Differentiation Quiz ... Set 6

Question 10: Find the derivative of the following function:

$$f(x) = 1963$$

(A) $+\infty$

(B) 1963

(C) $-\infty$

(D) 0

(E) None of the above

Answer: (D) The derivative of a constant function is always zero.

Question 11: Find the derivative of the following function:

$$f(x) = x^2 + 6x + 9$$

(A) $f'(x) = 2x + 6 + 9$

(B) $f'(x) = x^2 + 6$

(C) $f'(x) = 2x + 6$

(D) $f'(x) = 2x$

(E) None of the above

Answer: (C) Remember that 1) the derivative of a sum of functions is simply the sum of the derivatives of each of the functions, and 2) the power rule for derivatives says that if $f(x) = kx^n$, then $f'(x) = nkx^{n-1}$. Thus $f'(x) = 2x^{2-1} + 6x^{1-1} + 0 = 2x + 6$.

Question 12: Find the derivative of the following function:

$$f(x) = x^{\frac{1}{2}}$$

(A) $f'(x) = -\frac{1}{2\sqrt{x}}$

(B) $f'(x) = \frac{1}{\sqrt{x}}$

(C) $f'(x) = \frac{1}{2\sqrt{x}}$

(D) $f'(x) = \sqrt{x}$

(E) None of the above

Answer: (C) Remember that the power rule for derivatives works with fractional exponents as well! Thus $f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$.

Question 13: Find the derivative of the following function:

$$f(x) = 5x^2(x + 47)$$

(A) $f'(x) = 15x^2 + 470x$

(B) $f'(x) = 5x^2 + 470x$

(C) $f'(x) = 10x$

(D) $f'(x) = 15x^2 - 470x$

(E) None of the above

Answer: (A) Ideally, you would solve this problem by applying the product rule. Set $g(x) = 5x^2$ and $h(x) = (x + 47)$, then $f(x) = g(x)h(x)$. Apply the product rule:

$$\begin{aligned}
f'(x) &= g'(x)h(x) + g(x)h'(x) \\
&= 10x(x + 47) + 5x^2(1) \\
&= 10x^2 + 470x + 5x^2 \\
&= 15x^2 + 470x
\end{aligned}$$

Question 14: Find the derivative of the following function:

$$f(x) = \frac{5x^2}{x + 47}$$

(A) $f'(x) = \frac{5x^2 - 470x}{(x+47)^2}$

(B) $f'(x) = \frac{10x^2 + 470x}{(x+47)}$

(C) $f'(x) = 10x$

(D) $f'(x) = \frac{5x^2 + 470}{(x+47)^2}$

(E) None of the above

Answer: (E) Ideally, you would solve this problem by applying the quotient rule. Set $g(x) = 5x^2$ and $h(x) = (x + 47)$, then $f(x) = \frac{g(x)}{h(x)}$. Apply the quotient rule:

$$\begin{aligned}
f'(x) &= \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2} \\
&= \frac{10x(x + 47) - 5x^2(1)}{(x + 47)^2} \\
&= \frac{10x^2 + 470x - 5x^2}{(x + 47)^2} \\
&= \frac{5x^2 + 470x}{(x + 47)^2}
\end{aligned}$$

Question 15: Find the derivative of the following function:

$$f(x) = 5(x + 47)^2$$

(A) $f'(x) = 15x^2 + 470x$

(B) $f'(x) = 10x - 470$

(C) $f'(x) = 10x + 470$

(D) $f'(x) = 15x^2 - 470x$

(E) None of the above

Answer: (C) Ideally, you would solve this problem by applying the chain rule. Set $g(h) = 5h^2$ and $h(x) = (x+47)$, then $f(x) = g(h(x))$. Apply the chain rule:

$$\begin{aligned} f'(x) &= g'(h)h'(x) \\ &= 10h \\ &= 10(x + 47) \\ &= 10x + 470 \end{aligned}$$

Question 16: Find the derivative of the following function:

$$f(x) = (7x - 4)(3x + 8)^4$$

Answer: Combine the product rule and the chain rule:

$$\begin{aligned} f'(x) &= 7(3x + 8)^4 + (7x - 4)(4)(3)(3x + 8)^3 \\ &= 7(3x + 8)^4 + 12(7x - 4)(3x + 8)^3 \\ &= 7(3x + 8)^4 + (84x - 48)(3x + 8)^3 \end{aligned}$$

Question 17: Find the derivative of the following function:

$$f(x) = (122x^3 - 49)^{-4}$$

Answer: Use the chain rule:

$$\begin{aligned} f'(x) &= -4 * (122)(3)x^2(122x^3 - 49)^{-5} \\ &= -\frac{1464x^2}{(122x^3 - 49)^5} \end{aligned}$$

Question 18: Find the derivative of the following function:

$$f(x) = \frac{8x^2 + 3x - 9}{7x^2 - 4}$$

Answer: The easiest way is to solve this is to get rid of the fraction, and then combine the product rule with chain rule:

$$f(x) = (8x^2 + 3x - 9)(7x^2 - 4)^{-1}$$
$$f'(x) = (8(2)x + 3)(7x^2 - 4)^{-1} + (8x^2 + 3x - 9)(-1)(7x^2 - 4)^{-2}$$
$$= \frac{16x + 3}{7x^2 - 4} - \frac{8x^2 + 3x - 9}{(7x^2 - 4)^2}$$

Question 19: Find the derivative of the following function:

$$f(x) = (22 - 9x^6)^{\frac{1}{2}}$$

Answer: Use the chain rule:

$$f'(x) = \frac{1}{2}(22 - 9x^6)^{-\frac{1}{2}}(9)(6)x^5$$
$$= 7(3x + 8)^4 + 12(7x - 4)(3x + 8)^3$$
$$= \frac{27x^5}{2(22 - 9x^6)^{\frac{1}{2}}}$$

Question 20: Find the derivative of the following function:

$$f(x) = (18x^2 + 23)^{\frac{1}{3}}$$

Answer: Use the chain rule:

$$f'(x) = \frac{1}{3}(2)(18)x(18x^2 + 23)^{-\frac{1}{3}}$$
$$= \frac{12x}{(18x^2 + 23)^{\frac{1}{3}}}$$

Question 21: Find the derivative of the following function:

$$f(x) = 5x^2(4x - 9)^3$$

Answer: Combine the product rule and the chain rule:

$$f'(x) = 5(2)x(4x - 9)^3 + 5x^2(3)(4)(4x - 9)^2$$
$$= 10x(4x - 9)^3 + 60x^2(4x - 9)^2$$