

Chain Rule (... set 1)

Chain Rule Practice

Differentiate each function with respect to x .

1) $y = (5x^4 + 1)^2$

2) $y = \sqrt[5]{-x^3 - 4}$

3) $f(x) = (4x^5 - 1)\sqrt[3]{x + 1}$

4) $y = \sqrt{-x^4 - 1}(-x - 2)$

5) $y = (3x - 1)(-3x^2 - 4)^{-3}$

6) $f(x) = \left(\frac{5x^5 - 3}{-3x^3 + 1}\right)^3$

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Answers

$$1) \frac{dy}{dx} = 2(5x^4 + 1) \cdot 20x^3 \\ = 40x^3(5x^4 + 1)$$

$$2) \frac{dy}{dx} = \frac{1}{5}(-x^3 - 4)^{-\frac{4}{5}} \cdot -3x^2 \\ = -\frac{3x^2}{5(-x^3 - 4)^{\frac{4}{5}}}$$

$$3) f'(x) = (4x^5 - 1) \cdot \frac{1}{3}(x+1)^{-\frac{2}{3}} + (x+1)^{\frac{1}{3}} \cdot 20x^4 \\ = \frac{64x^5 + 60x^4 - 1}{3(x+1)^{\frac{2}{3}}}$$

$$4) \frac{dy}{dx} = (-x^4 - 1)^{\frac{1}{2}} \cdot -1 + (-x - 2) \cdot \frac{1}{2}(-x^4 - 1)^{-\frac{1}{2}} \cdot -4x^3 \\ = \frac{(x+1)^2(3x^2 - 2x + 1)}{(-x^4 - 1)^{\frac{1}{2}}}$$

$$5) \frac{dy}{dx} = (3x - 1) \cdot -3(-3x^2 - 4)^{-4} \cdot -6x + (-3x^2 - 4)^{-3} \cdot 3 \\ = \frac{3(15x^2 - 6x - 4)}{(-3x^2 - 4)^4}$$

$$6) f'(x) = 3 \cdot \left(\frac{5x^5 - 3}{-3x^3 + 1} \right)^2 \cdot \frac{(-3x^3 + 1) \cdot 25x^4 - (5x^5 - 3) \cdot -9x^2}{(-3x^3 + 1)^2} \\ = \frac{3x^2(5x^5 - 3)^2(-30x^5 + 25x^2 - 27)}{(-3x^3 + 1)^4}$$

Chain Rule (... set 1)

$$7) f(x) = \left(\frac{x^5 + 4}{x^2 - 5} \right)^{\frac{1}{5}}$$

$$8) f(x) = \frac{\sqrt[5]{x^2 - 3}}{-x - 5}$$

$$9) y = \sec 2x^4$$

$$10) f(x) = (-3x^3 - 1)\csc 5x^4$$

$$11) f(x) = \cos 3x^2 \cdot \sqrt[3]{5x^3 - 1}$$

$$12) f(x) = \sin 4x^3$$

$$13) f(x) = \frac{4x^4 + 5}{\tan 3x^5}$$

$$14) y = \cot \sqrt[3]{-5x^3 - 2}$$

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Answers

$$\begin{aligned} 7) f'(x) &= \frac{1}{5} \cdot \left(\frac{x^5 + 4}{x^2 - 5} \right)^{-\frac{4}{5}} \cdot \frac{(x^2 - 5) \cdot 5x^4 - (x^5 + 4) \cdot 2x}{(x^2 - 5)^2} \\ &= \frac{x(3x^5 - 25x^3 - 8)}{5(x^5 + 4)^{\frac{4}{5}} \cdot (x^2 - 5)^{\frac{6}{5}}} \end{aligned}$$

$$\begin{aligned} 8) f'(x) &= \frac{(-x - 5) \cdot \frac{1}{5}(x^2 - 3)^{-\frac{4}{5}} \cdot 2x + (x^2 - 3)^{\frac{1}{5}}}{(-x - 5)^2} & 9) \frac{dy}{dx} &= \sec 2x^4 \cdot \tan 2x^4 \cdot 8x^3 \\ &= \frac{3x^2 - 15 - 10x}{5(-x - 5)^2 \cdot (x^2 - 3)^{\frac{4}{5}}} & &= 8x^3 \sec 2x^4 \cdot \tan 2x^4 \end{aligned}$$

$$\begin{aligned} 10) f'(x) &= (-3x^3 - 1) \cdot -\csc 5x^4 \cot 5x^4 \cdot 20x^3 + \csc 5x^4 \cdot -9x^2 \\ &= x^2 \csc 5x^4 \cdot (60x^4 \cot 5x^4 + 20x \cot 5x^4 - 9) \end{aligned}$$

$$\begin{aligned} 11) f'(x) &= \cos 3x^2 \cdot \frac{1}{3}(5x^3 - 1)^{-\frac{2}{3}} \cdot 15x^2 + (5x^3 - 1)^{\frac{1}{3}} \cdot -\sin 3x^2 \cdot 6x \\ &= \frac{x(-30x^3 \sin 3x^2 + 6 \sin 3x^2 + 5x \cos 3x^2)}{(5x^3 - 1)^{\frac{2}{3}}} \end{aligned}$$

$$\begin{aligned} 12) f'(x) &= \cos 4x^3 \cdot 12x^2 \\ &= 12x^2 \cos 4x^3 \end{aligned}$$

$$\begin{aligned} 13) f'(x) &= \frac{\tan 3x^5 \cdot 16x^3 - (4x^4 + 5) \cdot \sec^2 3x^5 \cdot 15x^4}{\tan^2 3x^5} \\ &= \frac{x^3(16 \tan 3x^5 - 60x^5 \cdot \sec^2 3x^5 - 75x \cdot \sec^2 3x^5)}{\tan^2 3x^5} \end{aligned}$$

$$\begin{aligned} 14) \frac{dy}{dx} &= -\csc^2(-5x^3 - 2)^{\frac{1}{3}} \cdot \frac{1}{3}(-5x^3 - 2)^{-\frac{2}{3}} \cdot -15x^2 \\ &= \frac{5x^2 \cdot \csc^2(-5x^3 - 2)^{\frac{1}{3}}}{(-5x^3 - 2)^{\frac{2}{3}}} \end{aligned}$$