Differentiation - Product Rule

Differentiate each function with respect to *x*.

1)
$$y = -x^3(3x^4 - 2)$$

2) $f(x) = x^2(-3x^2 - 2)$

3)
$$y = (-2x^4 - 3)(-2x^2 + 1)$$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$

5)
$$f(x) = (5x^5 + 5)(-2x^5 - 3)$$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

7)
$$y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$$

8)
$$y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$$

Answers

Differentiate each function with respect to *x*.

1)
$$y = -x^{3}(3x^{4} - 2)$$

$$\frac{dy}{dx} = -x^{3} \cdot 12x^{3} + (3x^{4} - 2) \cdot -3x^{2}$$

$$= -21x^{6} + 6x^{2}$$
2) $f(x) = x^{2}(-3x^{2} - 2)$
 $f'(x) = x^{2} \cdot -6x + (-3x^{2} - 2) \cdot 2x$
 $= -12x^{3} - 4x$

3)
$$y = (-2x^4 - 3)(-2x^2 + 1)$$

 $\frac{dy}{dx} = (-2x^4 - 3) \cdot -4x + (-2x^2 + 1) \cdot -8x^3$
 $= 24x^5 - 8x^3 + 12x$

4)
$$f(x) = (2x^4 - 3)(x^2 + 1)$$

 $f'(x) = (2x^4 - 3) \cdot 2x + (x^2 + 1) \cdot 8x^3$
 $= 12x^5 + 8x^3 - 6x$

5)
$$f(x) = (5x^5 + 5)(-2x^5 - 3)$$

 $f'(x) = (5x^5 + 5) \cdot -10x^4 + (-2x^5 - 3) \cdot 25x^4$
 $= -100x^9 - 125x^4$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$
 $f'(x) = (-3 + x^{-3}) \cdot -12x^2 + (-4x^3 + 3) \cdot -3x^{-4}$
 $= 36x^2 - \frac{9}{x^4}$

7)
$$y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$$

 $\frac{dy}{dx} = (-2x^4 + 5x^2 + 4) \cdot -6x + (-3x^2 + 2)(-8x^3 + 10x)$
 $= 36x^5 - 76x^3 - 4x$

8)
$$y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$$

 $\frac{dy}{dx} = (x^4 + 3)(-20x^4 + 20x^3) + (-4x^5 + 5x^4 + 5) \cdot 4x^3$
 $= -36x^8 + 40x^7 - 60x^4 + 80x^3$

9)
$$y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$$

10)
$$f(x) = (-10x^2 - 7\sqrt[5]{x^2} + 9)(2x^3 + 4)$$

11)
$$y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$$

12)
$$y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$$

13)
$$f(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)(x^2 + 10)$$

Answers

9)
$$y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$$

 $\frac{dy}{dx} = (5x^4 - 3x^2 - 1) \cdot -10x + (-5x^2 + 3)(20x^3 - 6x)$
 $= -150x^5 + 120x^3 - 8x$

10)
$$f(x) = (-10x^2 - 7\sqrt[5]{x^2} + 9)(2x^3 + 4)$$
$$f'(x) = (-10x^2 - 7x^{\frac{2}{5}} + 9) \cdot 6x^2 + (2x^3 + 4)(-20x - \frac{14}{5}x^{-\frac{3}{5}})$$
$$= -100x^4 - \frac{238x^{\frac{12}{5}}}{5} + 54x^2 - 80x - \frac{56}{5x^{\frac{3}{5}}}$$

11)
$$y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$$

$$\frac{dy}{dx} = (5 + 3x^{-2})(20x^4 + 18x^2) + (4x^5 + 6x^3 + 10) \cdot -6x^{-3}$$

$$= 100x^4 + 126x^2 + 18 - \frac{60}{x^3}$$

12)
$$y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$$

 $\frac{dy}{dx} = (-6x^4 + 2 + 6x^{-4}) \cdot 24x^3 + (6x^4 + 7)(-24x^3 - 24x^{-5})$
 $= -288x^7 - 120x^3 - \frac{168}{x^5}$

13)
$$f(x) = \left(-7x^{4} + 10x^{\frac{2}{5}} + 8\right)(x^{2} + 10)$$
$$f'(x) = \left(-7x^{4} + 10x^{\frac{2}{5}} + 8\right) \cdot 2x + (x^{2} + 10)\left(-28x^{3} + 4x^{-\frac{3}{5}}\right)$$
$$= -42x^{5} - 280x^{3} + 24x^{\frac{7}{5}} + 16x + \frac{40}{x^{\frac{3}{5}}}$$

Critical thinking question:

14) A classmate claims that $(f \cdot g)' = f' \cdot g'$ for any functions f and g. Show an example that proves your classmate wrong.

Answers

Critical thinking question:

14) A classmate claims that $(f \cdot g)' = f' \cdot g'$ for any functions f and g. Show an example that proves your classmate wrong.

Many answers. Ex: f = 2x, g = 4, $8 \neq 0$