

# Elementary Functions

## Which functions are called **elementary**

In our calculus course, we are going to deal mostly with *elementary functions*.

They are

power functions ( $x^2, \sqrt{x}, x^{1/3}, \dots$ ),

exponential functions ( $2^x, \pi^x, e^x, \dots$ ),

logarithmic functions ( $\ln x, \log_2 x, \dots$ ),

trigonometric functions ( $\sin x, \cos x, \tan x, \dots$ ),

inverse trigonometric functions ( $\arcsin x, \arctan x, \dots$ )

and their sums, differences, products, quotients, and compositions.

For example,

$f(x) = \frac{\arcsin \sqrt{x^2 - 3}}{\ln(x^4 + 5) - \tan e^{\cos x}}$  is an elementary function.

There are many non-elementary functions, for example

$f(x) = \int_0^x \frac{\sin t}{t} dt$  is **not** an elementary function.

# Elementary Functions

Elementary functions of a single variable  $x$  include:

- **Constant functions:**  $2$ ,  $\pi$ ,  $e$ , etc.
- **Rational powers of  $x$ :**  $x$ ,  $x^2$ ,  $\sqrt{x}$  ( $x^{\frac{1}{2}}$ ),  $x^{\frac{2}{3}}$ , etc.
- more general **algebraic functions:**  $f(x)$  satisfying  $f(x)^5 + f(x) + x = 0$ , which is not expressible through  $n$ -th roots or rational powers of  $x$  alone
- **Exponential functions:**  $e^x$ ,  $a^x$
- **Logarithms:**  $\ln x$ ,  $\log_a x$
- **Trigonometric functions:**  $\sin x$ ,  $\cos x$ ,  $\tan x$ , etc.
- **Inverse trigonometric functions:**  $\arcsin x$ ,  $\arccos x$ , etc.
- **Hyperbolic functions:**  $\sinh x$ ,  $\cosh x$ , etc.
- **Inverse hyperbolic functions:**  $\operatorname{arsinh} x$ ,  $\operatorname{arcosh} x$ , etc.
- All functions obtained by adding, subtracting, multiplying or dividing a finite number of any of the previous functions<sup>[6]</sup>
- All functions obtained by root extraction of a polynomial with coefficients in elementary functions<sup>[7]</sup>