4.1

Global max and min on [a,b]

Ch 4 goal: what does the graph of $f(x) = \frac{x^2}{x^2-4}$ look like?

We need to find:

1) Domain
2) asymptotes
3) Where is f increasing / decreasing?
4) Where is f concave up / down?
5) Does f have any peaks / valley?
[local max / min]
6) Does f have any global max min?

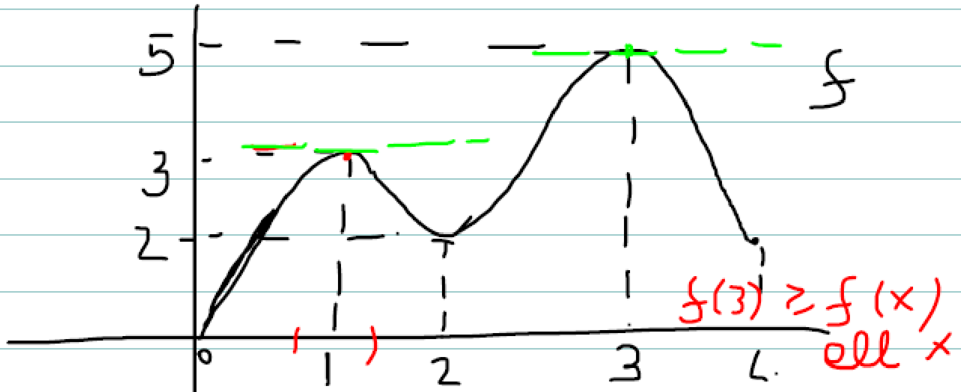
4.1 / 4.3 maxima and minima

4.5 putting info together

4.7 maxima / minima word problems

4.6 more limits

1. The **Domain** of a function $f(x)$ is all allowed values of x
2. **Interval**: all numbers between two given numbers (geometrically a segment). Examples
   - $(-2, 3)$ means all x satisfying $-2 < x < 3$
   - $[-2, 3]$ means all x satisfying $-2 \leq x \leq 3$
   - $(4, +\infty)$ means all x satisfying $x > 4$
   - $(-\infty, 4]$ means all x satisfying $x \leq 4$
3. A **local maximum** for the function $f(x)$ is a number c (x value), in the domain of f, such that there is an open interval I containing c and contained in the domain of f with $f(c) \geq f(x)$ for all x in I. (If the domain of f is $[1, 2]$, can 1 or 2 be local maxima for f?) $f(c)$ is called **local maximum value**.
4. A **local minimun** for the function $f(x)$ is a number c (x value), in the domain of f, such that there is an open interval I containing c and contained in the domain of f with $f(c) \leq f(x)$ for all x in I. (If the domain of f is $[1, 2]$, can 1 or 2 be local maxima for f?)$f(c)$ is called **local minimum value**.
5. A **global or absolute maximum** for the function $f(x)$ is a number c (x value) in the domain of f such that $f(c) \geq f(x)$ for all x in the domain of f. $f(c)$ (y value) is the **maximum value of** f (is a global maximum also a local maximum ?)
6. A **global or absolute minimum** for the function $f(x)$ is a number c (x value) in the domain of f such that $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ (y value) is the **minimum value of** f
7. A **critical number** for the function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.
8. An **inflection point** for f is a point $(c, f(c))$ on the curve $y = f(x)$ where the curve changes concavity. We require f to be continuous at c.
9. The line $y = c$ is called an **horizontal asymptote** for $y = f(x)$ if $\lim_{x\to\infty} f(x) = c$ or $\lim_{x\to-\infty} f(x) = c$ (see sec. 2.6 of the textbook).
10. The line $x = d$ is called a **vertical asymptote** for $y = f(x)$ if $\lim_{x\to d^+} f(x) = \infty$ or $\lim_{x\to d^-} f(x) = \infty$ (see sec. 2.2 of the textbook).

