

Differentiation Rules ... Set 2

Derivatives in Your Head!

Differentiate with respect to x :

	A	B	C	D	E	F	G
1.	x^3	$4x^3$	$\frac{8x^2}{3}$	$x^{\frac{1}{2}}$	$3x^{\frac{1}{3}}$	x^{q+2}	\sqrt{x}
2.	x^{10}	$8x$	$\frac{-7x^2}{2}$	$x^{\frac{3}{4}}$	$4x^{\frac{3}{4}}$	x^{3m}	$\sqrt[5]{x}$
3.	x^4	$5x^4$	$\frac{5x^3}{4}$	$x^{\frac{2}{3}}$	$-6x^{\frac{2}{3}}$	x^{p+q}	$-\sqrt[7]{x^2}$
4.	x^5	$-3x^2$	$\frac{2x^5}{3}$	$x^{\frac{5}{4}}$	$-10x^{-\frac{3}{5}}$	$-x^{171}$	$\sqrt[4]{x^3}$
5.	x^6	$2x$	$-\frac{6x^7}{7}$	$x^{\frac{3}{2}}$	$9x^{\frac{2}{3}}$	x^{n+1}	$\sqrt[3]{x^8}$
6.	x^2	8	$\frac{2}{9}x^3$	$x^{1.8}$	$14x^{-\frac{3}{7}}$	x^{nr+n+1}	$-\sqrt{x^{1.8}}$
7.	x^7	$-7x^6$	$\frac{11}{5}x^5$	$x^{6.1}$	$-2x^{-\frac{4}{5}}$	$-x^{5d+k+6}$	$\sqrt[3]{x^{2.4}}$
8.	x^9	$9x^5$	$-\frac{3}{4}x^6$	$x^{3.4}$	$4x^{-\frac{5}{3}}$	$x^{\sqrt{2}+1}$	$\sqrt[3]{x^{-5.1}}$
9.	x	$-12x^7$	$\frac{3}{7}x^4$	$x^{-\frac{1}{2}}$	$-7x^{\frac{2}{3}}$	$x^{3\pi+4}$	$\sqrt{\frac{1}{x^{7.6}}}$
10.	x^0	-14	$\frac{5}{8}x^3$	$x^{-\frac{7}{2}}$	$\frac{4x^{\frac{5}{4}}}{5}$	x^{3i-2}	$-\frac{1}{\sqrt[4]{x^3}}$
11.	x^{-2}	$-8x^0$	$\frac{-7x^{-3}}{3}$	$x^{\frac{4}{3}}$	$-\frac{2}{3}x^{\frac{3}{2}}$	π^3	$\frac{7}{\sqrt[5]{x^2}}$
12.	x^{-1}	$11x^{-7}$	$\frac{2x^{-6}}{9}$	$x^{-\frac{5}{4}}$	$\frac{4}{21}x^{\frac{7}{2}}$	$-5x^{2n+1}$	$-\sqrt[3]{\frac{1}{x^2}}$
13.	x^{-4}	$9x^{-8}$	$-\frac{4x^{-5}}{15}$	$x^{-5.3}$	$-\frac{9}{4}x^{-\frac{4}{3}}$	$3x^{5k+4}$	$\frac{1}{\sqrt{x^3}}$
14.	x^{-3}	$-14x^{-10}$	$\frac{5x^{-14}}{7}$	$x^{-2.6}$	$-\frac{2x^{\frac{5}{4}}}{5}$	$-4x^{\sqrt{3}+7}$	$x\sqrt{x}$
15.	x^{-10}	$4x^{-3}$	$-\frac{2}{9}x^{-3}$	$x^{-8.3}$	$\frac{4}{3}x^{\frac{7}{3}}$	$\sqrt{3}x^{\pi-2}$	$-\frac{5}{3\sqrt{x}}$
16.	x^{-7}	$5x^{-1}$	$\frac{1}{5}x^{-3}$	$x^{\frac{11}{5}}$	$\frac{2}{7}x^{-\frac{2}{5}}$	$-2x^{5t+1}$	$\sqrt[4]{\frac{1}{x^{-1.2}}}$
17.	x^{-16}	$-2x^{-6}$	$\frac{-8}{5}x^{-4}$	$x^{\frac{5}{7}}$	$-\frac{9x^{-\frac{8}{5}}}{4}$	$2.5x^{2e+3}$	$-\frac{3}{7\sqrt{x^{1.4}}}$
18.	x^{-9}	$3x^{-5}$	$\frac{4}{7}x^{-6}$	$x^{-\frac{9}{2}}$	$-\frac{3}{14}x^{\frac{7}{3}}$	$0.4x^{6f}$	$\frac{14}{3\sqrt[2]{x^3}}$
19.	x^{-6}	$-7x^{-4}$	$\frac{5}{8}x^3$	$-7x^{-4}$	$\frac{10}{9}x^{-\frac{3}{5}}$	$2x^{a+b}$	$x^2\sqrt[3]{x}$
20.	x^{-11}	$6x^{-2}$	$-\frac{5}{14}x^{-7}$	$6x^{-2}$	$\frac{7x^{-\frac{5}{3}}}{2}$	$3.5x^{4w+2}$	$-\frac{2x^3}{\sqrt[5]{x}}$

Differentiation Rules ... Set 2

Answers

	A	B	C	D	E	F	G
1.	$3x^2$	$12x^2$	$\frac{16x}{3}$	$\frac{x^{-\frac{1}{2}}}{2}$	$x^{-\frac{2}{3}}$	$(q+2)x^{q+1}$	$\frac{x^{-\frac{1}{2}}}{2} = \frac{1}{2\sqrt{x}}$
2.	$10x^9$	8	$-7x$	$\frac{3x^{-\frac{1}{4}}}{4}$	$3x^{-\frac{1}{4}}$	$3mx^{3m-1}$	$\frac{x^{-\frac{4}{5}}}{5} = \frac{1}{5\sqrt[5]{x^4}}$
3.	$4x^3$	$20x^3$	$\frac{15x^2}{4}$	$\frac{2x^{-\frac{1}{3}}}{3}$	$-4x^{-\frac{1}{3}}$	$(p+q)x^{p+q-1}$	$-\frac{2x^{-\frac{5}{7}}}{7} = -\frac{2}{7\sqrt[7]{x^5}}$
4.	$5x^4$	$-6x$	$\frac{10x^4}{3}$	$\frac{5x^{\frac{1}{4}}}{4}$	$6x^{-\frac{8}{5}}$	$-171x^{170}$	$\frac{3x^{\frac{1}{4}}}{4} = \frac{3}{4\sqrt[4]{x}}$
5.	$6x^5$	2	$-6x^6$	$\frac{3x^{\frac{1}{2}}}{2}$	$6x^{-\frac{1}{3}}$	$(n+1)x^n$	$\frac{8x^{\frac{5}{3}}}{3} = \frac{8\sqrt[3]{x^5}}{3}$
6.	$2x$	0	$\frac{2}{3}x^2$	$1.8x^{0.8}$	$-6x^{-\frac{10}{7}}$	$(m+n+1)x^{m+n}$	$-0.9x^{-0.1}$
7.	$7x^6$	$-42x^5$	$11x^4$	$6.1x^{5.1}$	$\frac{8x^{-\frac{9}{5}}}{5}$	$-(5d+k+6)x^{5d-k+5}$	$0.8x^{-0.2}$
8.	$9x^8$	$45x^4$	$-\frac{9}{2}x^5$	$3.4x^{2.4}$	$-\frac{20x^{-\frac{8}{3}}}{3}$	$(\sqrt{2}+1)x^{\sqrt{2}+1}$	$-1.7x^{-2.7}$
9.	1	$-84x^6$	$\frac{12}{7}x^3$	$-\frac{x^{-\frac{3}{2}}}{2}$	$-\frac{14x^{-\frac{1}{3}}}{3}$	$(3\pi+4)x^{3\pi+3}$	$-3.8x^{-4.8}$
10.	0	0	$\frac{15}{8}x^2$	$-\frac{7x^{-\frac{9}{2}}}{2}$	$\frac{4x^{\frac{5}{4}}}{5}$	$(3i-2)x^{3i-3}$	$\frac{3x^{\frac{7}{4}}}{4} = \frac{3}{4\sqrt[4]{x^7}}$
11.	$-2x^{-3}$	0	$7x^{-4}$	$-\frac{4x^{-\frac{7}{3}}}{3}$	$-x^{-\frac{5}{2}}$	0	$\frac{14x^{-\frac{7}{5}}}{5} = \frac{14}{5\sqrt[5]{x^7}}$
12.	$-x^{-2}$	$-77x^{-8}$	$-\frac{4x^{-7}}{3}$	$-\frac{5x^{-\frac{9}{4}}}{4}$	$\frac{2}{3}x^{\frac{5}{2}}$	$-5(2n+1)x^{2n}$	$\frac{2x^{-\frac{5}{3}}}{3} = \frac{2}{3\sqrt[3]{x^5}}$
13.	$-4x^{-5}$	$-72x^{-9}$	$\frac{4x^{-6}}{3}$	$-5.3x^{-6.3}$	$3x^{-\frac{7}{3}}$	$3(5k+4)x^{5k+3}$	$-\frac{3x^{-\frac{5}{2}}}{2} = -\frac{3}{2\sqrt{x^5}}$
14.	$-3x^{-4}$	$140x^{-11}$	$-10x^{-15}$	$-2.6x^{-3.6}$	$-\frac{x^{\frac{1}{4}}}{2}$	$-4(\sqrt{3}+7)x^{\sqrt{3}+6}$	$\frac{3x^{\frac{1}{2}}}{2} = \frac{3\sqrt{x}}{2}$
15.	$-10x^{-11}$	$-12x^{-4}$	$\frac{2}{3}x^{-4}$	$-8.3x^{-9.3}$	$\frac{28}{9}x^{\frac{4}{3}}$	$\sqrt{3}(\pi-2)x^{\pi-2}$	$\frac{5x^{-\frac{3}{2}}}{6} = \frac{5}{6\sqrt{x^3}}$
16.	$-7x^{-8}$	$-5x^{-2}$	$-\frac{3}{5}x^{-4}$	$\frac{11x^{\frac{6}{5}}}{5}$	$-\frac{4}{35}x^{-\frac{7}{5}}$	$-2(5i+1)x^{5i}$	$0.3x^{-0.7}$
17.	$-16x^{-17}$	$12x^{-7}$	$\frac{32}{5}x^{-5}$	$\frac{5x^{-\frac{2}{7}}}{7}$	$\frac{18x^{-\frac{13}{5}}}{45}$	$2.5(2e+3)x^{2e+2}$	$0.3x^{-1.7}$
18.	$-9x^{-10}$	$-15x^{-6}$	$-\frac{24}{7}x^{-7}$	$-\frac{9x^{-\frac{11}{2}}}{2}$	$-\frac{1}{2}x^{\frac{4}{3}}$	$2.4fx^{6f-1}$	$-2x^{-\frac{10}{7}} = -\frac{2}{\sqrt[7]{x^{10}}}$
19.	$-6x^{-7}$	$28x^{-5}$	$\frac{15}{8}x^2$	$28x^{-5}$	$-\frac{2}{3}x^{-\frac{8}{5}}$	$2(a+b)x^{a+b-1}$	$\frac{7x^{\frac{4}{3}}}{3} = \frac{7\sqrt[3]{x^4}}{3}$
20.	$-11x^{-12}$	$-12x^{-3}$	$\frac{5}{2}x^{-8}$	$-12x^{-3}$	$-\frac{35x^{-\frac{8}{3}}}{6}$	$(14w+7)x^{4w+1}$	$\frac{28x^{\frac{9}{5}}}{5} = \frac{28\sqrt[5]{x^9}}{5}$

