# The 2nd Derivative Test

## ... Set 1

**For each problem, find all points of relative minima and maxima.**

1) $f(x) = -x^3 + 3x^2 - 2$

2) $f(x) = x^4 - 2x^2 - 1$

3) $f(x) = -x^3 + 6x^2 - 9x + 7$

4) $f(x) = x^3 - 3x^2 - 3$

5) $f(x) = -x^3 - 9x^2 - 24x - 21$

6) $f(x) = -x^4 + 2x^2 + 4$

7) $f(x) = x^3 - 3x^2 + 5$

8) $f(x) = x^4 - 2x^2 + 4$

**For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.**

9) $f(x) = -x^3 + 4x^2 - 2$

10) $f(x) = -x^3 + x^2 + 5x - 3$

11) $f(x) = -x^4 + x^3 + 3x^2 - 3$

12) $f(x) = x^4 - 3x^3 + 5x + 1$

13) $f(x) = \dfrac{2x}{x+1}$

14) $f(x) = \dfrac{2}{x+1}$

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## Answers

1) Relative minimum: $(0, -2)$
   Relative maximum: $(2, 2)$

2) Relative minima: $(-1, -2), (1, -2)$
   Relative maximum: $(0, -1)$

3) Relative minimum: $(1, 3)$
   Relative maximum: $(3, 7)$

4) Relative minimum: $(2, -7)$
   Relative maximum: $(0, -3)$

5) Relative minimum: $(-4, -5)$
   Relative maximum: $(-2, -1)$

6) Relative minimum: $(0, 4)$
   Relative maxima: $(-1, 5), (1, 5)$

7) Relative minimum: $(2, 1)$
   Relative maximum: $(0, 5)$

8) Relative minima: $(-1, 3), (1, 3)$
   Relative maximum: $(0, 4)$

9) Inflection point at: $x = \dfrac{4}{3}$

   Concave up: $\left(-\infty, \dfrac{4}{3}\right)$ Concave down: $\left(\dfrac{4}{3}, \infty\right)$

10) Inflection point at: $x = \dfrac{1}{3}$

    Concave up: $\left(-\infty, \dfrac{1}{3}\right)$ Concave down: $\left(\dfrac{1}{3}, \infty\right)$

11) Inflection points at: $x = -\dfrac{1}{2}, 1$

    Concave up: $\left(-\dfrac{1}{2}, 1\right)$ Concave down: $\left(-\infty, -\dfrac{1}{2}\right), (1, \infty)$

12) Inflection points at: $x = 0, \dfrac{3}{2}$

    Concave up: $(-\infty, 0), \left(\dfrac{3}{2}, \infty\right)$ Concave down: $\left(0, \dfrac{3}{2}\right)$

13) No inflection points exist.
    Concave up: $(-\infty, -1)$ Concave down: $(-1, \infty)$

14) No inflection points exist.
    Concave up: $(-1, \infty)$ Concave down: $(-\infty, -1)$

# The 2<sup>nd</sup> Derivative Test

## ... Set 1

**For each problem, find all points of relative minima and maxima.**

15) $f(x) = -x^4 + 2x^2 + 3$

**For each problem, find the open intervals where the function is increasing and decreasing.**

16) $f(x) = -x^4 + 4x^2$

**For each problem, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of $x$ and $y$.**

17) $5x + y^3 = 3y$

18) $2y^2 = 2x^3 + 2y$

19) $5y = 5x^2 - 3y^3$

20) $2y = 2x^2 - y^3$

21) $-4y^3 + 2xy = 5x^3$

22) $4x^2 - 2y^2 = 2x^3y^3$

23) $2x = 2xy + 1$

24) $2 = 3x - 2x^3y^2$

**For each problem, find the indicated derivative with respect to $x$.**

25) $f(x) = -x^4 + x^3 + x$   Find $f''$

26) $f(x) = -3x^3$   Find $f^{(4)}$

27) $f(x) = -x^5 - x^4 + 3x^2$   Find $f'''$

28) $f(x) = -4x^2$   Find $f''$

**Differentiate each function with respect to $x$.**

29) $f(x) = (-5x^3 - 3)(-3x^3 + 4)$

30) $f(x) = \dfrac{3}{x^2 + 4}$

... Set 1

<span style="color:red">Answers</span>

15) Relative minimum: $(0, 3)$
Relative maxima: $(-1, 4), (1, 4)$

16) Increasing: $(-\infty, -\sqrt{2}), (0, \sqrt{2})$ Decreasing: $(-\sqrt{2}, 0), (\sqrt{2}, \infty)$

17) $\dfrac{dy}{dx} = -\dfrac{5}{3y^2 - 3}$

18) $\dfrac{dy}{dx} = \dfrac{3x^2}{2y - 1}$

19) $\dfrac{dy}{dx} = \dfrac{10x}{5 + 9y^2}$

20) $\dfrac{dy}{dx} = \dfrac{4x}{2 + 3y^2}$

21) $\dfrac{dy}{dx} = \dfrac{15x^2 - 2y}{-12y^2 + 2x}$

22) $\dfrac{dy}{dx} = \dfrac{3x^2 y^3 - 4x}{-2y - 3y^2 x^3}$

23) $\dfrac{dy}{dx} = \dfrac{-y + 1}{x}$

24) $\dfrac{dy}{dx} = \dfrac{3 - 6x^2 y^2}{4x^3 y}$

25) $f''(x) = -12x^2 + 6x$

26) $f^{(4)}(x) = 0$

27) $f'''(x) = -60x^2 - 24x$

28) $f''(x) = -8$

29) $f'(x) = (-5x^3 - 3) \cdot -9x^2 + (-3x^3 + 4) \cdot -15x^2$
$= 90x^5 - 33x^2$

30) $f'(x) = -\dfrac{3 \cdot 2x}{(x^2 + 4)^2}$
$= -\dfrac{6x}{x^4 + 8x^2 + 16}$