Derivative of Sine and Cosine … Set 1

Calculus **Trig Derivatives Worksheet #1**

$\frac{d}{dx}\sin u = \cos u * u'$	$\frac{d}{dx}\cos u = -\sin u * u'$
$\frac{d}{dx}\tan u = \sec^2 u * u'$	$\frac{d}{dx}\cot u = -csc^2 u * u'$
$\frac{d}{dx}\sec u = \sec u \tan u * u'$	$\frac{d}{dx}\csc u = -\csc u \cot u * u'$

Finding a Derivative of a Trigonometric Function In Exercises 39–54, find the derivative of the trigonometric function.

39. $f(t) = t^2 \sin t$

42. $f(x) = \frac{\sin x}{x^3}$

43. $f(x) = -x + \tan x$

44. $y = x + \cot x$

51. $f(x) = x^2 \tan x$

52. $f(x) = \sin x \cos x$

53. $y = 2x \sin x + x^2 \cos x$

54. $h(\theta) = 5\theta \sec \theta + \theta \tan \theta$

Derivative of Sine and Cosine … Set 1

Answers

Calculus	Trig Derivatives
$\frac{d}{dx}\sin u = \cos u * u'$	$\frac{d}{dx}\cos u = -\sin u * u'$
$\frac{d}{dx}\tan u = \sec^2 u * u'$	$\frac{d}{dx}\cot u = -\csc^2 u * u'$
$\frac{d}{dx}\sec u = \sec u \tan u * u'$	$\frac{d}{dx}\csc u = -\csc u \cot u * u'$

Finding a Derivative of a Trigonometric Function In Exercises 39–54, find the derivative of the trigonometric function.

*product rule $f'g + fg'$

39. $f(t) = t^2 \sin t$

$f'(t) = \overbrace{2t}^{f'} \cdot \overbrace{\sin t}^{g} + \overbrace{t^2}^{f} \cdot \overbrace{\cos(t)}^{g'}$

$f'(t) = 2t\sin(t) + t^2\cos(t)$

*quotient rule $\frac{f'g - fg'}{g^2}$

42. $f(x) = \frac{\sin x}{x^3}$

$f'(x) = \frac{\overbrace{\cos x}^{f'} \cdot \overbrace{x^3}^{g} - \overbrace{\sin x}^{f} \cdot \overbrace{3x^2}^{g'}}{\overbrace{(x^3)^2}^{g^2}}$

$f'(x) = \frac{x^3\cos x - 3x^2\sin x}{x^6}$

$f'(x) = \frac{x^2(x\cos x - 3\sin x)}{x^6}$

$f'(x) = \frac{x\cos x - 3\sin x}{x^4}$

43. $f(x) = -x + \tan x$

$f'(x) = -1 + \sec^2 x$

44. $y = x + \cot x$

$y' = 1 - \csc^2 x$

51. $f(x) = x^2 \tan x$

$f'(x) = \overbrace{2x}^{f'} \overbrace{\tan x}^{g} + \overbrace{x^2}^{f} \overbrace{\sec^2 x}^{g'}$

$f'(x) = 2x\tan x + x^2\sec^2 x$

52. $f(x) = \sin x \cos x$

$f'(x) = \overbrace{(\cos x)}^{f'} \overbrace{(\cos x)}^{g} + \overbrace{(\sin x)}^{f} \overbrace{(-\sin x)}^{g'}$

$f'(x) = \cos^2 x - \sin^2 x$

53. $y = 2x \sin x + x^2 \cos x$

$y' = \overbrace{2}^{f'} \overbrace{\sin x}^{g} + \overbrace{2x}^{f} \overbrace{\cos x}^{g'} + \overbrace{2x}^{f'} \overbrace{\cos x}^{g} + \overbrace{x^2}^{f} \overbrace{(-\sin x)}^{g'}$

$y' = 4x\cos x + 2\sin x - x^2\sin x$

54. $h(\theta) = 5\theta \sec\theta + \theta\tan\theta$

$h'(\theta) = \overbrace{5}^{f'} \overbrace{\sec\theta}^{g} + \overbrace{5\theta}^{f} \overbrace{\sec\theta\tan\theta}^{g'} + \overbrace{1}^{f'} \overbrace{\tan\theta}^{g} + \overbrace{\theta}^{f} \overbrace{\sec^2\theta}^{g'}$

$h'(\theta) = 5\sec\theta + 5\theta\sec\theta\tan\theta + \tan\theta + \theta\sec^2\theta$

Derivative of Sine and Cosine … Set 1

Finding a Derivative In Exercises 43–64, find the derivative of the function.