Differentiation Rules ... Set 2

Derivatives of Powers

Find $\frac{dy}{dx}$ if:

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|---|-----------------------------------|--|
| 1. $y = x + \sqrt{x}$ | 2. $y = x^5 - 3\sqrt{x}$ | 3. $y = x^{\frac{5}{2}} - \frac{2}{x}$ |
| 4. $y = 3x^4 - \frac{2}{x} + \frac{6}{x^2}$ | 5. $y = (x+5)(x+2)$ | 6. $y = (3x+1)(5x-3)$ |
| 7. $y = (5x^2-3)(4x^3+x)$ | 8. $y = (x^3+1)(2x+3)$ | 9. $y = (x^5-2x)^2$ |
| 10. $y = (x-2)(x+1)(3x+1)$ | 11. $y = (x-a)^3$ | 12. $y = (2x+3)^3$ |
| 13. $y = 2x(3x^2-7x+8)$ | 14. $y = 3x^2(x+1)(x-2)$ | 15. $y = (x + \frac{1}{x})^2$ |
| 16. $y = \frac{2x+5}{x}$ | 17. $y = \frac{x^3-2}{x}$ | 18. $y = \frac{x^2-4x+7}{x}$ |
| 19. $y = \frac{x^3-4x^2+3x-2}{x^2}$ | 20. $y = \frac{3x^7-7x+11}{2x^3}$ | 21. $y = \frac{(2x+3)(2x-3)}{x}$ |
| 22. $y = \frac{x+6}{x^3}$ | 23. $y = \frac{2x^3+x+4}{2x^5}$ | 24. $y = \frac{x-3}{\sqrt{x}}$ |

Find the derivative if:

- | | | |
|-----------------------------------|---|-------------------------------------|
| 25. $f(x) = ax^3 + bx^2 + cx + d$ | 26. $k = \frac{1}{a}(x^2 + \frac{b}{x} + c)$ | 27. $b = -3m^{-8} + 3\sqrt{7}$ |
| 28. $f = ax^4 + bx^2 + c$ | 29. $r = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} - t + 4$ | 30. $g = -8h^5 + 3h^{-2} + h^{1.6}$ |

Find the derivative with respect to the variable indicated:

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|---|--|---|
| 31. $C = 2\pi r$ [r] | 32. $y = mx + b$ [x] | 33. $v = u + at$ [t] |
| 34. $A = \pi r^2$ [r] | 35. $E = \frac{1}{2}mv^2$ [v] | 36. $P = \frac{V^2}{R}$ [V] |
| 37. $s = ut + \frac{1}{2}at^2$ [t] | 38. $P = RI^2$ [I] | 39. $V = \frac{4\pi r^3}{3}$ [r] |
| 40. $F = \frac{\pi r^4 P}{8\eta l}$ [r] | 41. $T = 2\pi\sqrt{\frac{l}{g}}$ [l] | 42. $v = \sqrt{\frac{2GM}{r}}$ [r] |
| 43. $F = \frac{GMm}{r^2}$ [r] | 44. $E = \frac{q}{4\pi\epsilon_0 r^2}$ [r] | 45. $\Lambda = \sqrt{\frac{h^2}{2\pi m k T}}$ [T] |

Calculate the first, second, third, and fourth derivatives of:

- | | | |
|---------------------------|-------------------------|--------------------------|
| 46. $y = 4x^4 + 2x^3 + 3$ | 47. $k = x^3 + 7x - 11$ | 48. $b = -m^{-2} + 3m^3$ |
|---------------------------|-------------------------|--------------------------|

Find the gradient of the tangent to the curve at the point indicated:

- | | | |
|---------------------------|---------------------------|--|
| 49. $y = x^2 + 3x$ (2,10) | 50. $y = 2x^3 - 4$ (3,50) | 51. $y = -x^2 + \frac{1}{x}$ (-2,-4.5) |
|---------------------------|---------------------------|--|

Find the equation of the tangent to the curve at the point indicated:

- | | | |
|--------------------------|----------------------------|-----------------------------------|
| 52. $y = 3x^2 - x$ (1,2) | 53. $y = x^3 + 4x$ (-1,-5) | 54. $y = x^2 - \frac{1}{x}$ (1,2) |
|--------------------------|----------------------------|-----------------------------------|

Find the value of x that satisfies the equation given:

- | | |
|---------------------------------|--|
| 55. $y = x^4 + 3x + 1$ | $y^{IV} - y^{III} + \frac{2y^{II}}{x^2} = 0$ |
| 56. $y = \frac{1}{x}, x \neq 0$ | $x^3 y^{II} + x^3 y^I + xy = 0$ |

Differentiation Rules ... Set 2

Answers

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|--|--|---|
| 1. $\frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x}}$ | 2. $\frac{dy}{dx} = 5x^4 - \frac{3}{2\sqrt{x}}$ | 3. $\frac{dy}{dx} = \frac{5x^{\frac{3}{2}}}{2} + \frac{2}{x^2}$ |
| 4. $y' = 12x^3 + \frac{2}{x^2} - \frac{12}{x^3}$ | 5. $y' = 2x + 7$ | 6. $y' = 30x - 4$ |
| 7. $y' = 100x^4 - 21x^2 - 3$ | 8. $y' = 8x^3 + 9x^2 + 2$ | 9. $y' = 10x^9 - 24x^5 + 8x$ |
| 10. $y' = 9x^2 - 4x - 7$ | 11. $y' = 3x^2 - 6ax + 3a^2$ | 12. $y' = 24x^2 + 72x + 54$ |
| 13. $\frac{dy}{dx} = 18x^2 - 28x + 16$ | 14. $\frac{dy}{dx} = 12x^3 - 9x^2 - 12x$ | 15. $\frac{dy}{dx} = 2x - \frac{2}{x^3}$ |
| 16. $\frac{dy}{dx} = -\frac{5}{x^2}$ | 17. $\frac{dy}{dx} = 2x + \frac{2}{x^2}$ | 18. $\frac{dy}{dx} = 1 - \frac{7}{x^2}$ |
| 19. $\frac{dy}{dx} = 1 - \frac{3}{x^2} + \frac{4}{x^3}$ | 20. $\frac{dy}{dx} = 6x^3 + \frac{7}{x^3} - \frac{33}{2x^4}$ | 21. $\frac{dy}{dx} = 4 + \frac{9}{x^2}$ |
| 22. $\frac{dy}{dx} = -\frac{2}{x^3} - \frac{18}{x^4}$ | 23. $\frac{dy}{dx} = -\frac{2}{x^3} - \frac{2}{x^5} - \frac{10}{x^6}$ | 24. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{3}{2\sqrt{x^3}}$ |
| 25. $f'(x) = 3ax^2 + 2bx + c$ | 26. $\frac{dk}{dx} = \frac{1}{a}(2x - \frac{b}{x^2})$ | 27. $\frac{db}{dm} = 24m^{-9}$ |
| 28. $\frac{df}{dx} = 4ax^3 + 2bx$ | 29. $\frac{dr}{dt} = t^3 - t^2 + t - 1$ | 30. $\frac{dg}{dh} = -40h^4 - \frac{6}{h^3} + 1.6h^{0.6}$ |
| 31. $\frac{dC}{dr} = 2\pi$ | 32. $\frac{dy}{dx} = m$ | 33. $\frac{dv}{dt} = a$ |
| 34. $\frac{dA}{dr} = 2\pi r$ | 35. $\frac{dE}{dv} = mv$ | 36. $\frac{dP}{dV} = \frac{2V}{R}$ |
| 37. $\frac{ds}{dt} = u + at$ | 38. $\frac{dP}{dI} = 2RI$ | 39. $\frac{dV}{dr} = 4\pi r^2$ |
| 40. $\frac{dF}{dr} = \frac{\pi r^3 P}{2\eta t}$ | 41. $\frac{dT}{dI} = \frac{\pi}{\sqrt{gI}}$ | 42. $\frac{dv}{dr} = -\sqrt{\frac{GM}{2r^3}}$ |
| 43. $\frac{dF}{dr} = -\frac{2GMm}{r^3}$ | 44. $\frac{dE}{dr} = -\frac{q}{2\pi\epsilon_0 r^2}$ | 45. $\frac{d\Delta}{dT} = -\frac{h}{\sqrt{8\pi m k T^3}}$ |
| 46. $y' = 16x^3 + 6x^2$
$y'' = 48x^2 + 12x$
$y''' = 96x + 12$
$y^{IV} = 96$ | 47. $\frac{dk}{dx} = 3x^2 + 7$
$\frac{d^2k}{dx^2} = 6x$
$\frac{d^3k}{dx^3} = 6$
$\frac{d^4k}{dx^4} = 0$ | 48. $\frac{db}{dm} = 2m^{-3} + 9m^2$
$\frac{d^2b}{dm^2} = -6m^{-4} + 18m$
$\frac{d^3b}{dm^3} = 24m^{-5} + 18$
$\frac{d^4b}{dm^4} = -120m^{-6}$ |
| 49. $\frac{dy}{dx} = 2x + 3 = 7$ at (2,1) | 50. $\frac{dy}{dx} = 6x^2 = 54$ at (3,-1) | 51. $y' = -2x - \frac{1}{x^2} = \frac{15}{4}$ at (-2,3) |
| 52. $y = 5x - 3$ | 53. $y = 7x + 2$ | 54. $y = 3x - 1$ |
| 55. $y' = 4x^3 + 3, y'' = 12x^2, y''' = 24x, y^{IV} = 24$ and
$x = 2$ | | |
| 56. $y' = -\frac{1}{x^2}, y'' = \frac{2}{x^3}$ and
$x = 3$ | | |

