Differentiation … The Chain Rule

… Set 1

Chain Rule Practice

Differentiate each function with respect to x.

1) $y = (5x^4 + 1)^2$

2) $y = \sqrt[5]{-x^3 - 4}$

3) $f(x) = (4x^5 - 1)\sqrt[3]{x + 1}$

4) $y = \sqrt{-x^4 - 1}\,(-x - 2)$

5) $y = (3x - 1)(-3x^2 - 4)^{-3}$

6) $f(x) = \left(\dfrac{5x^5 - 3}{-3x^3 + 1}\right)^3$

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Answers

1) $\frac{dy}{dx} = 2(5x^4 + 1) \cdot 20x^3$

$= 40x^3(5x^4 + 1)$

2) $\frac{dy}{dx} = \frac{1}{5}(-x^3 - 4)^{-\frac{4}{5}} \cdot -3x^2$

$= -\frac{3x^2}{5(-x^3 - 4)^{\frac{4}{5}}}$

3) $f'(x) = (4x^5 - 1) \cdot \frac{1}{3}(x + 1)^{-\frac{2}{3}} + (x + 1)^{\frac{1}{3}} \cdot 20x^4$

$= \frac{64x^5 + 60x^4 - 1}{3(x + 1)^{\frac{2}{3}}}$

4) $\frac{dy}{dx} = (-x^4 - 1)^{\frac{1}{2}} \cdot -1 + (-x - 2) \cdot \frac{1}{2}(-x^4 - 1)^{-\frac{1}{2}} \cdot -4x^3$

$= \frac{(x + 1)^2(3x^2 - 2x + 1)}{(-x^4 - 1)^{\frac{1}{2}}}$

5) $\frac{dy}{dx} = (3x - 1) \cdot -3(-3x^2 - 4)^{-4} \cdot -6x + (-3x^2 - 4)^{-3} \cdot 3$

$= \frac{3(15x^2 - 6x - 4)}{(-3x^2 - 4)^4}$

6) $f'(x) = 3 \cdot \left(\frac{5x^5 - 3}{-3x^3 + 1}\right)^2 \cdot \frac{(-3x^3 + 1) \cdot 25x^4 - (5x^5 - 3) \cdot -9x^2}{(-3x^3 + 1)^2}$

$= \frac{3x^2(5x^5 - 3)^2(-30x^5 + 25x^2 - 27)}{(-3x^3 + 1)^4}$

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7) $f(x) = \left(\frac{x^5 + 4}{x^2 - 5}\right)^{\frac{1}{5}}$

8) $f(x) = \frac{\sqrt[5]{x^2 - 3}}{-x - 5}$

9) $y = \sec 2x^4$

10) $f(x) = (-3x^3 - 1)\csc 5x^4$

11) $f(x) = \cos 3x^2 \cdot \sqrt[3]{5x^3 - 1}$

12) $f(x) = \sin 4x^3$

13) $f(x) = \frac{4x^4 + 5}{\tan 3x^5}$

14) $y = \cot \sqrt[3]{-5x^3 - 2}$

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Answers

7) $f'(x) = \frac{1}{5} \cdot \left(\frac{x^5+4}{x^2-5}\right)^{-\frac{4}{5}} \cdot \frac{(x^2-5)\cdot 5x^4 - (x^5+4)\cdot 2x}{(x^2-5)^2}$

$= \frac{x(3x^5 - 25x^3 - 8)}{5(x^5+4)^{\frac{4}{5}} \cdot (x^2-5)^{\frac{6}{5}}}$

8) $f'(x) = \frac{(-x-5)\cdot \frac{1}{5}(x^2-3)^{-\frac{4}{5}} \cdot 2x + (x^2-3)^{\frac{1}{5}}}{(-x-5)^2}$

$= \frac{3x^2 - 15 - 10x}{5(-x-5)^2 \cdot (x^2-3)^{\frac{4}{5}}}$

9) $\frac{dy}{dx} = \sec 2x^4 \cdot \tan 2x^4 \cdot 8x^3$

$= 8x^3 \sec 2x^4 \cdot \tan 2x^4$

10) $f'(x) = (-3x^3 - 1) \cdot -\csc 5x^4 \cot 5x^4 \cdot 20x^3 + \csc 5x^4 \cdot -9x^2$

$= x^2 \csc 5x^4 \cdot (60x^4 \cot 5x^4 + 20x \cot 5x^4 - 9)$

11) $f'(x) = \cos 3x^2 \cdot \frac{1}{3}(5x^3-1)^{-\frac{2}{3}} \cdot 15x^2 + (5x^3-1)^{\frac{1}{3}} \cdot -\sin 3x^2 \cdot 6x$

$= \frac{x(-30x^3 \sin 3x^2 + 6\sin 3x^2 + 5x\cos 3x^2)}{(5x^3-1)^{\frac{2}{3}}}$

12) $f'(x) = \cos 4x^3 \cdot 12x^2$

$= 12x^2 \cos 4x^3$

13) $f'(x) = \frac{\tan 3x^5 \cdot 16x^3 - (4x^4+5)\cdot \sec^2 3x^5 \cdot 15x^4}{\tan^2 3x^5}$

$= \frac{x^3(16\tan 3x^5 - 60x^5 \cdot \sec^2 3x^5 - 75x \cdot \sec^2 3x^5)}{\tan^2 3x^5}$

14) $\frac{dy}{dx} = -\csc^2(-5x^3-2)^{\frac{1}{3}} \cdot \frac{1}{3}(-5x^3-2)^{-\frac{2}{3}} \cdot -15x^2$

$= \frac{5x^2 \cdot \csc^2(-5x^3-2)^{\frac{1}{3}}}{(-5x^3-2)^{\frac{2}{3}}}$