

Differentiating a Variety of Functions ... (set 1)

Example

Differentiate $\log_e (x^2 + 3x + 1)$.

Differentiating a Variety of Functions ... (set 1)

Answers

Solution

We solve this by using the chain rule and our knowledge of the derivative of $\log_e x$.

$$\begin{aligned}\frac{d}{dx} \log_e (x^2 + 3x + 1) &= \frac{d}{dx} (\log_e u) \quad (\text{where } u = x^2 + 3x + 1) \\ &= \frac{d}{du} (\log_e u) \times \frac{du}{dx} \quad (\text{by the chain rule}) \\ &= \frac{1}{u} \times \frac{du}{dx} \\ &= \frac{1}{x^2 + 3x + 1} \times \frac{d}{dx} (x^2 + 3x + 1) \\ &= \frac{1}{x^2 + 3x + 1} \times (2x + 3) \\ &= \frac{2x + 3}{x^2 + 3x + 1}.\end{aligned}$$

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Example

Find $\frac{d}{dx}(e^{3x^2})$.

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Answers

Solution

This is an application of the chain rule together with our knowledge of the derivative of e^x .

$$\begin{aligned}\frac{d}{dx}(e^{3x^2}) &= \frac{de^u}{dx} \quad \text{where } u = 3x^2 \\ &= \frac{de^u}{du} \times \frac{du}{dx} \quad \text{by the chain rule} \\ &= e^u \times \frac{du}{dx} \\ &= e^{3x^2} \times \frac{d}{dx}(3x^2) \\ &= 6xe^{3x^2}.\end{aligned}$$

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Example

Find $\frac{d}{dx}(e^{x^3+2x})$.

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Answers

Solution

Again, we use our knowledge of the derivative of e^x together with the chain rule.

$$\begin{aligned}\frac{d}{dx}(e^{x^3+2x}) &= \frac{de^u}{dx} \quad (\text{where } u = x^3 + 2x) \\ &= e^u \times \frac{du}{dx} \quad (\text{by the chain rule}) \\ &= e^{x^3+2x} \times \frac{d}{dx}(x^3 + 2x) \\ &= (3x^2 + 2) \times e^{x^3+2x}.\end{aligned}$$

Differentiating a Variety of Functions ... (set 1)

Example

Differentiate $\ln(2x^3 + 5x^2 - 3)$.

Differentiating a Variety of Functions ... (set 1)

Answers

Solution

We solve this by using the chain rule and our knowledge of the derivative of

$$\begin{aligned}\frac{d}{dx} \ln(2x^3 + 5x^2 - 3) &= \frac{d \ln u}{dx} \quad (\text{where } u = (2x^3 + 5x^2 - 3)) \\ &= \frac{d \ln u}{du} \times \frac{du}{dx} \quad (\text{by the chain rule}) \\ &= \frac{1}{u} \times \frac{du}{dx} \\ &= \frac{1}{2x^3 + 5x^2 - 3} \times \frac{d}{dx}(2x^3 + 5x^2 - 3) \\ &= \frac{1}{2x^3 + 5x^2 - 3} \times (6x^2 + 10x) \\ &= \frac{6x^2 + 10x}{2x^3 + 5x^2 - 3}.\end{aligned}$$

Differentiating a Variety of Functions ... (set 1)

Exercise 1

Differentiate the following functions.

a. $f(x) = \ln(2x^3)$

b. $f(x) = e^{x^7}$

c. $f(x) = \ln(11x^7)$

d. $f(x) = e^{x^2+x^3}$

e. $f(x) = \log_e(7x^{-2})$

f. $f(x) = e^{-x}$

g. $f(x) = \ln(e^x + x^3)$

h. $f(x) = \ln(e^x x^3)$

i. $f(x) = \ln\left(\frac{x^2 + 1}{x^3 - x}\right)$

Differentiating a Variety of Functions ... (set 1)

Answers

Solutions to Exercise 1

a. $f'(x) = \frac{6x^2}{2x^3} = \frac{3}{x}$

Alternatively write $f(x) = \ln 2 + 3 \ln x$ so that $f'(x) = 3 \frac{1}{x}$.

b. $f'(x) = 7x^6 e^{x^7}$

c. $f'(x) = \frac{7}{x}$

d. $f'(x) = (2x + 3x^2)e^{x^2+x^3}$

e. Write $f(x) = \log_e 7 - 2 \log_e x$ so that $f'(x) = -\frac{2}{x}$.

f. $f'(x) = -e^{-x}$

g. $f'(x) = \frac{e^x + 3x^2}{e^x + x^3}$

h. Write $f(x) = \ln e^x + \frac{3}{\ln x}$ so that $f'(x) = 1 + \frac{3}{x}$.

i. Write $f(x) = \ln(x^2 + 1) - \ln(x^3 - x)$ so that $f'(x) = \frac{2x}{x^2 + 1} - \frac{3x^2 - 1}{x^3 - x}$.