The 1st Derivative Test

… Set 3

Classify the relative extrema of $h(x) = (x^2 - 4)^3$. Then sketch a quick graph plotting only the critical points.

The 1st Derivative Test

… Set 3

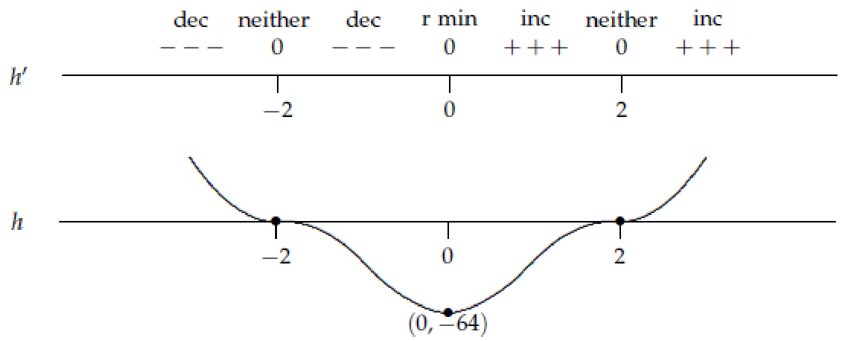
Answers

Classify the relative extrema of $h(x) = (x^2 - 4)^3$. Then sketch a quick graph plotting only the critical points.

SOLUTION. Use the First Derivative Test.

$$h'(x) = 3(x^2 - 4)^2(2x) = 0 \quad \text{at } x = 0, \pm 2.$$

It is easy to determine the sign of the derivative.



The 1st Derivative Test

… Set 3

Classify the relative extrema of $f(x) = \frac{1}{4}(x^2 - 9)^2$.

The 1st Derivative Test

… Set 3

Answers

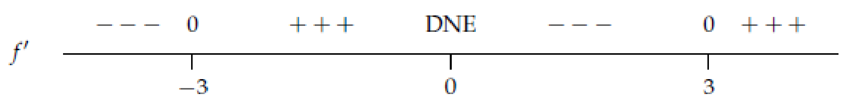
Classify the relative extrema of $f(x) = \frac{1}{4}(x^2 - 9)^2$.

[Answer: The critical values are at $x = 0, \pm 2$. Classify them.]

The 1st Derivative Test

… Set 3

Suppose that a function $f(x)$ is continuous and has the following number line that describes its first derivative. Interpret this information to find where f is increasing, decreasing, and has relative extrema. Then draw a graph of the original function f that satisfies these conditions.



The 1st Derivative Test

… Set 3

Answers

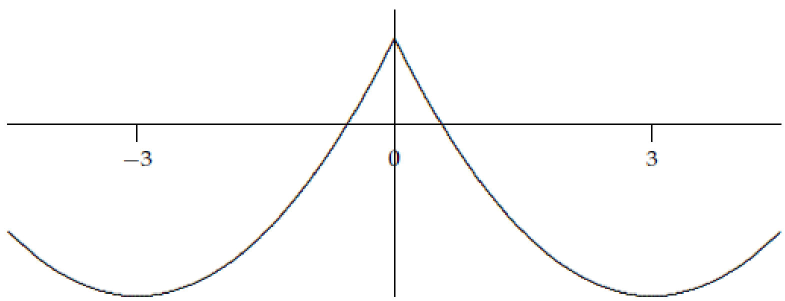
Suppose that a function $f(x)$ is continuous and has the following number line that describes its first derivative. Interpret this information to find where f is increasing, decreasing, and has relative extrema. Then draw a graph of the original function f that satisfies these conditions.

	$---$	0	$+++$	DNE	$---$	0	$+++$
f'							
		-3		0		3	

SOLUTION. Use the First Derivative Test to determine what type of extrema we have.

	dec	r min	inc	r max	dec	r min	inc
	$---$	0	$+++$	DNE	$---$	0	$+++$
f'							
		-3		0		3	

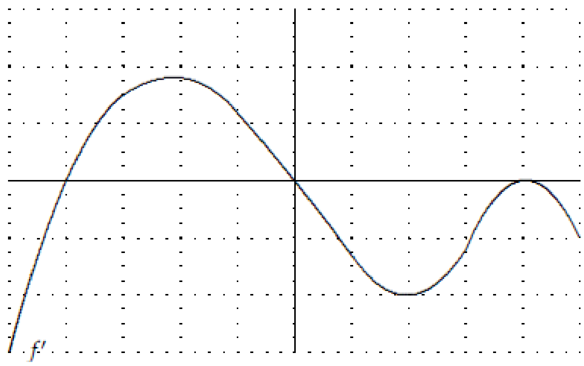
We can use this information to graph one possible solution for f. Note the function is cannot be differentiable at 0 (but should be elsewhere).



The 1st Derivative Test

… Set 3

Suppose that the graph of f' is given below. Translate this information into 'number line' form and then attempt to graph the original function f.