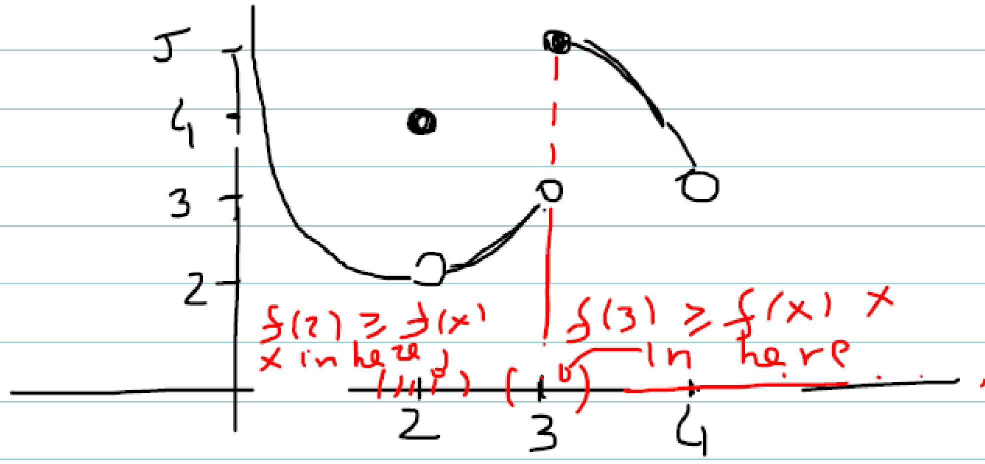


$f(x) \le f(1)$ in a small interval around 1

f defined on $[0, 4]$

endpoints cannot be local max y

local max	local max value
1, 3	3, 5
	y
global max	global max value
3	5
local min	local min value
2	2
global min	global min value
0	0



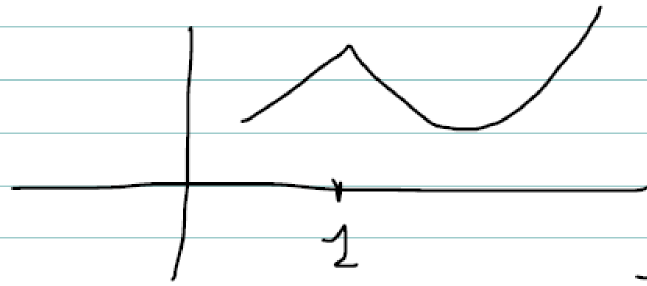
$y = 0$ vertical asymptote

f fined on $(0, 4)$

local max	local max value
3, 2	5, 4
global max	global max value
None	None
local min	local min value
None	None
global min	global min value
None	None

- A continuous function defined on a closed and bounded interval [a,b] always has an absolute max and and an absolute min.
  global
  global
- If f has a local min or max at c, then c is a critical number for f. i.e $f'(c)=0$ or $f'(c)$ DNE
- If f is continuous on [a,b], the absolute min and max are either critical numbers for f or they are equal to a or b.

Global Minimum and Local Maximum … Set 3

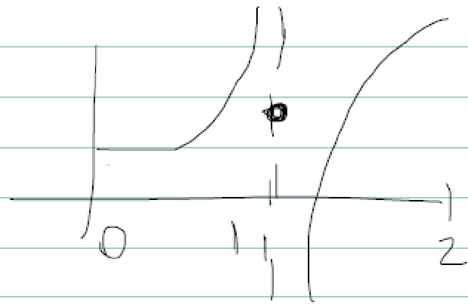


f

$f'(\underline{1}) = DNE$

$\underline{1}$ local max

f continuous



f not continuous on [0, 2]

f defined on [0, 2]
f has no absolute max/min values on [0, 2]

Critical number for f is an x value in the domain of f s.t $f'(x)=0$ or $f'(x)$ DNE

numbers

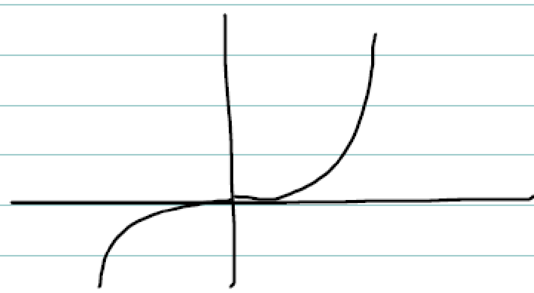
- Find the critical ~~points~~ numbers of