Differentiation Rules ... Set 2

Chain Rule

Differentiate:

1. $(x + 3)^4$

4. $(7x - 2)^6$

7. $(x^2 - 1)^{100}$

10. $3(x + 13)^2$

13. $2(x^7 + 3x^2 - 1)^6$

16. $\frac{(x - x^2 - x^4)^5}{5}$

19. $(9 - 4x)^{-3}$

22. $(x^{-1} - 2x^{-2})^{-3}$

25. $(5x + 7)^{\frac{3}{2}}$

28. $(4x^2 - 6x + 1)^{\frac{7}{3}}$

2. $(2x + 5)^3$

5. $(x^2 + 1)^3$

8. $(3x^2 - 2x)^2$

11. $(3x^2 + 7x)^4$

14. $3(x^5 - 2x)^2$

17. $\frac{(5 - x)^{-2}}{2}$

20. $(4x^2 - 3x^3 + x)^{-2}$

23. $(3x + 1)^{\frac{1}{2}}$

26. $(x^3 - 5x^2 + x)^{\frac{3}{4}}$

29. $(2x^3 - 9x + 12)^{-\frac{2}{3}}$

3. $(1 - x)^7$

6. $(x^6 + x^3)^{20}$

9. $2(5x - 3)^8$

12. $(x^2 + 7x - 1)^8$

15. $(5x^2 + 4)^{11}$

18. $(2x + 1)^{-1}$

21. $5(x^2 - 9)^{-3}$

24. $(6x + 1)^{\frac{1}{3}}$

27. $(x^5 - 5x)^{\frac{1}{5}}$

30. $(x^{-4} + 7x^{-2} + 8)^{-\frac{5}{2}}$

Find y' :

31. $y = \frac{1}{3x - 1}$

34. $y = \frac{3}{(7x^2 - 3x + 7)^{10}}$

37. $y = \frac{3}{4(2x - 5)^8}$

40. $y = \sqrt{2x + 5}$

43. $y = \sqrt[3]{x^3 - 3x}$

46. $y = \sqrt[3]{(11 - 3x)^2}$

32. $y = \frac{2}{3x^2 - x + 5}$

35. $y = \frac{1}{9 - x^2}$

38. $y = \frac{1}{x^4 + 5x^3 - 2x}$

41. $y = \sqrt{x^2 - 3}$

44. $y = \sqrt{4 - x^2}$

47. $y = \sqrt[3]{(4x - 1)^4}$

33. $y = \frac{1}{x^3 + x^2 + x + 1}$

36. $y = \frac{7}{3(5x^2 + 2)^3}$

39. $y = \frac{1}{12(4x - 1)^3}$

42. $y = \sqrt[3]{9x - 4}$

45. $y = \sqrt[3]{8x^3 + 27}$

48. $y = \sqrt[5]{7x^3 - 2x^2 + 5}$

Find the derivative:

49. $y = \sqrt[4]{7m^3 - 4m^2 + 2}$

52. $d = \frac{3}{\sqrt{x + 2}}$

55. $k(n) = \frac{4}{\sqrt{n^2 + 6}}$

50. $c = \sqrt[3]{(4k^2 + 3)^2}$

53. $f = \frac{2}{\sqrt{4e + 5}}$

56. $p(r) = \frac{12}{\sqrt[4]{(7 - r)^5}}$

51. $r = \sqrt[4]{(4w + 3)^5}$

54. $g(x) = \frac{1}{\sqrt{1 - 2x}}$

57. $q(z) = \frac{5}{\sqrt[5]{z^5 - 32}}$

Calculate $\frac{dy}{dx}$ for the following functions:

58. $y = (x - 3)\sqrt{(x - 3)}$

61. $y = (x - \frac{1}{x})^4$

64. $y = (\frac{x^3}{3} + \frac{x^2}{2} + x)^{-k}$

67. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

59. $y = \sqrt{4 - \sqrt{x + 2}}$

62. $y = (x^2 + \frac{1}{x^2})^3$

65. $y = [(2x + 1)^2 + (x + 1)^2]^3$

68. $y = [(1 + \frac{1}{x})^{-1} + 1]^{-1}$

60. $y = [(2x + 1)^{10} + 1]^{10}$

63. $y = (3x - 1)^{2k+1}$

66. $y = (7x + \sqrt{x^2 + 3})^6$

69. $y = [x^2 + (x^2 + 9)^{\frac{1}{2}}]^{\frac{1}{2}}$

Differentiation Rules ... Set 2

Answers

- | | | |
|--|---|--|
| 1. $4(x+3)^3$ | 2. $6(2x+5)^2$ | 3. $-7(1-x)^6$ |
| 4. $42(7x-2)^5$ | 5. $6x(x^2+1)^2$ | 6. $60x^{59}(2x^3+1)(x^3+1)^{19}$ |
| 7. $200x(x^2-1)^{99}$ | 8. $4x(3x-1)(3x-2)$ | 9. $80(5x-3)^7$ |
| 10. $6(x+13)$ | 11. $4x^3(6x+7)(3x+7)^3$ | 12. $8(x^2+7)(x^2+7x-1)^7$ |
| 13. $12x(7x^5+6)(x^7+3x^2-1)^5$ | 14. $6x(x^4-2)(5x^4-2)$ | 15. $110x(5x^2+4)^{10}$ |
| 16. $(1-2x-4x^3)(x-x^2-x^4)^4$ | 17. $(5-x)^{-3}$ | 18. $-2(2x+1)^{-2}$ |
| 19. $12(9-4x)^{-4}$ | 20. $-\frac{2(8x-9x^2+1)}{(4x^2-3x^3+x)^3}$ | 21. $-30x(x^2-9)^{-4}$ |
| 22. $-3x^5(4-x)(x-2)^{-4}$ | 23. $\frac{3}{2\sqrt{3x+1}}$ | 24. $2(6x+1)^{-\frac{2}{3}}$ |
| 25. $\frac{15(5x+7)^{\frac{1}{2}}}{2}$ | 26. $\frac{3(3x^2-10x+1)}{4\sqrt[4]{x^3-5x^2+x}}$ | 27. $(x^4-1)(x^5-5x)^{-\frac{4}{5}}$ |
| 28. $\frac{14(4x-3)(4x^2-6x+1)^{\frac{4}{3}}}{3}$ | 29. $-\frac{2(2x^2-3)}{\sqrt[3]{(2x^3-9x+12)^5}}$ | 30. $\frac{5(2-7x^2)}{x^5\sqrt{(x^{-4}+7x^{-2}+8)^7}}$ |
| 31. $y' = -\frac{3}{3x-1^2}$ | 32. $y' = -\frac{2(6x-1)}{3x^2-x+5^2}$ | 33. $y' = -\frac{(3x^2+2x+1)}{x^3+x^2+x+1^2}$ |
| 34. $y' = -\frac{30(14x-3)}{(7x^2-3x+7)^{11}}$ | 35. $y' = \frac{2x}{(9-x^2)^2}$ | 36. $y' = -\frac{70x}{(5x^2+2)^4}$ |
| 37. $y' = -\frac{12}{(2x-5)^9}$ | 38. $y' = \frac{(4x^3+15x-2)}{(x^4+5x^3-2x)^2}$ | 39. $y' = -\frac{1}{(4x-1)^4}$ |
| 40. $y' = \frac{1}{\sqrt{2x+5}}$ | 41. $y' = \frac{x}{\sqrt{x^2-3}}$ | 42. $y' = \frac{3}{\sqrt[3]{(9x-4)^2}}$ |
| 43. $y' = \frac{x^2-1}{\sqrt[3]{(x^3-3x)^2}}$ | 44. $y' = -\frac{x}{\sqrt{4-x^2}}$ | 45. $y' = \frac{8x^2}{\sqrt[3]{(8x^3+27)^2}}$ |
| 46. $y' = -\frac{2}{\sqrt[3]{11-3x}}$ | 47. $y' = \frac{16\sqrt[3]{4x-1}}{3}$ | 48. $y' = \frac{x(21x-4)}{5^5\sqrt{(7x^3-2x^2+5)^4}}$ |
| 49. $\frac{dy}{dm} = \frac{21m^2-8m}{4^4\sqrt{(7m^3-4m^2+2)^3}}$ | 50. $\frac{dc}{dk} = \frac{16k}{3^3\sqrt[3]{4k^2+3}}$ | 51. $\frac{dr}{dw} = 5\sqrt[4]{4w+3}$ |
| 52. $\frac{dd}{dx} = -\frac{3}{2\sqrt{(x+2)^3}}$ | 53. $\frac{df}{de} = -\frac{4}{\sqrt{(4e+5)^3}}$ | 54. $g'(x) = \frac{1}{\sqrt{(1-2x)^3}}$ |
| 55. $k'(n) = -\frac{4n}{\sqrt{(n^2+6)^3}}$ | 56. $p'(r) = \frac{15}{\sqrt[4]{(7-r)^9}}$ | 57. $q'(z) = -\frac{5z^4}{\sqrt{(z^5-32)^6}}$ |
| 58. $\frac{dy}{dx} = \frac{3\sqrt{(x-3)}}{2}$ | 59. $\frac{dy}{dx} = \frac{1}{4\sqrt{4-\sqrt{x}+2}\sqrt{x+2}}$ | 60. $\frac{dy}{dx} = \frac{200(2x+1)^9}{[(2x+1)^{10}+1]^9}$ |
| 61. $\frac{dy}{dx} = 4\left(1+\frac{1}{x^2}\right)\left(x-\frac{1}{x}\right)^3$ | 62. $\frac{dy}{dx} = 6\left(x-\frac{1}{x^3}\right)\left(x^2+\frac{1}{x^2}\right)^2$ | 63. $\frac{dy}{dx} = 3(2k+1)(3x-1)^{2k}$ |
| 64. $\frac{dy}{dx} = -\frac{k(x^2+x+1)}{\left(\frac{x^3}{3}+\frac{x^2}{2}+x\right)^{k+1}}$ | 65. $\frac{dy}{dx} = \frac{6(5x+3)}{[(2x+1)^2+(x+1)^2]^2}$ | 66. $\frac{dy}{dx} = \frac{6(7x+\sqrt{x^2+3})^5}{\left[7+\frac{x}{\sqrt{x^2+3}}\right]}$ |
| 67. $\frac{dy}{dx} = \frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}$ | 68. $\frac{dy}{dx} = \frac{1}{x^2\left(1+\frac{1}{x}\right)^2\left[\left(1+\frac{1}{x}\right)^{-1}+1\right]^2}$ | 69. $\frac{dy}{dx} = \frac{2x+\frac{x}{\sqrt{x^2+9}}}{\sqrt{x^2+(x^2+9)^{\frac{1}{2}}}}$ |