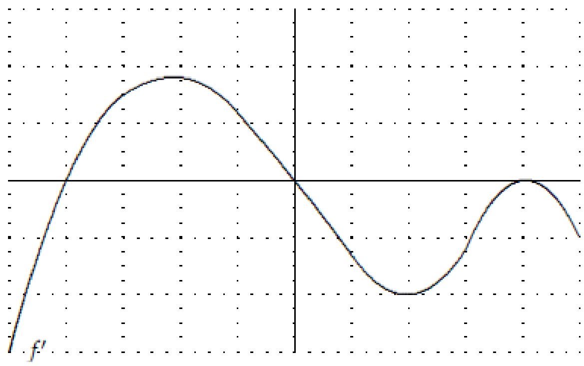


The 1st Derivative Test

… Set 3

Answers

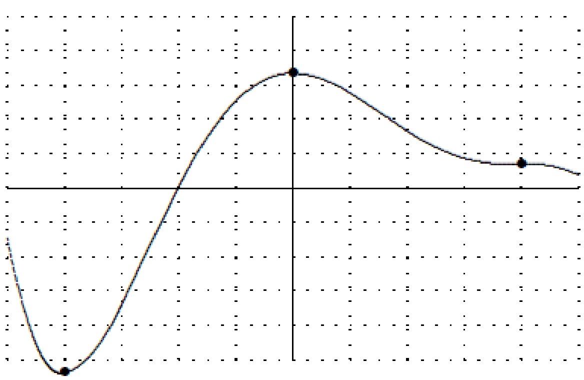
Suppose that the graph of f' is given below. Translate this information into number line' form and then attempt to graph the original function f.



SOLUTION. All we need to do is pay attention to the sign of the derivative.

dec	r min	inc	r max	dec	none	dec
$--$	0	$+++$	0	$----$	0	$--$
	-4		0		4	

Here's one function that satisfies these conditions. Notice that $x = 4$ is not a relative extreme point.



The 1st Derivative Test

… Set 3

Let $f(x) = xe^{2x}$. Where is f increasing? Decreasing? Where does it have relative extrema?

The 1st Derivative Test

… Set 3

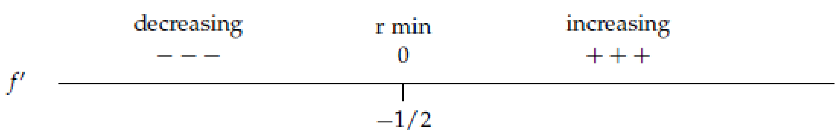
Answers

Let $f(x) = xe^{2x}$. Where is f increasing? Decreasing? Where does it have relative extrema?

SOLUTION. Use the Increasing/Decreasing Test. Find the derivative and the critical numbers.

$$f'(x) = e^{2x} + 2xe^{2x} = e^{2x}[1 + 2x] = 0 \quad \text{at} \quad x = -1/2.$$

Set up the number line and determine the sign of $f'(x)$ on either side of the critical point. $f'(-1) = -e^{-2} < 0$ and $f'(0) = 1$.



Using interval notation: f is increasing on $(-1/2, \infty)$ and it is decreasing on $(-\infty, -1/2)$. From the First Derivative Test, there is a relative min at $x = -1/2$.

The 1st Derivative Test

… Set 3

Let $f(x) = x - \sin x$. Where is f increasing? Decreasing? Where does it have relative extrema?

The 1st Derivative Test

… Set 3

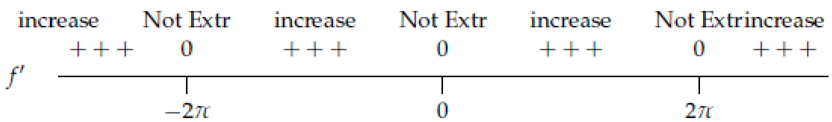
Answers

Let $f(x) = x - \sin x$. Where is f increasing? Decreasing? Where does it have relative extrema?

SOLUTION. Use the Increasing/Decreasing Test. Find the derivative and the critical numbers.

$$f'(x) = 1 - \cos x = 0 \text{ at } x = 0, \pm 2\pi, \pm 4\pi \ldots.$$

Since $\cos x \leq 1$ the sign of $f'(x)$ between the critical points is always positive. So the $f(x)$ is always increasing and by the First Derivative Test and there are no relative extrema.



The 1st Derivative Test

… Set 3

Let $f(x) = (x^2 - 4)^{2/3}$.

Where is f increasing?

Decreasing?

Where does it have relative extrema?

The 1st Derivative Test

… Set 3

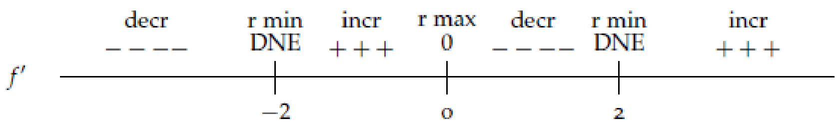
Answers

Let $f(x) = (x^2 - 4)^{2/3}$. Where is f increasing? Decreasing? Where does it have relative extrema?

SOLUTION. Use the Increasing/Decreasing Test. Find the derivative and the critical points.

$$f'(x) = \frac{2}{3}(x^2-4)^{-1/3}2x = \frac{4x}{3(x^2-4)^{1/3}} = 0 \text{ at } x = 0 \text{ DNE at } x = \pm 2.$$

Determine the sign of $f'(x)$ between and beyond the critical points. $f'(-3) < 0$, $f'(-1) > 0$, $f'(1) < 0$ and $f'(3) > 0$.



Using the First Derivative Test, f has relative mins at $x = \pm 2$ and a relative max at 0. Can you sketch the shape of the graph based on the information?