Definition of Derivative (… set 4)

1. Use the limit definition of the derivative to find $g'(x)$ if $g(x) = 3x^2$.

Answers

1. Find $g'(x)$ if $g(x) = 3x^2$.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 3x^2}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{3x^2 + 6hx + 3h^2 - 3x^2}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{6hx + 3h^2}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{h(6x + 3h)}{h}$$

$$g'(x) = \lim_{h \to 0} (6x + 3h)$$

Answer: $g'(x) = 6x$

Definition of Derivative (… set 4)

2. The alternate definition of the derivative at a point $x = a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Use the alternate definition to find $f'(4)$ if $f(x) = 8x - 7$.

Answers

2. Find $f'(4)$ if $f(x) = 8x - 7$ using

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(4) = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$$

$$f'(4) = \lim_{x \to 4} \frac{(8x - 7) - (8 \cdot 4 - 7)}{x - 4}$$

$$f'(4) = \lim_{x \to 4} \frac{8x - 7 - (32 - 7)}{x - 4}$$

$$f'(4) = \lim_{x \to 4} \frac{8x - 7 - (25)}{x - 4}$$

$$f'(4) = \lim_{x \to 4} \frac{8x - 32}{x - 4}$$

$$f'(4) = \lim_{x \to 4} \frac{8(x - 4)}{x - 4}$$

$$f'(4) = \lim_{x \to 4} 8$$

Answer: $f'(4) = 8$

Definition of Derivative (… set 4)

3. Use the limit definition of the derivative to find $h'(x)$ if $h(x) = 2x^2 + 8x - 5$.

Answers

3. Find $h'(x)$ if $h(x) = 2x^2 + 8x - 5$.

$$h'(x) = \lim_{h \to 0} \frac{h(x+h) - h(x)}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + 8(x+h) - 5 - (2x^2 + 8x - 5)}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) + 8x + 8h - 5 - 2x^2 - 8x + 5}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{2x^2 + 4hx + 2h^2 + 8x + 8h - 5 - 2x^2 - 8x + 5}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{4hx + 2h^2 + 8h}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{h(4x + 2h + 8)}{h}$$

$$h'(x) = \lim_{h \to 0} (4x + 2h + 8)$$

Answer: $h'(x) = 4x + 8$

4. The alternate definition of the derivative at a point $x = a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Use the alternate definition to find $f'(1)$ if $f(x) = x^3$.

Answers

4. Find $f'(1)$ if $f(x) = x^3$ using

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$f'(1) = \lim_{x \to 1} \frac{x^3 - (1^3)}{x - 1}$$

$$f'(1) = \lim_{x \to 1} \frac{x^3 - 1}{x - 1}$$

$$f'(1) = \lim_{x \to 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$$

$$f'(1) = \lim_{x \to 1} (x^2 + x + 1)$$

$$f'(1) = 1^2 + 1 + 1$$

$$f'(1) = 1 + 1 + 1$$

Answer: $f'(1) = 3$

Definition of Derivative (… set 4)

5. Use the limit definition of the derivative to find $g'(3)$ if $g(x) = x^2 + 9x + 7$.

Answers

5. Find $g'(3)$ if $g(x) = x^2 + 9x + 7$.

$$g'(3) = \lim_{h \to 0} \frac{g(3+h) - g(3)}{h}$$

$$g'(3) = \lim_{h \to 0} \frac{(3+h)^2 + 9(3+h) + 7 - (3^2 + 9 \cdot 3 + 7)}{h}$$

$$g'(3) = \lim_{h \to 0} \frac{(9 + 6h + h^2) + 27 + 9h + 7 - (9 + 27 + 7)}{h}$$

$$g'(3) = \lim_{h \to 0} \frac{9 + 6h + h^2 + 27 + 9h + 7 - 9 - 27 - 7}{h}$$

$$g'(3) = \lim_{h \to 0} \frac{6h + h^2 + 9h}{h}$$

$$g'(3) = \lim_{h \to 0} \frac{h(6 + h + 9)}{h}$$

$$g'(3) = \lim_{h \to 0} (6 + h + 9)$$

Answer: $g'(3) = 15$

6. The alternate definition of the derivative at a point $x = a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Use the alternate definition to find $h'(2)$ if $h(x) = x^3 + x^2 - x$.

Answers

6. Find $h'(2)$ if $h(x) = x^3 + x^2 - x$ using

$$h'(a) = \lim_{x \to a} \frac{h(x) - h(a)}{x - a}$$

$$h'(2) = \lim_{x \to 2} \frac{h(x) - h(2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{x^3 + x^2 - x - (2^3 + 2^2 - 2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{x^3 + x^2 - x - (8 + 4 - 2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{x^3 + x^2 - x - (10)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{x^3 + x^2 - x - 10}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{(x - 2)(x^2 + 3x + 5)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} (x^2 + 3x + 5)$$

$$h'(2) = 2^2 + 3 \cdot 2 + 5$$

$$h'(2) = 4 + 6 + 5$$

Answer: $h'(2) = 15$

Definition of Derivative (… set 4)

7. Use the limit definition of the derivative to find $g'(x)$ if $g(x) = x^4$.

Answers

7. Find $g'(x)$ if $g(x) = x^4$.

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - x^4}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$g'(x) = \lim_{h \to 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$

$$g'(x) = \lim_{h \to 0} (4x^3 + 6x^2h + 4xh^2 + h^3)$$

Answer: $g'(x) = 4x^3$

Definition of Derivative (… set 4)

8. The alternate definition of the derivative at a point $x = a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Use the alternate definition to find $f'(6)$ if $f(x) = 9x + 18$.