Differentiation Rules ... Set 2

Product Rule

Expand and differentiate, and compare by differentiating using the product rule:

- | | | |
|-----------------------|-------------------|----------------------------|
| 1. $(3x - 1)(4x + 3)$ | 2. $5x(6x - 1)$ | 3. $x^3(3x + 2)$ |
| 4. $4x^2(2x^3 - 1)$ | 5. $3x^4(7x - 2)$ | 6. $(2x^2 - 5)(3x^2 + 8x)$ |

Differentiate mentally without simplification (check your answers below):

- | | | |
|----------------------------|------------------------------------|-------------------------------------|
| 7. $(3x - 2)(5x + 7)$ | 8. $(4 - x^2)(3x + 5)$ | 9. $(5x - 2)(x - 3)$ |
| 10. $(5x - 2)(x - 1)$ | 11. $(x^2 - 1)(x + 7)$ | 12. $(x^4 + 8)(6 - 5x)$ |
| 13. $(2x + 9)(x^2 - 4)$ | 14. $(3x + 5)(8x - 1)$ | 15. $(9x^2 - 5)(3x - 8)$ |
| 16. $5x^2(3x - 8)$ | 17. $4x^5(2x^2 - 5x + 3)$ | 18. $(x^2 - 7x + 1)(3x - 4)$ |
| 19. $(5x^3 + 2)(4x - x^3)$ | 20. $(x^2 + 3x + 1)(x^3 - 4x + 5)$ | 21. $(x^{100} - 5)(5x^8 - 11x + 1)$ |

Differentiate on paper without simplification:

- | | | |
|------------------------------|------------------------------------|--------------------------------|
| 22. $(x + 2)(x^2 - 2x + 7)$ | 23. $(1 - x^3)(7x + 4)$ | 24. $(3x - 5)(x^3 + 2x^2 - 8)$ |
| 25. $(x^2 - 2)(5x - x^3)$ | 26. $(x^2 + 3x - 1)(x^3 - 4x + 7)$ | 27. $(x^3 - 2x + 8)(6 - 5x)$ |
| 28. $(8x^2 - 5x)(13x^2 - 4)$ | 29. $(x^5 - 2x^3)(7x^2 + x - 8)$ | 30. $(3 - x^3)(8x + 1)$ |
| 31. $(x + 1)(x + 2)(x + 3)$ | 32. $(x + 1)(x^2 + 2)(x^3 + 3)$ | 33. $4x(x - 1)(2x - 3)$ |

Differentiate (using an embedded chain rule):

- | | | |
|-----------------------------------|--------------------------------------|--|
| 34. $x^2(x + 1)^3$ | 35. $8x(3x - 2)^5$ | 36. $2x^4(3 - x)^3$ |
| 37. $(x + 1)(2x - 5)^4$ | 38. $(x^3 - 4x^2 + 1)(x^2 + 3)^5$ | 39. $(3x^2 - 2x - 1)(x^2 + 5x - 2)^2$ |
| 40. $4x^3(2x - 1)^{-3}$ | 41. $2x^8(11 - x)^{-7}$ | 42. $(4x - 3)(5x + 3)^{-2}$ |
| 43. $2x(x + 3)^{\frac{1}{2}}$ | 44. $(3x - 1)(6 - x)^{-\frac{3}{4}}$ | 45. $4x^3(8x - 1)^{\frac{4}{5}}$ |
| 46. $(2x + 9)\sqrt{x^2 - 4}$ | 47. $x\sqrt{11 - x}$ | 48. $x^2\sqrt[3]{4x - 7}$ |
| 49. $(3x + 8)^3(x + 1)^4$ | 50. $(2x^2 - 3x + 1)^2(5x - 1)^4$ | 51. $(5x + 3)^4(x - 2)^{-\frac{1}{2}}$ |
| 52. $(8x + 1)^3\sqrt[4]{9 - x^3}$ | 53. $(2x + 7)^8\sqrt{(x + 4)^5}$ | 54. $(6x + 1)^8(3x - 7)^{-5}$ |
| 55. $\frac{x}{(2x - 9)^5}$ | 56. $(x + x^{-1})(x - x^{-2})$ | 57. $(x + \sqrt{x})(1 + \sqrt{x})$ |

Find the equation of the tangent to the following curves at the point indicated):

- | | | |
|-------------------------------|-----------------------------|---------------------------------------|
| 58. $y = x^2(x + 1)^3$ (-1,0) | 59. $y = x(3x - 2)^2$ (1,1) | 60. $y = (x + 1)\sqrt{x + 3}$ (-2,-1) |
|-------------------------------|-----------------------------|---------------------------------------|

Differentiation Rules ... Set 2

Answers

1. $24x + 5$ 2. $60x - 5$ 3. $12x^3 + 6x^2$
4. $40x^4 - 8x$ 5. $105x^5 - 24x^3$ 6. $24x^3 + 48x^2 - 30x - 40$

Answers with structure, but no simplification:

22. $\frac{1(x^2 - 2x + 7) + (x + 2)(2x - 2)}{(x + 2)(2x - 2)}$ 23. $\frac{-3x^2(7x + 4) + (1 - x^3)(7)}{(1 - x^3)(7)}$ 24. $\frac{3(x^3 + 2x^2 - 8) + (3x - 5)(3x^2 + 4x)}{(3x - 5)(3x^2 + 4x)}$
25. $\frac{2x(5x - x^3) + (x^2 - 2)(5 - 3x^2)}{(x^2 - 2)(5 - 3x^2)}$ 26. $\frac{(2x + 3)(x^3 - 4x + 7) + (x^2 + 3x - 1)(3x^2 - 4)}{(x^2 + 3x - 1)(3x^2 - 4)}$ 27. $\frac{(3x^2 - 2)(6 - 5x) + (x^3 - 2x + 8)(-5)}{(x^3 - 2x + 8)(-5)}$
28. $\frac{(16x - 5)(13x^2 - 4) + (8x^2 - 5x)(26x)}{(8x^2 - 5x)(26x)}$ 29. $\frac{(5x^4 - 6x^2)(7x^2 + x - 8) + (x^5 - 2x^3)(14x + 1)}{(x^5 - 2x^3)(14x + 1)}$ 30. $\frac{-3x^2(8x + 1) + (3 - x^3)(8)}{(3 - x^3)(8)}$
31. $\frac{(1)(x + 2)(x + 3) + (x + 1)(1)(x + 3) + (x + 1)(x + 2)(1)}{(x + 1)(x + 2)(1)}$ 32. $\frac{(1)(x^2 + 2)(x^3 + 3) + (x + 1)(2x)(x^3 + 3) + (x + 1)(x^2 + 2)(3x^2)}{(x + 1)(x^2 + 2)(3x^2)}$ 33. $\frac{4(x - 1)(2x - 3) + 4x(1)(2x - 3) + 4x(x - 1)(2)}{4x(x - 1)(2)}$

Simplified answers (equivalent expressions use or remove radicals or negative indices):

