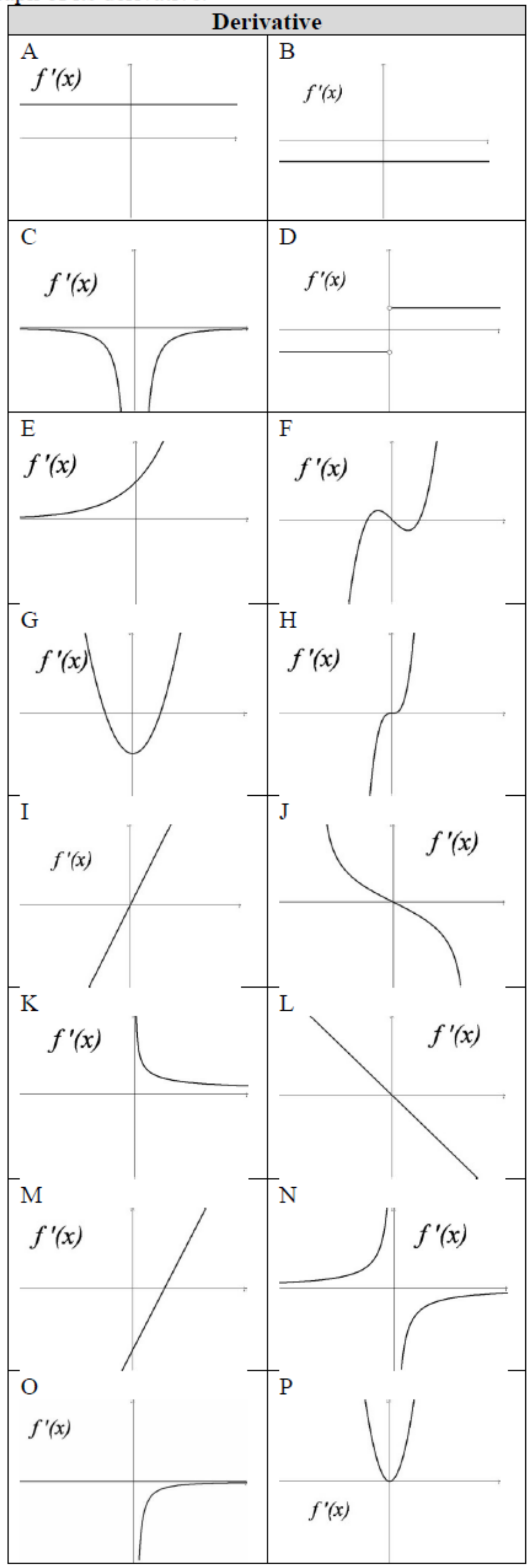
Match each function with the graph of its derivative.

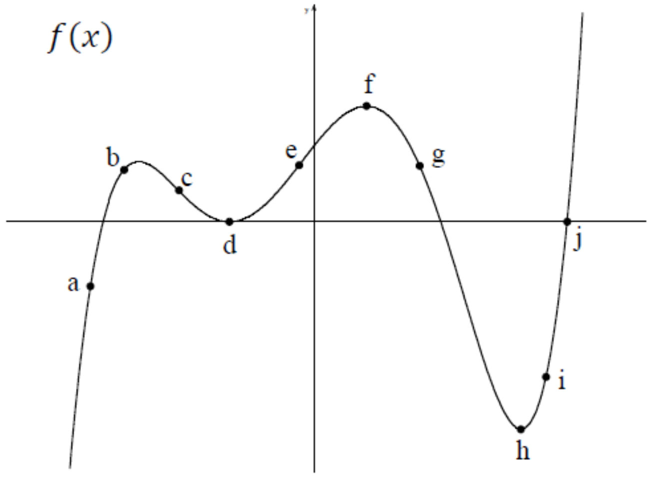


5. I
6. G
7. B
8. D
9. P
10. O
11. H
12. F
13. L
14. M
15. J
16. E
17. N
18. A
19. K
20. C



Derivative Graphs … Set 2

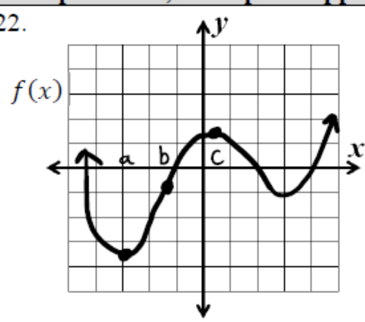
21. Using the figure below, complete the chart by indicating whether each value is positive (+), negative (-), or zero (0) at the indicated points. For these problems, if the point appears to be a max or min, assume it is. If it appears to be a point of inflection, assume it is.



