

Logarithmic Differentiation ... Set 1

Logarithmic Differentiation

Use logarithmic differentiation to differentiate each function with respect to x .

1) $y = 2x^{2x}$

2) $y = 5x^{5x}$

3) $y = 3x^{3x}$

4) $y = 4x^{x^4}$

5) $y = (3x^4 + 4)^3 \sqrt{5x^3 + 1}$

6) $y = (x^5 + 5)^2 \sqrt{2x^2 + 3}$

7) $y = \frac{(3x^4 - 2)^5}{(3x^3 + 4)^2}$

8) $y = \sqrt{3x^2 + 1} (3x^4 + 1)^3$

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Answers

Use logarithmic differentiation to differentiate each function with respect to x .

1) $y = 2x^{2x}$

$$\begin{aligned}\frac{dy}{dx} &= y(2 \ln x + 2) \\ &= 4x^{2x}(\ln x + 1)\end{aligned}$$

2) $y = 5x^{5x}$

$$\begin{aligned}\frac{dy}{dx} &= y(5 \ln x + 5) \\ &= 25x^{5x}(\ln x + 1)\end{aligned}$$

3) $y = 3x^{3x}$

$$\begin{aligned}\frac{dy}{dx} &= y(3 \ln x + 3) \\ &= 9x^{3x}(\ln x + 1)\end{aligned}$$

4) $y = 4x^{x^4}$

$$\begin{aligned}\frac{dy}{dx} &= y(4x^3 \ln x + x^3) \\ &= 4x^{x^4+3}(4 \ln x + 1)\end{aligned}$$

5) $y = (3x^4 + 4)^3 \sqrt{5x^3 + 1}$

$$\begin{aligned}\frac{dy}{dx} &= y \left(\frac{36x^3}{3x^4 + 4} + \frac{15x^2}{10x^3 + 2} \right) \\ &= \frac{3x^2(3x^4 + 4)^2(135x^4 + 24x + 20)}{2\sqrt{5x^3 + 1}}\end{aligned}$$

6) $y = (x^5 + 5)^2 \sqrt{2x^2 + 3}$

$$\begin{aligned}\frac{dy}{dx} &= y \left(\frac{10x^4}{x^5 + 5} + \frac{2x}{2x^2 + 3} \right) \\ &= \frac{2x(x^5 + 5)(11x^5 + 15x^3 + 5)}{\sqrt{2x^2 + 3}}\end{aligned}$$

7) $y = \frac{(3x^4 - 2)^5}{(3x^3 + 4)^2}$

$$\begin{aligned}\frac{dy}{dx} &= y \left(\frac{60x^3}{3x^4 - 2} - \frac{18x^2}{3x^3 + 4} \right) \\ &= \frac{6x^2(3x^4 - 2)^4(21x^4 + 40x + 6)}{(3x^3 + 4)^3}\end{aligned}$$

8) $y = \sqrt{3x^2 + 1} (3x^4 + 1)^3$

$$\begin{aligned}\frac{dy}{dx} &= y \left(\frac{3x}{3x^2 + 1} + \frac{36x^3}{3x^4 + 1} \right) \\ &= \frac{3x(3x^4 + 1)^2(39x^4 + 1 + 12x^2)}{\sqrt{3x^2 + 1}}\end{aligned}$$

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$$9) y = \frac{\sqrt{2x^3 + 3}}{(x^4 - 3)^3}$$

$$10) y = (2x^2 - 5)^3 \sqrt{x^2 - 2}$$

Use logarithmic differentiation to differentiate each function with respect to x . You do not need to simplify or substitute for y .

$$11) y = \frac{(5x - 4)^4}{(3x^2 + 5)^5 \cdot (5x^4 - 3)^3}$$

$$12) y = (x + 2)^4 \cdot (2x - 5)^2 \cdot (5x + 1)^3$$

$$13) y = (5x^5 + 2)^2 \cdot (3x^3 - 1)^3 \cdot (3x - 1)^4$$

$$14) y = \frac{(x^2 + 3)^4}{(5x^5 - 2)^5 \cdot (3x^2 - 5)^2}$$

$$15) y = (3x^3 - 4)^5 \cdot (3x - 1)^3 \cdot (5x^3 - 2)^2 \cdot (x + 3)^4$$

$$16) y = \frac{(4x^2 - 5)^2}{(2x - 3)^4 \cdot (5x^4 - 2)^5 \cdot (3x^2 - 4)^3}$$

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Answers

$$9) y = \frac{\sqrt{2x^3 + 3}}{(x^4 - 3)^3}$$

$$\begin{aligned} \frac{dy}{dx} &= y \left(\frac{3x^2}{2x^3 + 3} - \frac{12x^3}{x^4 - 3} \right) \\ &= \frac{3x^2(-7x^4 - 3 - 12x)}{(x^4 - 3)^4 \sqrt{2x^3 + 3}} \end{aligned}$$

$$10) y = (2x^2 - 5)^3 \sqrt{x^2 - 2}$$

$$\begin{aligned} \frac{dy}{dx} &= y \left(\frac{12x}{2x^2 - 5} + \frac{x}{x^2 - 2} \right) \\ &= \frac{x(2x^2 - 5)^2(14x^2 - 29)}{\sqrt{x^2 - 2}} \end{aligned}$$

Use logarithmic differentiation to differentiate each function with respect to x . You do not need to simplify or substitute for y .

$$11) y = \frac{(5x - 4)^4}{(3x^2 + 5)^5 \cdot (5x^4 - 3)^3}$$

$$\frac{dy}{dx} = y \left(\frac{20}{5x - 4} - \frac{30x}{3x^2 + 5} - \frac{60x^3}{5x^4 - 3} \right)$$

$$12) y = (x + 2)^4 \cdot (2x - 5)^2 \cdot (5x + 1)^3$$

$$\frac{dy}{dx} = y \left(\frac{4}{x + 2} + \frac{4}{2x - 5} + \frac{15}{5x + 1} \right)$$

$$13) y = (5x^5 + 2)^2 \cdot (3x^3 - 1)^3 \cdot (3x - 1)^4$$

$$\frac{dy}{dx} = y \left(\frac{50x^4}{5x^5 + 2} + \frac{27x^2}{3x^3 - 1} + \frac{12}{3x - 1} \right)$$

$$14) y = \frac{(x^2 + 3)^4}{(5x^5 - 2)^5 \cdot (3x^2 - 5)^2}$$

$$\frac{dy}{dx} = y \left(\frac{8x}{x^2 + 3} - \frac{125x^4}{5x^5 - 2} - \frac{12x}{3x^2 - 5} \right)$$

$$15) y = (3x^3 - 4)^5 \cdot (3x - 1)^3 \cdot (5x^3 - 2)^2 \cdot (x + 3)^4$$

$$\frac{dy}{dx} = y \left(\frac{45x^2}{3x^3 - 4} + \frac{9}{3x - 1} + \frac{30x^2}{5x^3 - 2} + \frac{4}{x + 3} \right)$$

$$16) y = \frac{(4x^2 - 5)^2}{(2x - 3)^4 \cdot (5x^4 - 2)^5 \cdot (3x^2 - 4)^3}$$

$$\frac{dy}{dx} = y \left(\frac{16x}{4x^2 - 5} - \frac{8}{2x - 3} - \frac{100x^3}{5x^4 - 2} - \frac{18x}{3x^2 - 4} \right)$$