Natural domain $(-\infty, +\infty)$

$$\frac{x^3}{3} - \frac{5}{2}x^2 + 6x$$

1) Calculate $f'(x)$: $f'(x) = x^2 - 5x + 6$
2) Set $f'(x)=0$ $\quad x^2 - 5x + 6 = 0$ for $x = 3, 2$
3) List all x in the domain of f s.t. $f'(x)$ is not defined : NONE

$f(x) = x^3$



$f'(x) = 3x^2$

0 is critical number
for f but <u>not</u> local max/
min

Global max/min

- To find the global max/min of a continuous function f(x) defined on a closed interval [a,b]:
- List all critical numbers for f .
- Find the values of f at all critical numbers and at a and b.
- The biggest/smallest value is the global max /min value for f and the critical number or a or b that produces this value is the global max/min for f

Find global max and min for

1) $f(x)=x^3$ on $[-2,1]$

1) Find critical numbers

$f'(x) = 3x^2$

$3x^2 = 0$ for $\boxed{x=0}$

No x s.t. $f'(x)$ is not defined

2) Compute f at critical points and endpoints

$f(0) = 0$

$f(-2) = -8$

$f(1) = 1$

Global min -2, Global min value -8

Global max 1, Global max value 1

2) $g(x)= x/ (x^2+1)$ on [0,2]

1) Find critical numbers

$$f'(x) = \frac{x^2+1 - x \cdot 2x}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}, \text{ solve } f'(x)=0$$

$-x^2+1=0 \qquad x = \pm 1 \qquad \boxed{x=1}$

$f'(x)$ not defined for: no x

2) Evaluate f

$f(0)= 0$ 0 global min, 0 global min value

$f(2)= \frac{2}{5}$

$f(1) = \frac{1}{2}$ 1 global max, $\frac{1}{2}$ global max value

3) $f(x) = |x|/(x^2+1)$ on [-2,2] $\begin{cases} \dfrac{x}{x^2+1} & \text{if } x \geq 0 \\ -\dfrac{x}{x^2+1} & \text{if } x < 0 \end{cases}$

1) Find critical points $\boxed{\text{Is } f \text{ continuous?}}$

for $x > 0$ $f'(x) = \dfrac{x^2+1 - x \cdot 2x}{(x^2+1)^2} = \dfrac{-x^2+1}{x^2+1}$ (f_+)

for $x < 0$ $f'(x) = \dfrac{-(x^2+1) - (-x)(2x)}{(x^2+1)^2} = \dfrac{x^2-1}{x^2+1}$ (f_-)

$f_+'(x) = 0$ if $-x^2+1 = 0$ $x = 1$

$f_-'(x) = 0$ if $x^2-1 = 0$ $x - 1$

f_+' defined for all $x > 0$, f_-' defined for all $x < 0$

What about $x=0$?
Note: even if $x=0$ was not a critical point, it would not hurt to add it to my list of critical points and endpoints

However, you should be able to answer the question: is f differentiable at 0 and if so what is $f'(0)$?

$$f'_+(0) = 1$$

$$f'_-(0) = -1$$

$f'(0)$ DNE

Note

if $f(x) = \begin{cases} g(x) & x \leq a \\ k(x) & x \geq a \end{cases}$

and $g'(a)$ and $k'(a)$ exist then

1) if $g'(a) \neq k'(a)$ then $f'(a)$ DNE

2) if $g'(a) = k'(a) = c$ then $f'(a) = c$

Or

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{h \to 0^+} \frac{h}{(h^2+1)h} = 1$$

$$\lim_{h \to 0^-} \frac{-h}{(h^2+1)h} = -1$$

$$f'(0) = DNE$$

To find global max / min
evaluate

$f(0) = 0$ global min value at $x = 0$

$f(1) = 1/2$

$f(-1) = 1/2$

global max value at $x = -1, 1$

$f(-2) = 2/5$

$f(2) = 2/5$

Remember $f(x) = \dfrac{|x|}{x^2+1}$