34. $x(5x + 2)(x + 1)^2$ 35. $16(9x - 1)(3x - 2)^4$ 36. $-2x^3(7x - 12)(3 - x)^2$
37. $(10x + 3)(2x - 5)^3$ 38. $\frac{x(13x^3 - 48x^2 + 9x - 14)}{(x^2 + 3)^4}$ 39. $\frac{2(x^2 + 5x - 2)}{(5x^3 + 29x^2 + 10x - 8)}$
40. $-12x^2(2x - 1)^{-4}$ 41. $-2x^7(x - 88)(11 - x)^{-8}$ 42. $-2(10x - 21)(5x + 3)^{-3}$
43. $3(x + 2)(x + 3)^{-\frac{1}{2}}$ 44. $\frac{-3(4x^2 - 27x + 1)(6 - x)^{-\frac{7}{4}}}{4}$ 45. $\frac{4x^2(152x - 15)(8x - 1)^{-\frac{1}{5}}}{5}$
46. $\frac{4x^2 + 9x - 8}{\sqrt{x^2 - 4}}$ 47. $\frac{(3x - 22)}{2\sqrt{11 - x}}$ 48. $\frac{14x(2x - 3)}{3^3\sqrt{(4x - 7)^2}}$
49. $\frac{(21x + 41)(3x + 8)^2}{(x + 1)^3}$ 50. $\frac{2(2x^2 - 3x + 1)}{(40x^2 - 49x + 13)(5x - 1)^3}$ 51. $\frac{(35x - 43)(5x + 3)^3}{2\sqrt{(x - 1)^3}}$
52. $-\frac{(8x + 1)^2(120x^3 + 3x^2 - 864)}{4\sqrt{(9 - x^3)^3}}$ 53. $\frac{1}{3}(58x + 227)(2x + 7)^7 \cdot \sqrt[3]{(x + 4)^2}$ 54. $\frac{3(18x - 117)(6x + 1)^7}{(3x - 7)^6}$
55. $-\frac{8x + 9}{(2x - 9)^6}$ 56. $2x + \frac{1}{x^2} + \frac{3}{x^4}$ 57. $2 + \frac{3\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}$

The equations of the tangents are:

58. $y = 0$ 59. $y = 7x - 6$ 60. $y = \frac{x}{2}$

Differentiation Rules ... Set 2

Quotient Rule

Divide each term of the numerator by the denominator before differentiating. Compare this with the result you get by using the quotient rule:

1. $\frac{x+6}{x}$

2. $\frac{x^3-1}{x^2}$

3. $\frac{3x-1}{x^2}$

4. $\frac{x^3-3x^2}{x}$

5. $\frac{x^4+2x^3}{x^2}$

6. $\frac{2x^2+5x-1}{x}$

Using negative indices, differentiate by using the product rule. Compare this with the result you get by using the quotient rule:

7. $\frac{x+5}{2x+1}$

8. $\frac{x-3}{5x+2}$

9. $\frac{3x+8}{x-5}$

10. $\frac{x^2-2}{x^2+9}$

11. $\frac{x^3}{x+4}$

12. $\frac{x+1}{3x^2-7}$

Differentiate mentally without simplification (check your answers on the following page):

13. $\frac{1}{2x-1}$

14. $\frac{x^3}{x^2-4}$

15. $\frac{x+4}{x-6}$

16. $\frac{2x+5}{4x-3}$

17. $\frac{x}{2x^2-8}$

18. $\frac{x-7}{x^2}$

19. $\frac{x^2+4x-1}{x+3}$

20. $\frac{x^2-9x+11}{2x+5}$

21. $\frac{3x-1}{x^2+12}$

22. $\frac{6x+7}{x^2-x+3}$

23. $\frac{x^3+x}{x^2-x-1}$

24. $\frac{5x^2-2x}{3x+1}$

Differentiate (using an embedded chain rule):

25. $\frac{2x}{(x+1)^{\frac{1}{2}}}$

26. $\frac{(2x+7)^3}{4x-1}$

27. $\frac{x-1}{(7x+3)^4}$

28. $\frac{(3x-4)^5}{(2x+1)^3}$

29. $\frac{2x-5}{\sqrt{x+1}}$

30. $\frac{\sqrt{x-1}}{4x+1}$

31. $\frac{\sqrt{x^2+1}}{(x-8)^2}$

32. $\frac{x-4}{\sqrt[3]{x}}$

33. $\frac{(x+3)^4}{x^2}$

Find the derivative of:

34. $y = \frac{x+3}{\sqrt{x}+2}$

35. $f = \frac{p^{\frac{2}{3}}}{2p+1}$

36. $b = \frac{\sqrt[3]{w}}{w^2+5}$

37. $m = \frac{h^3-1}{h^3+1}$

38. $g = \frac{7t^4+11}{t+8}$

39. $e = \left(\frac{4y+3}{5y-1}\right)^3$

40. $k = \frac{8n^2-5n+11}{n+2}$

41. $r = \frac{v-6}{\sqrt{(v+1)^5}}$

42. $z = \frac{4a-9}{(a+5)^{\frac{3}{4}}}$

Find the equation of the tangent to the following curves at the point indicated):

43. $y = \frac{x+3}{x-1}$

(2,5)

44. $y = \frac{x+3}{3x-2}$

(1,4)

45. $y = \frac{x}{x-2}$

(3,3)

46. $y = \frac{x^2}{x-1}$

(-1, -1/2)

47. $y = \frac{x^2+1}{x^2-3}$

(2,5)

48. $y = \frac{x-7}{x^2}$

(1,-6)

Differentiation Rules ... Set 2

Answers

You should discover that the results are the same and that it is advisable to simplify expressions first!

1. $-\frac{6}{x^2}$	2. $\frac{x^3 + 2}{x^3} = 1 + \frac{2}{x^3}$	3. $-\frac{3x - 2}{x^3}$
4. $2x - 3$	5. $2x + 2$	6. $\frac{2x^2 + 1}{x^2} = 2 + \frac{1}{x^2}$

You should discover that the results are the same and that the quotient rule is (usually) simpler to use.

7. $-\frac{9}{(2x + 1)^2}$	8. $\frac{17}{(5x + 2)^2}$	9. $-\frac{23}{(x - 5)^2}$
10. $\frac{22x}{(x^2 + 9)^2}$	11. $\frac{2x^2(x + 6)}{(x + 4)^2}$	12. $-\frac{3x^2 + 6x + 7}{(3x^2 - 7)^2}$

You should have found the following structures:

13. $\frac{0(2x - 1) - 1(2)}{(2x - 1)^2}$	14. $\frac{3x^2(x^2 - 4) - x^3(2x)}{(x^2 - 4)^2}$	15. $\frac{1(x - 6) - (x + 4)1}{(x - 6)^2}$
16. $\frac{2(4x - 3) - (2x + 5)4}{(4x - 3)^2}$	17. $\frac{1(2x^2 - 8) - x(4x)}{(2x^2 - 8)^2}$	18. $\frac{1(x^2) - (x - 7)(2x)}{x^4}$
19. $\frac{(2x + 4)(x + 3) - (x^2 + 4x - 1)1}{(x + 3)^2}$	20. $\frac{(2x - 9)(2x + 5) - (x^2 - 9x + 11)2}{(2x + 5)^2}$	21. $\frac{3(x^2 + 12) - (3x - 1)(2x)}{(x^2 + 12)^2}$
22. $\frac{6(x^2 - x - 3) - (6x + 7)(2x - 1)}{(x^2 - x + 3)^2}$	23. $\frac{(3x^2 + 1)(x^2 - x - 1) - (x^3 + x)(2x - 1)}{(x^2 - x - 1)^2}$	24. $\frac{(10x - 2)(3x + 1) - (5x^2 - 2x)3}{(3x + 1)^2}$

The derivatives are:

25. $\frac{x + 2}{(x + 1)^{\frac{3}{2}}}$	26. $\frac{2(8x - 17)(2x + 7)^2}{(4x - 1)^2}$	27. $-\frac{21x - 31}{(7x + 3)^5}$
28. $\frac{3(4x + 13)(3x - 4)^4}{(2x + 1)^4}$	29. $\frac{2x + 9}{2\sqrt{(x + 1)^3}}$	30. $-\frac{4x - 9}{2\sqrt{x - 1}(4x + 1)^2}$
31. $-\frac{x^2 + 8x + 2}{\sqrt{x^2 + 1}(x - 8)^3}$	32. $\frac{2(x + 2)}{3^3\sqrt{x^4}}$	33. $\frac{2(x - 3)(x + 3)^3}{x^3}$

The derivatives are:

34. $\frac{dy}{dx} = \frac{x + 4\sqrt{x} - 3}{2\sqrt{x}(\sqrt{x} + 2)^2}$	35. $\frac{df}{dp} = -\frac{2(p - 1)}{3^2\sqrt{p}(2p + 1)^2}$	36. $\frac{db}{dw} = -\frac{5(w^2 - 1)}{3w^{\frac{2}{3}}(w^2 + 5)^2}$
37. $\frac{dm}{dh} = \frac{6h^2}{(h^3 + 1)^2}$	38. $\frac{dg}{dt} = \frac{21t^4 + 224t^3 - 11}{(t + 8)^2}$	39. $\frac{de}{dy} = -\frac{57(4y + 3)^2}{(5y - 1)^4}$
40. $\frac{dk}{dn} = \frac{8n^2 + 32n - 21}{(n + 2)^2}$	41. $\frac{dr}{dv} = -\frac{3v - 32}{2\sqrt{(v + 1)^7}}$	42. $\frac{dz}{da} = \frac{4a + 107}{4(a + 5)^{\frac{7}{4}}}$

The equations of the tangents (in gradient-intercept form) are:

43. $y = -4x + 13$	44. $y = -11x + 15$	45. $y = -2x + 9$
46. $y = \frac{3x}{4} + \frac{1}{4}$	47. $y = -16x + 37$	48. $y = 13x - 19$

Differentiation Rules ... Set 2

Exponential Functions

Differentiate with respect to x :

- | | | |
|------------------------------|--------------------------------------|------------------------------|
| 1. $y = e^x$ | 2. $y = 3e^x$ | 3. $y = e^{4x}$ |
| 4. $y = 2e^{5x}$ | 5. $y = e^{5x} - e^{2x}$ | 6. $y = 2e^{3x} + e^{-x}$ |
| 7. $y = e^{3.5x} + e^{1.9x}$ | 8. $y = 6e^{2x} - \frac{e^{-2x}}{2}$ | 9. $y = e^{2x} \cdot e^{7x}$ |
| 10. $y = e^{x^2}$ | 11. $y = e^{x^2-2x+7}$ | 12. $y = 3e^{-x^4}$ |
| 13. $y = 8e^{5x-1}$ | 14. $y = x^3e^{2x}$ | 15. $y = (2x + 1)e^{-x}$ |