$\frac{d}{dx}\sin u = \cos u * u'$	$\frac{d}{dx}\cos u = -\sin u * u'$
$\frac{d}{dx}\tan u = \sec^2 u * u'$	$\frac{d}{dx}\cot u = -csc^2 u * u'$
$\frac{d}{dx}\sec u = \sec u \tan u * u'$	$\frac{d}{dx}\csc u = -\csc u \cot u * u'$

43. $y = \cos 4x$

53. $y = 4\sec^2 x$

54) $g(x) = 5\cos^3 \pi x$

58. $h(t) = 2\cot^2(\pi t + 2)$

Finding a Derivative In Exercises 1–16, find dy/dx by implicit differentiation.

11. $\sin x + 2\cos 2y = 1$

13. $\sin x = x(1 + \tan y)$

14. $\cot y = x - y$

15. $y = \sin xy$

Derivative of Sine and Cosine … Set 1

Answers

$\frac{d}{dx}\sin u = \cos u * u'$	$\frac{d}{dx}\cos u = -\sin u * u'$
$\frac{d}{dx}\tan u = \sec^2 u * u'$	$\frac{d}{dx}\cot u = -\csc^2 u * u'$
$\frac{d}{dx}\sec u = \sec u \tan u * u'$	$\frac{d}{dx}\csc u = -\csc u \cot u * u'$

Finding a Derivative In Exercises 43–64, find the derivative of the function.

*chain rule

43. $y = \cos 4x$

$y' = -\sin(4x) \cdot 4$

$y' = -4\sin(4x)$

53. $y = 4\sec^2 x$ *rewrite expression

$y = 4[\sec x]^2$ out: $4[\ ]^2$ in: $\sec u$ inner: x

$y' = 8(\sec x) \cdot \sec x \tan x \cdot 1$

$y' = 8\sec^2 x \tan x$

57) $g(x) = 5\cos^3 \pi x$ out: $5(\ )^3$ in: $\cos u$ inner: πx

$g(x) = 5[\cos(\pi x)]^3$

$g'(x) = 15[\cos \pi x]^2 \cdot -\sin(\pi x) \cdot \pi$

$g'(x) = -15\pi \cos^2 \pi x \sin \pi x$

58. $h(t) = 2\cot^2(\pi t + 2)$ out: $2(\ )^2$ in: $\cot u$ inner: $\pi t + 2$

$h(t) = 2[\cot(\pi t + 2)]^2$

$h'(x) = 4(\cot(\pi t + 2)) \cdot -\csc^2(\pi t + 2) \cdot \pi$

$h'(t) = -4\pi \cot(\pi t + 2)\csc^2(\pi t + 2)$

Finding a Derivative In Exercises 1–16, find dy/dx by implicit differentiation.

*implicit *product rule

11. $\sin x + 2\cos 2y = 1$

$\cos x - 2\sin(2y) \cdot 2\left(\frac{dy}{dx}\right) = 0$

$-4\sin 2y \left(\frac{dy}{dx}\right) = -\cos x$

$\frac{dy}{dx} = \frac{-\cos x}{-4\sin 2y} = \frac{\cos x}{4\sin 2y}$

13. $\sin x = x(1 + \tan y)$

$\sin x = x + x\tan y$

$\cos x = 1 + \overset{f'}{1} \cdot \overset{g}{\tan y} + \overset{f}{x} \cdot \overset{g'}{\sec^2 y}\left(\frac{dy}{dx}\right)$

$\cos x - 1 - \tan(y) = x\sec^2 y \left(\frac{dy}{dx}\right)$

$\frac{\cos x - 1 - \tan(y)}{x\sec^2 y} = \frac{dy}{dx}$

14. $\cot y = x - y$

$-\csc^2 y \left(\frac{dy}{dx}\right) = 1 - \left(\frac{dy}{dx}\right)$

$-\csc^2 y \left(\frac{dy}{dx}\right) + 1\left(\frac{dy}{dx}\right) = 1$

$\frac{dy}{dx}\left(1 - \csc^2 y\right) = 1$

$\frac{dy}{dx} = \frac{1}{1 - \csc^2 y}$

15. $y = \sin xy$

$1\left(\frac{dy}{dx}\right) = \cos(xy) \cdot \left[1 \cdot y + x\left(\frac{dy}{dx}\right)\right]$

$1\left(\frac{dy}{dx}\right) = y\cos(xy) + x\cos(xy)\left(\frac{dy}{dx}\right)$

$1\left(\frac{dy}{dx}\right) - x\cos(xy)\left(\frac{dy}{dx}\right) = y\cos(xy)$

$\frac{dy}{dx}\left(1 - x\cos(xy)\right) = y\cos(xy)$

$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$