Answers

8. Find $f'(6)$ if $f(x) = 9x + 18$ using

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6}$$

$$f'(6) = \lim_{x \to 6} \frac{9x + 18 - (9 \cdot 6 + 18)}{x - 6}$$

$$f'(6) = \lim_{x \to 6} \frac{9x + 18 - (54 + 18)}{x - 6}$$

$$f'(6) = \lim_{x \to 6} \frac{9x + 18 - (72)}{x - 6}$$

$$f'(6) = \lim_{x \to 6} \frac{9x + 18 - 72}{x - 6}$$

$$f'(6) = \lim_{x \to 6} \frac{9x - 54}{x - 6}$$

$$f'(6) = \lim_{x \to 6} \frac{9(x - 6)}{x - 6}$$

$$f'(6) = \lim_{x \to 6} (9)$$

Answer: $f'(6) = 9$

Definition of Derivative (… set 4)

9. Use the limit definition of the derivative to find $k'(x)$ if $k(x) = 5x^2 + 3x$.

Answers

9. Find $k'(4)$ if $k(x) = 5x^2 + 3x$.

$$k'(4) = \lim_{h\to 0} \frac{k(4+h) - k(4)}{h}$$

$$k'(4) = \lim_{h\to 0} \frac{5(4+h)^2 + 3(4+h) - (5\cdot 4^2 + 3\cdot 4)}{h}$$

$$k'(4) = \lim_{h\to 0} \frac{5(4+h)^2 + 3(4+h) - (80+12)}{h}$$

$$k'(4) = \lim_{h\to 0} \frac{5(16+8h+h^2) + 12 + 3h - 80 - 12}{h}$$

$$k'(4) = \lim_{h\to 0} \frac{80 + 40h + 5h^2 + 12 + 3h - 80 - 12}{h}$$

$$k'(4) = \lim_{h\to 0} \frac{40h + 5h^2 + 3h}{h}$$

$$k'(4) = \lim_{h\to 0} \frac{h(40 + 5h + 3)}{h}$$

$$k'(4) = \lim_{h\to 0} (43 + 5h)$$

Answer: $k'(4) = 43$

Definition of Derivative (… set 4)

10. The alternate definition of the derivative at a point $x = a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Use the alternate definition to find $f'(-5)$ if $f(x) = -2x^2 + 4x + 1$.

Definition of Derivative (… set 4)

Answers

10. Find $f'(-5)$ if $f(x) = -2x^2 + 4x + 1$ using

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(-5) = \lim_{x \to -5} \frac{f(x) - f(-5)}{x - (-5)}$$

$$f'(-5) = \lim_{x \to -5} \frac{-2x^2 + 4x + 1 - (-2 \cdot (-5)^2 + 4(-5) + 1)}{x + 5}$$

$$f'(-5) = \lim_{x \to -5} \frac{-2x^2 + 4x + 1 - (-50 - 20 + 1)}{x + 5}$$

$$f'(-5) = \lim_{x \to -5} \frac{-2x^2 + 4x + 1 - (-69)}{x + 5}$$

$$f'(-5) = \lim_{x \to -5} \frac{-2x^2 + 4x + 1 + 69}{x + 5}$$

$$f'(-5) = \lim_{x \to -5} \frac{-2x^2 + 4x + 70}{x + 5}$$

$$f'(-5) = \lim_{x \to -5} \frac{-2(x - 7)(x + 5)}{x + 5}$$

$$f'(-5) = \lim_{x \to -5} [-2(x - 7)]$$

$$f'(-5) = \lim_{x \to -5} (-2x + 14)$$

$$f'(-5) = -2(-5) + 14$$

Answer: $f'(-5) = 24$

Definition of Derivative (… set 4)

11. Use the limit definition of the derivative to find $f'(x)$ if

$$f(x) = 5x^2 - 14x + 12$$

Answers

11. Find $f'(x)$ if $f(x) = 5x^2 - 14x + 12$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{5(x+h)^2 - 14(x+h) + 12 - (5x^2 - 14x + 12)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{5(x^2 + 2hx + h^2) - 14x - 14h + 12 - 5x^2 + 14x - 12}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{5x^2 + 10hx + 5h^2 - 14x - 14h + 12 - 5x^2 + 14x - 12}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{10hx + 5h^2 - 14h}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{h(10x + 5h - 14)}{h}$$

$$f'(x) = \lim_{h\to 0} (10x + 5h - 14)$$

Answer: $f'(x) = 10x - 14$

12. The alternate definition of the derivative at a point $x = a$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Use the alternate definition to find $g'(-2)$ if $g(x) = 6x^2 - 8x + 3$.

Answers

12. Find $g'(-2)$ if $g(x) = 6x^2 - 8x + 3$ using

$$g'(a) = \lim_{x \to a} \frac{g(x) - g(a)}{x - a}$$

$$g'(-2) = \lim_{x \to -2} \frac{g(x) - g(-2)}{x - (-2)}$$

$$g'(-2) = \lim_{x \to -2} \frac{6x^2 - 8x + 3 \ - (6 \cdot (-2)^2 - 8(-2) + 3)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{6x^2 - 8x + 3 \ - (24 + 16 + 3)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{6x^2 - 8x + 3 \ - (43)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{6x^2 - 8x + 3 - 43}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{6x^2 - 8x - 40}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} \frac{2(3x - 10)(x + 2)}{x + 2}$$

$$g'(-2) = \lim_{x \to -2} [2(3x - 10)]$$

$$g'(-2) = \lim_{x \to -2} (6x - 20)$$

$$g'(-2) = 6(-2) - 20$$

Answer: $g'(-2) = -32$