

## Logarithmic Differentiation ... Set 2

For each of the following, differentiate the function first using any rule you want, then using logarithmic differentiation:

1.  $y = x^2$

*Solution.* If  $y = x^2$ , then

$$\ln y = \ln(x^2) = 2 \ln x.$$

Differentiating,

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x},$$

so

$$\frac{dy}{dx} = \frac{2y}{x} = \frac{2x^2}{x} = 2x.$$

This is the same answer that we could have gotten with the power rule.

2.  $y = e^x$

*Solution.*

$$y = e^x \implies \ln y = x$$

$$\frac{1}{y} \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = y = e^x.$$

3.  $y = \sqrt{x^2 + 1}$

*Solution.*

$$y = \sqrt{x^2 + 1} \implies \ln y = \frac{1}{2} \ln(x^2 + 1)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \frac{2x}{x^2 + 1}$$

$$\frac{dy}{dx} = \frac{xy}{x^2 + 1} = \frac{x\sqrt{x^2 + 1}}{x^2 + 1} = \frac{x}{\sqrt{x^2 + 1}}.$$

Again, notice this is the same answer we could have gotten without logarithmic differentiation.

4.  $y = x \sin x$

*Solution.*

$$\ln y = \ln x + \ln(\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} + \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \left( \frac{1}{x} + \frac{\cos x}{\sin x} \right) (x \sin x)$$

$$= \sin x + x \cos x.$$

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5.  $y = \frac{x}{x^2+2}$

*Solution.*

$$\begin{aligned}\ln y &= \ln x - \ln(x^2 + 2) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{1}{x} - \frac{2x}{x^2 + 2} \\ \frac{dy}{dx} &= \left( \frac{1}{x} - \frac{2x}{x^2 + 2} \right) \frac{x}{x^2 + 2} \\ &= \frac{(x^2 + 2) - x(2x)}{(x^2 + 2)^2}.\end{aligned}$$

6.  $y = \sqrt{(x^2 + 1)(x - 1)^2}$ .

*Solution.*

$$\begin{aligned}\ln y &= \frac{1}{2} (\ln(x^2 + 1) + 2 \ln(x - 1)) \\ \frac{1}{y} \frac{dy}{dx} &= \frac{x}{x^2 + 1} + \frac{1}{x - 1} \\ \frac{dy}{dx} &= \left( \frac{x}{x^2 + 1} + \frac{1}{x - 1} \right) \sqrt{(x^2 + 1)(x - 1)^2}\end{aligned}$$

Use logarithmic differentiation to find the following derivatives:

7.  $y = (x + 1)^x$

*Solution.*

$$\begin{aligned}\ln y &= x \ln(x + 1) \\ \frac{1}{y} \frac{dy}{dx} &= \ln(x + 1) + \frac{x}{x + 1} \\ \frac{dy}{dx} &= \left( \ln(x + 1) + \frac{x}{x + 1} \right) (x + 1)^x.\end{aligned}$$

9.  $y = (\sqrt{x})^x$

*Solution.*

$$\begin{aligned}\ln y &= x \ln \sqrt{x} = \frac{1}{2} x \ln x \\ \frac{1}{y} \frac{dy}{dx} &= \frac{1}{2} (\ln x + 1) \\ \frac{dy}{dx} &= \frac{1}{2} (\ln x + 1) (\sqrt{x})^x.\end{aligned}$$

8.  $y = x^{x+1}$

*Solution.*

$$\begin{aligned}\ln y &= (x + 1) \ln x \\ \frac{1}{y} \frac{dy}{dx} &= \ln x + \frac{x + 1}{x} \\ \frac{dy}{dx} &= \left( \ln x + \frac{x + 1}{x} \right) x^{x+1}.\end{aligned}$$

10.  $y = x^{\sqrt{x}}$

*Solution.*

$$\begin{aligned}\ln y &= \sqrt{x} \ln x \\ \frac{1}{y} \frac{dy}{dx} &= \frac{1}{2\sqrt{x}} \ln x + \frac{\sqrt{x}}{x} \\ \frac{dy}{dx} &= \frac{(\ln x + 2)x^{\sqrt{x}}}{2\sqrt{x}}.\end{aligned}$$