Definition of Derivative (… set 3)

Definition of Derivative Practice Worksheet

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

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Answers

1) Use the Limit Definition of a derivative to find f'(x) if f(x) = $2x^2 - 3x + 1$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $\quad f(\;) = 2(\;)^2 - 3(\;) + 1$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) + 1 - (2x^2 - 3x + 1)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2(x^2+2xh+h^2) - 3x - 3h + 1 - 2x^2 + 3x - 1}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 \cancel{-3x} - 3h \cancel{+1} \cancel{-2x^2} \cancel{+3x} \cancel{-1}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = 4x + 2(0) - 3$$

$$\boxed{f'(x) = 4x - 3}$$

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2) Use the Alternative definition of the derivative to find $f'(2)$ if $f(x) = \sqrt{2-x}$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

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Answers

2) Use the Alternative definition of the derivative to find $H'(2)$ if $H(x) = \sqrt{3-x}$

$$f'(\underline{c}) = \lim_{x \to \underline{c}} \frac{f(x) - f(\underline{c})}{x - \underline{c}}$$

$c = 2$ $\quad h(2) = \sqrt{3-2} = 1$

$$h'(2) = \lim_{x \to 2} \frac{h(x) - h(2)}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{\sqrt{3-x} - 1}{x - 2}$$

$$h'(2) = \lim_{x \to 2} \frac{(\sqrt{3-x} - 1)}{(x-2)} \frac{(\sqrt{3-x} + 1)}{(\sqrt{3-x} + 1)}$$

$$h'(2) = \lim_{x \to 2} \frac{3 - x - 1}{(x-2)(\sqrt{3-x} + 1)}$$

$$h'(2) = \lim_{x \to 2} \frac{\cancel{(2-x)}(-1)}{\cancel{(x-2)}(\sqrt{3-x} + 1)}$$

$$h'(2) = \frac{-1}{\sqrt{3-2} + 1} = \frac{-1}{1+1} = \boxed{-\frac{1}{2}}$$

Definition of Derivative (… set 3)

3) Use the Limit Definition of a Derivative to find f'(x) if $f(x) = \sqrt{2x-1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Answers

3) Use the Limit Definition of a Derivative to find $f'(x)$ if $f(x) = \sqrt{2x-1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(\ ) = \sqrt{2(\ ) - 1}$$

$$f(x+h) = \sqrt{2(x+h) - 1}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h} \cdot \frac{(\sqrt{2x+2h-1} + \sqrt{2x-1})}{\sqrt{2x+2h-1} + \sqrt{2x-1}}$$

$$f'(x) = \lim_{h \to 0} \frac{2x+2h-1-(2x-1)}{h[\sqrt{2x+2h-1} + \sqrt{2x-1}]}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{2x}+2h\cancel{-1}\cancel{-2x}\cancel{+1}}{h[\sqrt{2x+2h-1} + \sqrt{2x-1}]}$$

$$f'(x) = \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}(\sqrt{2x+2h-1} + \sqrt{2x-1})}$$

$$= \frac{2}{\sqrt{2x-1} + \sqrt{2x-1}} = \frac{2}{2\sqrt{2x-1}} = \boxed{\frac{1}{\sqrt{2x-1}}}$$

Definition of Derivative (… set 3)

4) Use the Limit Definition of a derivative to find f ’(3) if f(x) = $\frac{2}{5-x}$

Definition of Derivative (… set 3)

Answers

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$f(\ ) = \frac{2}{5-(\ \ )}$

4) Use the Limit Definition of a derivative to find f'(3) if $f(x) = \frac{2}{5-x}$

$f(x+h) = \frac{2}{5-(x+h)}$

$$f'(x) = \lim_{h\to 0}\frac{\frac{2}{5-(x+h)} - \frac{2}{5-x}}{h}$$

$$f'(x) = \lim_{h\to 0}\frac{\frac{2}{5-x-h} - \frac{2}{5-x}}{h}\cdot(5-x)(5-x-h)$$

$$\lim_{h\to 0}\frac{2(5-x)-2(5-x-h)}{h(5-x)(5-x-h)}$$

$$\lim_{h\to 0}\frac{\cancel{10}-\cancel{2x}-\cancel{10}+\cancel{2x}+2h}{h(5-x)(5-x-h)}$$

$$\lim_{h\to 0}\frac{2\cancel{h}}{\cancel{h}(5-x)(5-x-h)}$$

$$f'(x) = \lim_{h\to 0}\frac{2}{(5-x)(5-x-h)} = \frac{2}{(5-x)(5-x)}$$

$$\boxed{f'(x) = \frac{2}{(5-x)^2}}$$

$$f'(3) = \frac{2}{(5-3)^2} = \frac{2}{(2)^2}$$

$$\boxed{f'(3) = \frac{1}{2}}$$

$f(-1) = 2(-1)-3 = -5$

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $y - y_1 = m(x - x_1)$

Definition of Derivative (… set 3)

Answers

5) Use either general or alternative method above to find the equation of the tangent line to $f(x) = 2x - 3x^2$ at $x = -1$. $\quad y - y_1 = m(x - x_1)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$f(\ ) = 2(\ ) - 3(\ )^2$

$f(x+h) = 2(x+h) - 3(x+h)^2$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h) - 3(x+h)^2 - (2x - 3x^2)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{2x + 2h - 3(x^2 + 2xh + h^2) - 2x + 3x^2}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x} + 2h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{2x} + \cancel{3x^2}}{h}$$

$$\lim_{h\to 0} \frac{2h - 6xh - 3h^2}{h} \rightarrow \lim_{h\to 0} \frac{\cancel{h}(2 - 6x - 3h)}{\cancel{h}}$$

$f'(x) = 2 - 6x - 0$

$\boxed{f'(x) = 2 - 6x}$

$f'(-1) = 2 - 6(-1)$

$= 2 + 6$

$\underline{f'(-1) = 8}$

Alt. method $\quad f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$

$$f'(-1) = \lim_{x\to -1} \frac{f(x) - f(-1)}{x - (-1)}$$

$$f'(-1) = \lim_{x\to -1} \frac{2x - 3x^2 - (-5)}{x + 1} = \frac{2x - 3x^2 + 5}{x + 1}$$

$$= \lim_{x\to -1} \frac{-1(3x^2 - 2x - 5)}{x + 1}$$

$$f'(-1) = \lim_{x\to -1} \frac{-1(3x - 5)\cancel{(x + 1)}}{\cancel{(x + 1)}}$$

$\underline{f'(-1) = -1(-3 - 5) = 8}$

point: $f(-1) = -2 - 3 = -5$

slope: $m = 8$

point: $(-1, -5)$

slope: $m = 8$

$\boxed{y + 5 = 8(x + 1)}$