Find the derivative function:

- | | | |
|------------------------------|------------------------|-----------------------------------|
| 16. $y = xe^{-2x}$ | 17. $y = x^3e^{-x}$ | 18. $y = x^3 - xe^{4x}$ |
| 19. $y = (x^2 - 6)e^{8x}$ | 20. $y = \sqrt{x}e^x$ | 21. $y = 4e^{2x^2}$ |
| 22. $y = xe^{x^2}$ | 23. $y = e^{(e^x)}$ | 24. $y = \frac{e^{2x+1}}{2x+7}$ |
| 25. $y = \frac{e^{3x}}{x^2}$ | 26. $y = e^{\sqrt{x}}$ | 27. $y = \frac{e^x + 1}{e^x - 1}$ |

Calculate $\frac{dy}{dx}$:

- | | | |
|--|---------------------------------|---|
| 28. $y = \frac{x}{e^{-x}}$ | 29. $y = (e^x + 2)^8$ | 30. $y = e^{4\sqrt{x}} - e^{\frac{1}{x}}$ |
| 31. $y = e^{\sqrt{x}} + e^{\frac{5}{4}}$ | 32. $y = 4x^3 + 3x^2 - e^{-2x}$ | 33. $y = 2e^{1-x}$ |
| 34. $y = (e^x + x)^{10}$ | 35. $y = e^{x^3+1}$ | 36. $y = x^e e^x$ |

Find the derivative of:

- | | | |
|----------------------------------|---------------------------|----------------------------------|
| 37. $y = x^e e^{x-e}$ | 38. $p = m^2 e^{-\pi m}$ | 39. $a = \frac{k-1}{e^{2k}-1}$ |
| 40. $v = (t^2 - 3t)e^{8t}$ | 41. $j = \sqrt{d}e^{d+4}$ | 42. $b = \frac{e^{2q}}{e^q + 4}$ |
| 43. $f = \frac{e^g - e^{-g}}{2}$ | 44. $h = A + Be^{-6w}$ | 45. $l = \frac{1-n^2}{2e^n}$ |

Given the function on the left, demonstrate that the relationship on the right is true:

- | | |
|------------------------------|------------------------|
| 46. $y = e^x + e^{-x}$ | $y'' = y$ |
| 47. $y = 4e^{-x} + 5e^{-3x}$ | $y'' + 4y' + 3y = 0$ |
| 48. $y = e^{2x} + e^{8x}$ | $y'' - 10y' + 16y = 0$ |
| 49. $y = e^{2x} + e^{4x}$ | $y'' - 6y' + 8y = 0$ |
| 50. $y = (x + 1)e^{5x}$ | $y'' - 10y' + 25y = 0$ |
| 51. $y = A + Be^{-4x}$ | $y'' + 4y' = 0$ |

Find the equation of the tangent to the following curves at the point indicated):

- | | | |
|---------------------|------------------------|-------------------------|
| 52. $y = e^x$ (0,1) | 53. $y = e^{-x}$ (0,1) | 54. $y = e^{x-2}$ (3,e) |
|---------------------|------------------------|-------------------------|

Find the minimum value of each function (and its location) given:

- | | | |
|----------------------|---------------------|--------------------|
| 55. $y = (x - 2)e^x$ | 56. $y = -e^{-x^2}$ | 57. $y = xe^{x-1}$ |
|----------------------|---------------------|--------------------|

Differentiation Rules ... Set 2

Answers

- | | | |
|--|---|---|
| 1. $y' = e^x$ | 2. $y' = 3e^x$ | 3. $y' = 4e^{4x}$ |
| 4. $y' = 10e^{5x}$ | 5. $y' = 5e^{5x} - 2e^{2x}$ | 6. $y' = 6e^{3x} - e^{-x}$ |
| 7. $y' = 3.5e^{3.5x} + 1.9e^{1.9x}$ | 8. $y' = 12e^{2x} + e^{-2x}$ | 9. $y' = 9e^{9x}$ |
| 10. $y' = 2xe^{x^2}$ | 11. $y' = (2x - 2)e^{x^2 - 2x + 7}$ | 12. $y' = -12x^3e^{-x^4}$ |
| 13. $y' = 40e^{5x-1}$ | 14. $y' = 3x^2e^{2x} + 2x^3e^{2x}$ | 15. $y' = e^{-x} - 2xe^{-x}$ |
| 16. $y' = e^{-2x} - 2xe^{-2x}$ | 17. $y' = 3x^2e^{-x} - x^3e^{-x}$ | 18. $y' = 3x^2 - e^{4x} - 4xe^{4x}$ |
| 19. $y' = 2(4x^2 + x - 24)e^{8x}$ | 20. $y' = \frac{e^x}{2\sqrt{x}} + \sqrt{x}e^x$ | 21. $y' = 16xe^{2x^2}$ |
| 22. $y' = e^{x^2} + 2x^2e^{x^2}$ | 23. $y' = e^xe^{(e^x)}$ | 24. $y' = \frac{4(x+3)e^{2x+1}}{2x+7^2}$ |
| 25. $y' = \frac{(3x-2)e^{3x}}{x^3}$ | 26. $y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ | 27. $y' = -\frac{2e^x}{(e^x-1)^2}$ |
| 28. $\frac{dy}{dx} = e^x + xe^x$ | 29. $\frac{dy}{dx} = 8e^x(e^x+2)^7$ | 30. $\frac{dy}{dx} = \frac{e^{4\sqrt{x}}}{4\sqrt[4]{x^3}} - \frac{e^{-\frac{1}{x}}}{x^2}$ |
| 31. $\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ | 32. $\frac{dy}{dx} = 12x^2 + 6x + 2e^{-2x}$ | 33. $\frac{dy}{dx} = -2e^{1-x}$ |
| 34. $\frac{dy}{dx} = 10(e^x+1)(e^x+x)^9$ | 35. $\frac{dy}{dx} = 3x^2e^{x^3+1}$ | 36. $\frac{dy}{dx} = (x+e)x^{e-1}e^x$ |
| 37. $\frac{dy}{dx} = (x+e)x^{e-1}e^{x-e}$ | 38. $\frac{dp}{dm} = (2-\pi m)me^{-\pi m}$ | 39. $\frac{da}{dk} = \frac{3e^{2k} - 2ke^{2k} - 1}{(e^{2k} - 1)^2}$ |
| 40. $\frac{dv}{dt} = (8t^2 - 22t - 3)e^{8t}$ | 41. $\frac{dj}{dd} = \frac{(2d+1)e^{d+4}}{2\sqrt{d}}$ | 42. $\frac{db}{dq} = \frac{e^{3q} + 8e^{2q}}{(e^q + 4)^2}$ |
| 43. $\frac{df}{dg} = \frac{e^g + e^{-g}}{2}$ | 44. $\frac{dh}{dw} = -6Be^{-6w}$ | 45. $\frac{dl}{dn} = \frac{n^2 - 2n - 1}{2e^n}$ |

The derivatives required in order to demonstrate the relationships are:

- | | | |
|--|---------------------------------|----------------------------------|
| 46. $y = e^x + e^{-x}$ | 46. $y' = e^x - e^{-x}$ | 46. $y'' = e^x + e^{-x}$ |
| 47. $y = 4e^{-x} + 5e^{-3x}$ | 47. $y' = -4e^{-x} - 15e^{-3x}$ | 47. $y'' = 4e^{-x} + 45e^{-3x}$ |
| 48. $y = e^{2x} + e^{8x}$ | 48. $y' = 2e^{2x} + 8e^{8x}$ | 48. $y'' = 4e^{2x} + 64e^{8x}$ |
| 49. $y = e^{2x} + e^{4x}$ | 49. $y' = 2e^{2x} + 4e^{4x}$ | 49. $y'' = 4e^{2x} + 16e^{4x}$ |
| 50. $y = (x+1)e^{5x} = e^{5x} + xe^{5x}$ | 50. $y' = 6e^{5x} + 5xe^{5x}$ | 50. $y'' = 35e^{5x} + 25xe^{5x}$ |
| 51. $y = A + Be^{-4x}$ | 51. $y' = -4Be^{-4x}$ | 51. $y'' = 16Be^{-4x}$ |

The equations of the tangents (in gradient-intercept form) are:

- | | | |
|-----------------|------------------|-------------------|
| 52. $y = x + 1$ | 53. $y = -x + 1$ | 54. $y = ex - 2e$ |
|-----------------|------------------|-------------------|

The minimum values of the functions are:

- | | | |
|----------------------------|----------------------------|---|
| 55. $y = -e$ (at $x = 1$) | 56. $y = -1$ (at $x = 0$) | 57. $y = -\frac{1}{e^2}$ (at $x = -1$) |
|----------------------------|----------------------------|---|

Differentiation Rules ... Set 2

Logarithmic Functions

Differentiate with respect to x :

1. $y = \log_e x$

2. $y = \log_e(3x)$

3. $y = 2\log_e x$

4. $y = \ln(3x + 1)$

5. $y = \ln 3x + 1$

6. $y = \ln x + 2x$

7. $y = 5\ln(3x)$

8. $y = \ln(x^2)$

9. $y = \ln(x^5)$

10. $y = \ln(x^2 - 5)$

11. $y = 2\ln x + 5\ln(2x)$

12. $y = 7x - \ln(4x^3)$

Simplify, using logarithmic laws, before finding the derivative function:

13. $y = \ln\sqrt{x+9}$

14. $y = \ln\frac{x+1}{x+3}$

15. $y = \ln(x-5)(x+8)$

Calculate the derivative:

16. $y = \ln(2x-1)(x+8)$

17. $y = \ln(x+6)^4$

18. $y = \ln\frac{(x+1)(x+2)}{(x+3)}$

19. $y = \ln\frac{x}{x-2}$

20. $y = \ln\sqrt{x+4}$

21. $y = \ln\sqrt{(x+1)^3}$

22. $y = \ln\frac{1}{x}$

23. $y = \ln\frac{1}{3x+2}$

24. $y = \ln\frac{1}{2x^5}$

Calculate $\frac{dy}{dx}$:

25. $y = x\ln x$

26. $y = 2x^3\ln(x+4)$

27. $y = x\ln x - 3x$

28. $y = \ln(x^2)$

29. $y = (\ln x)^2$

30. $y = \ln(\ln x)$

31. $y = (1 + \ln x)^5$

32. $y = (\ln x - x)^9$

33. $y = (x^2 + \ln x)^6$

34. $y = \frac{\ln x}{x-2}$

35. $y = (2x+1)\ln x$

36. $y = x^3\ln(x+1)$

37. $y = \log(x)$

38. $y = \log_7(5x)$

39. $y = \log\left(\frac{2x^2-1}{\sqrt{x}}\right)$

40. $y = \frac{e^{2x}}{\ln x}$

41. $y = e^x \ln x$

42. $y = \ln\frac{e^x+1}{e^x-1}$

Given the function on the left, solve the equation on the right:

43. $y = \ln x$

$xy'' + (y')^2 = 2$

44. $y = (\ln x)^2$

$xy'' + y' = 1$

45. $y = x\ln x$

$y'' + xy' - y = 2$

46. $y = x^2\ln x$

$xy'' - y' = 8$

47. $y = (x+3)\ln x$

$xy'' + y' = 3$

48. $y = \ln(x^2 - 1)$

$2(x^2 - 1)y'' + 5y' = 0$

49. $y = e^x \ln x$

$y'' - y' = 0$

Find the equation of the tangent to the following curves at the point indicated):

50. $y = \ln\sqrt{2-x}$ (1,0)

51. $y = x\ln x$ (e,e)

52. $y = e^x + \ln x$

(1,e)

Find the minimum value of each function (and its location) given:

53. $y = (\ln x)^2$

54. $y = x^2 - \ln x$

55. $y = \frac{1}{x} + \ln x$

Differentiation Rules ... Set 2

Answers

- | | | |
|--|---|--|
| 1. $y' = \frac{1}{x}$ | 2. $y' = \frac{1}{x}$ | 3. $y' = \frac{2}{x}$ |
| 4. $y' = \frac{3}{3x+1}$ | 5. $y' = \frac{1}{x} + 1$ | 6. $y' = \frac{1}{x} + 2$ |
| 7. $y' = \frac{5}{x}$ | 8. $y' = \frac{2}{x}$ | 9. $y' = \frac{5}{x}$ |
| 10. $y' = \frac{2x}{x^2-5}$ | 11. $y' = \frac{7}{x}$ | 12. $y' = 7 - \frac{3}{x}$ |
| 13. $y' = \frac{1}{2(x+9)}$ | 14. $y' = \frac{1}{x+1} - \frac{1}{x+3}$ | 15. $y' = \frac{1}{x-5} + \frac{1}{x+8}$ |
| 16. $y' = \frac{2}{2x-1} + \frac{1}{x+8}$ | 17. $y' = \frac{4}{x+6}$ | 18. $y' = \frac{1}{x+1} + \frac{1}{x+2} - \frac{1}{x+3}$ |
| 19. $y' = \frac{1}{x} - \frac{1}{x-2}$ | 20. $y' = \frac{1}{2(x+4)}$ | 21. $y' = \frac{3}{2(x+1)}$ |
| 22. $y' = -\frac{1}{x}$ | 23. $y' = -\frac{3}{3x+2}$ | 24. $y' = -\frac{5}{x}$ |
| 25. $\frac{dy}{dx} = \ln x + 1$ | 26. $\frac{dy}{dx} = 6x^2 \ln(x+4) + \frac{2x^3}{x+4}$ | 27. $\frac{dy}{dx} = \ln x - 2$ |
| 28. $\frac{dy}{dx} = \frac{2}{x}$ | 29. $\frac{dy}{dx} = \frac{2 \ln x}{x}$ | 30. $\frac{dy}{dx} = \frac{1}{x \ln x}$ |
| 31. $\frac{dy}{dx} = \frac{5(1 + \ln x)^4}{x}$ | 32. $\frac{dy}{dx} = 9(\ln x - x)^8 \left(\frac{1}{x} - 1\right)$ | 33. $\frac{dy}{dx} = 6(x^2 + \ln x)^5 \left(2x + \frac{1}{x}\right)$ |
| 34. $\frac{dy}{dx} = \frac{(x-2) - x \ln x}{x(x-2)^2}$ | 35. $\frac{dy}{dx} = 2 \ln x + \frac{2x+1}{x}$ | 36. $\frac{dy}{dx} = 3x^2 \ln(x+1) + \frac{x^3}{x+1}$ |
| 37. $\frac{dy}{dx} = \frac{1}{x \ln 10}$ | 38. $\frac{dy}{dx} = \frac{1}{x \ln 7}$ | 39. $\frac{dy}{dx} = \frac{6x^2 + 1}{2 \ln 10 \cdot x(2x^2 - 1)}$ |
| 40. $\frac{dy}{dx} = \frac{(2x \ln x - 1)e^{2x}}{x \ln^2 x}$ | 41. $\frac{dy}{dx} = e^x \ln x + \frac{e^x}{x}$ | 42. $\frac{dy}{dx} = -\frac{2e^x}{e^{2x} - 1}$ |

The solutions to the equations are:

- | | | |
|-------------------------------|-------------------------------|------------------------|
| 43. $y = \ln x$ | 43. $xy'' + (y')^2 = 2$ | (½, -ln2) |
| 44. $y = (\ln x)^2 = \ln^2 x$ | 44. $xy'' + y' = 1$ | (2, ln ² 2) |
| 45. $y = x \ln x$ | 45. $y'' + xy' - y = 2$ | (1, 0) |
| 46. $y = x^2 \ln x$ | 46. $xy'' - y' = 8$ | (4, 16ln4) |
| 47. $y = (x+3) \ln x$ | 47. $xy'' + y' = 3$ | (e, e+3) |
| 48. $y = \ln(x^2 - 1)$ | 48. $2(x^2 - 1)y'' + 5y' = 0$ | (2, ln3) |
| 49. $y = e^x \ln x$ | 49. $y'' - y' = 0$ | (1, 0) |

The equations of the tangents (in gradient-intercept form) are:

- | | | |
|--------------------------------------|------------------|----------------------|
| 50. $y = -\frac{x}{2} + \frac{1}{2}$ | 51. $y = 2x - e$ | 52. $y = (e+1)x - 1$ |
|--------------------------------------|------------------|----------------------|

The minimum values of the functions are:

- | | | |
|---------------------------|---|---------------------------|
| 53. $y = 0$ (at $x = 1$) | 54. $y = \frac{1}{2}(1 + \ln 2)$ (at $x = \frac{1}{\sqrt{2}}$) | 55. $y = 1$ (at $x = 1$) |
|---------------------------|---|---------------------------|

Differentiation Rules ... Set 2

Trigonometric Functions

Differentiate with respect to x :

- | | | |
|--|--------------------------------------|--------------------------------|
| 1. $y = \tan x^2$ | 2. $y = \tan^2 x$ | 3. $y = \sin 3x$ |
| 4. $y = 2 \cos x$ | 5. $y = \sin x \cos x$ | 6. $y = \sin x + 5 \cos x$ |
| 7. $y = \tan 2x$ | 8. $y = x \sin x$ | 9. $y = \sin 2x \tan 3x$ |
| 10. $y = \frac{\sin x}{x}$ | 11. $y = \frac{x}{\cos x}$ | 12. $y = \frac{\sin x}{x^2}$ |
| 13. $y = \sin\left(x + \frac{\pi}{4}\right)$ | 14. $y = x^2 + \tan \frac{x}{2}$ | 15. $y = \cos \frac{\pi x}{3}$ |
| 16. $y = \frac{\cos x}{1 + \sin x}$ | 17. $y = \frac{3x + 4}{\sin 5x}$ | 18. $y = x^2 \cos x$ |
| 19. $y = \sin x^3$ | 20. $y = \cos^3 x$ | 21. $y = x^3 \tan 8x$ |
| 22. $y = \sin^2 4x$ | 23. $y = \cos(x^2 + 1)$ | 24. $y = \tan \pi x$ |
| 25. $y = \cos \sqrt{x}$ | 26. $y = \sqrt{\cos 2x}$ | 27. $y = \sin^4 \sqrt{x}$ |
| 28. $y = \tan^3 x$ | 29. $y = \sin^2 x + \cos^2 x$ | 30. $y = (\sin x + \cos x)^2$ |
| 31. $y = \tan(x^2 - 1)$ | 32. $y = \tan(\pi - x)$ | 33. $y = (2x + \tan 7x)^9$ |
| 34. $y = 7 \tan(x^2 + 5)$ | 35. $y = \sin(\cos x)$ | 36. $y = \cos(\sin x)$ |
| 37. $y = \sin x (1 + \cos x)$ | 38. $y = \sqrt[3]{\frac{\tan x}{x}}$ | 39. $y = \tan(\sqrt{\cos x})$ |

Calculate $\frac{dy}{dx}$:

- | | | |
|---------------------|------------------------|----------------------------------|
| 40. $y = \cot x$ | 41. $y = \sec x$ | 42. $y = \operatorname{cosec} x$ |
| 43. $y = \sec^2 4x$ | 44. $y = \tan x^\circ$ | 45. $y = \sin 3x^\circ$ |
| 46. $y = \sin bx$ | 47. $y = \cos(bx + c)$ | 48. $y = \tan(ax^2 + c)$ |

Find the derivative of:

- | | | |
|---------------------------|---------------------------------|--------------------------------|
| 49. $y = e^x \sin x$ | 50. $m = e^{\sin w}$ | 51. $j = e^{-a} \cos a$ |
| 52. $p = e^{4r} \sin 2r$ | 53. $s = \sin(e^t + t)$ | 54. $b = \cos(\ln s)$ |
| 55. $k = \ln(\cos q)$ | 56. $c = \ln(\sin 2n)$ | 57. $f = \ln(\tan h^2)$ |
| 58. $l = \sin(1 - \ln v)$ | 59. $z = \frac{e^{4u}}{\tan u}$ | 60. $q = \frac{\sin m^2}{e^m}$ |

Find the equation of the tangent to the following curves at the point indicated):

- | | | |
|-----------------------------|---|---|
| 61. $y = x \cos x$ $(0, 0)$ | 62. $y = \sin x \tan x$ $\left(\frac{\pi}{6}, \frac{1}{2\sqrt{3}}\right)$ | 63. $y = \sec x$ $\left(\frac{\pi}{4}, \sqrt{2}\right)$ |
| 64. $y = \sin x$ $(0, 0)$ | 65. $y = x + \tan x$ $(0, 0)$ | 66. $y = \csc x$ $\left(\frac{\pi}{3}, \frac{2}{\sqrt{3}}\right)$ |

Find the (relative) minimum value(s) of each function (and their locations) in the domain $0 \leq x \leq 2\pi$.