x	a	b	c	d	e	f	g	h	i	j
$f(x)$	−	+	+	0	+	+	+	−	−	0
$f'(x)$	+	+	−	0	+	0	−	0	+	+
$f''(x)$	−	−	0	+	0	−	−	+	+	+

Place the values of $f(x)$, $f'(x)$, and $f''(x)$ in increasing order for each point on the graph of $f(x)$. For these problems, if the point appears to be a max, min, or point of inflection assume it is.

22.



$f(a) < f'(a) < f''(a)$

$f(b) < f''(b) < f'(b)$

$f''(c) < f'(c) < f(c)$

Sketching Graphs of Derivatives

Test Prep

23. The graph of the function f is shown in the figure to the right. For which of the following values of x is $f'(x)$ negative and decreasing.

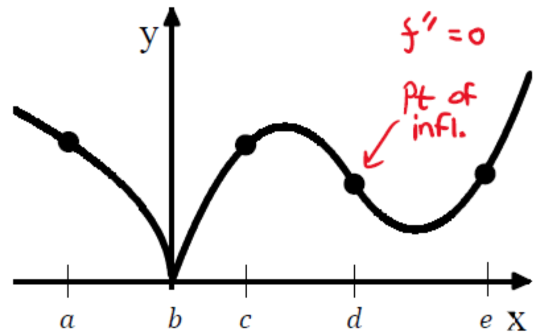
(A) a

(B) b

(C) c

(D) d

(E) e



Derivative Graphs … Set 2

24. Let f be a function that is continuous on the closed interval $[0, 4]$. The function f and its derivatives have the properties indicated in the table below.

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$	4
$f(x)$	1	Pos.	0	Neg.	-2	Neg.	0	Neg.	-1
$f'(x)$	0	Neg.	-20	Neg.	0	Pos.	DNE	Neg.	0
$f''(x)$	0	Neg.	0	Pos.	0	Pos.	DNE	Pos.	0

(a) Find the x-coordinate of each point at which f attains a maximum value or a minimum value.

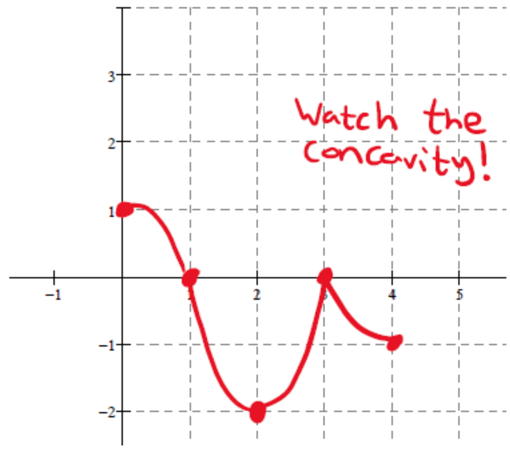
max at $x=0$ and $x=3$

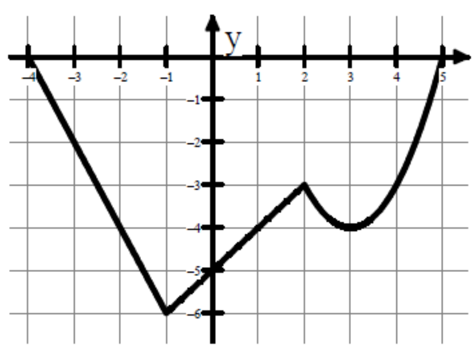
min at $x=2$ and $x=4$

(b) Find the x-coordinate of each point of inflection on the graph of f.

$x=1$

(c) In the xy-plane provided sketch the graph of a function with all the above characteristics of f.