- | | | |
|------------------------|------------------------------------|------------------------------------|
| 67. $y = x + 2 \sin x$ | 68. $y = \sqrt{3} \sin x + \cos x$ | 69. $y = 2 \sec x + \tan x$ |
| 70. $y = \sin^2 x$ | 71. $y = \tan x - 2x$ | 72. $y = \cos x - \sqrt{3} \sin x$ |

Differentiation Rules ... Set 2

Answers

- | | | |
|---|--|---|
| 1. $y' = 2x \sec^2 x^2$ | 2. $y' = 2 \tan x \sec^2 x$ | 3. $y' = 3 \cos 3x$ |
| 4. $y' = -2 \sin x$ | 5. $y' = \cos^2 x - \sin^2 x$ | 6. $y' = \cos x - 5 \sin x$ |
| 7. $y' = 2 \sec^2 2x$ | 8. $y' = \sin x + x \cos x$ | 9. $y' = 2 \cos 2x \tan 3x + 3 \sin 2x \sec^2 3x$ |
| 10. $y' = \frac{x \cos x - \sin x}{x^2}$ | 11. $y' = \frac{\cos x + x \sin x}{\cos^2 x}$ | 12. $y' = \frac{x \cos x - 2 \sin x}{x^3}$ |
| 13. $y' = \cos\left(x + \frac{\pi}{4}\right)$ | 14. $y' = 2x + \frac{1}{2} \sec^2 \frac{x}{2}$ | 15. $y' = -\frac{\pi}{3} \sin \frac{\pi x}{3}$ |
| 16. $y' = -\frac{1}{1 + \sin x}$ | 17. $y' = \frac{3 \sin 5x - 5(3x + 4) \cos 5x}{\sin^2 5x}$ | 18. $y' = 2x \cos x - x^2 \sin x$ |
| 19. $y' = 3x^2 \cos x^3$ | 20. $y' = -3 \sin x \cos^2 x$ | 21. $y' = 3x^2 \tan 8x + 8x^3 \sec^2 8x$ |
| 22. $y' = 8 \sin 4x \cos 4x$ | 23. $y' = -2x \sin(x^2 + 1)$ | 24. $y' = \pi \sec^2 \pi x$ |
| 25. $y' = -\frac{\sin \sqrt{x}}{2\sqrt{x}}$ | 26. $y' = -\frac{\sin 2x}{\sqrt{\cos 2x}}$ | 27. $y' = \frac{2 \sin^3 \sqrt{x} \cos \sqrt{x}}{\sqrt{x}}$ |
| 28. $y' = 3 \tan^2 x \sec^2 x$ | 29. $y' = 0$ | 30. $y' = 2(\cos^2 x - \sin^2 x)$ |
| 31. $y' = 2x \sec^2(x^2 - 1)$ | 32. $y' = -\sec^2(\pi - x)$ | 33. $y' = 9(2x + \tan 7x)^8 \cdot (2 + 7 \sec^2 7x)$ |
| 34. $y' = 14x \sec^2(x^2 + 5)$ | 35. $y' = -\sin x \cdot \cos(\cos x)$ | 36. $y' = -\cos x \cdot \sin(\sin x)$ |
| 37. $y' = \cos x + \cos^2 x - \sin^2 x$ | 38. $y' = \frac{x \sec^2 x - \tan x}{3x^{\frac{4}{3}} \tan^{\frac{2}{3}} x}$ | 39. $y' = -\frac{\sin x \sec^2(\sqrt{\cos x})}{2\sqrt{\cos x}}$ |
| 40. $\frac{dy}{dx} = -\csc^2 x$ | 41. $\frac{dy}{dx} = \tan x \sec x$ | 42. $\frac{dy}{dx} = -\cot x \operatorname{cosec} x$ |
| 43. $\frac{dy}{dx} = 8 \tan 4x \sec^2 4x$ | 44. $\frac{dy}{dx} = \frac{\pi}{180} \sec^2 x^\circ$ | 45. $\frac{dy}{dx} = \frac{\pi}{180} \cos 3x^\circ$ |
| 46. $\frac{dy}{dx} = b \cos bx$ | 47. $\frac{dy}{dx} = -b \sin(bx + c)$ | 48. $\frac{dy}{dx} = 2ax \sec^2(ax^2 + c)$ |
| 49. $\frac{dy}{dx} = e^x (\sin x + \cos x)$ | 50. $\frac{dm}{dw} = \cos w e^{\sin w}$ | 51. $\frac{dj}{da} = -\frac{\sin a + \cos a}{e^a}$ |
| 52. $\frac{dp}{dr} = 4e^{4r} \sin 2r + 2e^{4r} \cos 2r$ | 53. $\frac{ds}{dt} = (e^t + 1) \cos(e^t + t)$ | 54. $\frac{db}{ds} = -\frac{\sin(\ln s)}{s}$ |
| 55. $\frac{dk}{dq} = -\tan q$ | 56. $\frac{dc}{dn} = 2 \cot 2n$ | 57. $\frac{df}{dh} = \frac{2h}{\cos h^2 \sin h^2}$ |
| 58. $\frac{dl}{dv} = -\frac{\cos(1 - \ln v)}{v}$ | 59. $\frac{dz}{du} = e^{4u} (4 \cot u - \csc^2 u)$ | 60. $\frac{dq}{dm} = \frac{2m \cos m^2 - \sin m^2}{e^m}$ |

The equations of the tangents (in gradient-intercept form) are:

- | | | |
|-------------|--|---|
| 61. $y = x$ | 62. $y = \frac{7x}{6} + \frac{1}{2\sqrt{3}} - \frac{7\pi}{36}$ | 63. $y = \sqrt{2}x + \sqrt{2} - \frac{\sqrt{2}\pi}{4}$ |
| 64. $y = x$ | 65. $y = 2x$ | 66. $y = -\frac{2x}{3} + \frac{2}{\sqrt{3}} + \frac{2\pi}{9}$ |

The (relative) minimum values of the functions are:

- | | | |
|--|---|--|
| 67. $y = \frac{4\pi}{3} - \sqrt{3}$ (at $x = \frac{4\pi}{3}$) | 68. $y = -2$ (at $x = \frac{4\pi}{3}$) | 69. $y = \sqrt{3}$ (at $x = \frac{11\pi}{6}$) |
| 70. $y = 0$ (at $x = 0, \pi, 2\pi$) | 71. $y = 1 - \frac{\pi}{2}$ (at $x = \frac{\pi}{4}$) and $y = 1 - \frac{5\pi}{2}$ (at $x = \frac{5\pi}{4}$) | 72. $y = -2$ (at $x = \frac{2\pi}{3}$) |

Differentiation Rules ... Set 2

Differentiate mentally without simplification

7. $(3x - 2)(5x + 7)$

8. $(4 - x^2)(3x + 5)$

9. $(5x - 2)(x - 3)$

10. $(5x - 2)(x - 1)$

11. $(x^2 - 1)(x + 7)$

12. $(x^4 + 8)(6 - 5x)$

13. $(2x + 9)(x^2 - 4)$

14. $(3x + 5)(8x - 1)$

15. $(9x^2 - 5)(3x - 8)$

16. $5x^2(3x - 8)$

17. $4x^5(2x^2 - 5x + 3)$

18. $(x^2 - 7x + 1)(3x - 4)$

19. $(5x^3 + 2)(4x - x^3)$

20. $(x^2 + 3x + 1)(x^3 - 4x + 5)$

21. $(x^{100} - 5)(5x^8 - 11x + 1)$

Differentiation Rules ... Set 2

Answers

Mental Calculations

- | | | | | | |
|-----|--|-----|---|-----|---|
| 7. | $3(5x + 7) + (3x - 2)5$ | 8. | $-2x(3x + 5) + (4 - x^2)3$ | 9. | $5(x - 3) + (5x - 2)1$ |
| 10. | $5(x - 1) + (5x - 2)1$ | 11. | $2x(x + 7) + (x^2 - 1)1$ | 12. | $4x^3(6 - 5x) + (x^4 + 8)(-5)$ |
| 13. | $2(x^2 - 4) + (2x + 9)(2x)$ | 14. | $3(8x - 1) + (3x + 5)8$ | 15. | $18x(3x - 8) + (9x^2 - 5)3$ |
| 16. | $10x(3x - 8) + 5x^2(3)$ | 17. | $20x^4(2x^2 - 5x + 3) + 4x^5(4x - 5)$ | 18. | $(2x - 7)(3x - 4) + (x^2 - 7x + 1)3$ |
| 19. | $15x^2(4x - x^3) + (5x^3 + 2)(4 - 3x^2)$ | 20. | $(2x + 3)(x^3 - 4x + 5) + (x^2 + 3x + 1)(3x^2 - 4)$ | 21. | $100x^{99}(5x^8 - 11x + 1) + (x^{100} - 5)(40x^7 - 11)$ |