Graph of $g(x)$

Derivative Graphs … Set 2

25. The continuous function g is defined on the closed interval $[-4, 5]$. The graph of g consists of two line segments and a parabola. Let f be a function such that $f'(x) = g(x)$.

a. Fill in the missing entries in the table below to describe the behavior of g' and g''. Indicate Positive, Negative, or 0. Give reasons for your answers.

x	$-4 < x < -1$	$-1 < x < 2$	$2 < x < 3$	$3 < x < 5$
$g(x)$	Negative	Negative	Negative	Negative
$g'(x)$	**Negative**	**Positive**	**Negative**	**Positive**
$g''(x)$	**0**	**0**	**Positive**	**Positive**

$g'(x)$ is negative for $-4 < x < -1$ and $2 < x < 3$ because g is decreasing there.
$g'(x)$ is positive for $-1 < x < 2$ and $3 < x < 5$ because g is increasing there.
$g''(x) = 0$ for $-4 < x < -1$ and $-1 < x < 2$ the graph of g is linear there.
$g''(x)$ is positive for $2 < x < 3$ and $3 < x < 5$ because the graph of g is concave up there.

b. There is no value of x in the open interval $(0, 3)$ at which $g'(x) = \frac{g(3)-g(0)}{3-0}$. Explain why this does not violate the Mean Value Theorem.

The Mean Value Theorem can only be applied if g is differentiable on the interval. At $x = 2$, there is a sharp corner and g is not differentiable.

Therefore it cannot be applied on the interval $0 < x < 3$.

c. Find all values x in the open interval $(-4, 5)$ at which the graph of f has a point of inflection. Explain your reasoning.

The graph of f has a point of inflection at $x = -1, x = 2$, and $x = 3$.
$f'(x) = g(x)$ changes from increasing to decreasing at $x = 2$, and $f'(x) = g(x)$ changes from decreasing to increasing at $x = -1$ and $x = 3$.

d. At what value of x does f attain its absolute minimum on the closed interval $[-4, 5]$? Give a reason for your answer.

Because $f'(x) = g(x) < 0$ on the interval $(-4, 5)$, f is decreasing on the interval $(-4, 5)$. Therefore, the absolute minimum value of f on the closed interval $[-4, 5]$ occurs at the right endpoint $x = 5$.

Derivative Graphs … Set 2

25. The continuous function g is defined on the closed interval $[-4, 5]$. The graph of g consists of two line segments and a parabola. Let f be a function such that $f'(x) = g(x)$.

a. Fill in the missing entries in the table below to describe the behavior of g' and g''. Indicate Positive, Negative, or 0. Give reasons for your answers.

x	$-4 < x < -1$	$-1 < x < 2$	$2 < x < 3$	$3 < x < 5$
$g(x)$	Negative	Negative	Negative	Negative
$g'(x)$	**Negative**	**Positive**	**Negative**	**Positive**
$g''(x)$	**0**	**0**	**Positive**	**Positive**

$g'(x)$ is negative for $-4 < x < -1$ and $2 < x < 3$ because g is decreasing there.
$g'(x)$ is positive for $-1 < x < 2$ and $3 < x < 5$ because g is increasing there.
$g''(x) = 0$ for $-4 < x < -1$ and $-1 < x < 2$ the graph of g is linear there.
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b. There is no value of x in the open interval $(0, 3)$ at which $g'(x) = \frac{g(3)-g(0)}{3-0}$. Explain why this does not violate the Mean Value Theorem.

The Mean Value Theorem can only be applied if g is differentiable on the interval. At $x = 2$, there is a sharp corner and g is not differentiable. Therefore it cannot be applied on the interval $0 < x < 3$.

c. Find all values x in the open interval $(-4, 5)$ at which the graph of f has a point of inflection. Explain your reasoning.

The graph of f has a point of inflection at $x = -1, x = 2$, and $x = 3$.
$f'(x) = g(x)$ changes from increasing to decreasing at $x = 2$, and $f'(x) = g(x)$ changes from decreasing to increasing at $x = -1$ and $x = 3$.

d. At what value of x does f attain its absolute minimum on the closed interval $[-4, 5]$? Give a reason for your answer.

Because $f'(x) = g(x) < 0$ on the interval $(-4, 5)$, f is decreasing on the interval $(-4, 5)$. Therefore, the absolute minimum value of f on the closed interval $[-4, 5]$ occurs at the right endpoint $x = 5$.