Definite Integration … Set 4

Evaluating Definite Integrals

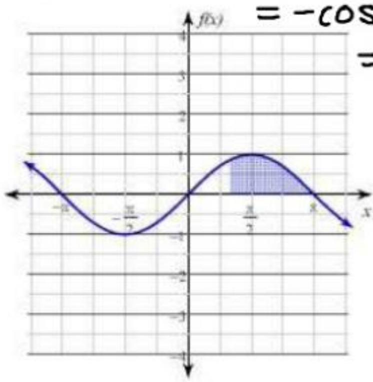
Evaluate each definite integral. Note: For problems 1-4, compare your numerical answer to the area shown to see if it makes sense. Remember, the definite integral represents the area between the function and the x-axis over the given interval. Area above the x-axis is positive. Area below the x-axis is negative.

1) $\int_{\frac{\pi}{3}}^{\pi} \sin x \, dx = -\cos x \Big|_{\frac{\pi}{3}}^{\pi}$

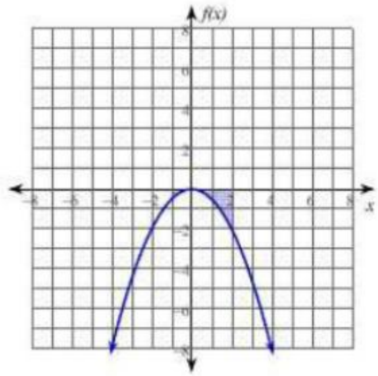
$= -\cos\pi + \cos\frac{\pi}{3}$

$= 1 + \frac{1}{2}$

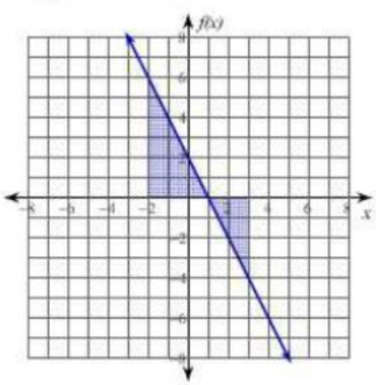
$= \frac{3}{2}$



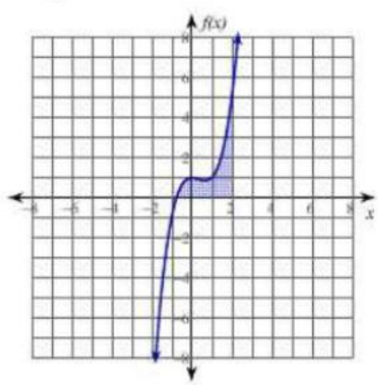
2) $\int_{-1}^{2} -\frac{x^2}{2} \, dx = -\frac{x^3}{6}\Big|_{-1}^{2} = \left(-\frac{8}{6} + \frac{-1}{6}\right)$



3) $\int_{-2}^{3} (-2x + 2) \, dx$



4) $\int_{-1}^{2} (x^3 - x^2 + 1) \, dx$



Evaluating Definite Integrals

1) $\frac{3}{2} = 1.5$

2) $-\frac{3}{2} = -1.5$

3) 5

4) $\frac{15}{4} = 3.75$

5) $-\frac{5}{2} = -2.5$

6) 0

7) 6

8) 0

9) $-\frac{\sqrt{3}}{2} \approx -0.866$

10) $\frac{e-1}{e} \approx 0.632$

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5) $\int_{-3}^{2} (-x-1)\,dx$

6) $\int_{-3}^{1} (2x+2)\,dx$

7) $\int_{0}^{3} (-2x^2+4x+2)\,dx$

8) $\int_{-2}^{2} 5x^{\frac{1}{3}}\,dx$

$\frac{5}{1}\left(\frac{3}{4}x^{\frac{4}{3}}\right)\Big|_{-2}^{2}$

$\frac{15}{4}(2)^{\frac{4}{3}} - \frac{15}{4}(-2)^{\frac{4}{3}}$

0

9) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} -\sin x\,dx = \cos x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$\cos\frac{\pi}{2} - \cos\frac{\pi}{6}$

$0 - \frac{\sqrt{3}}{2}$

$-\frac{\sqrt{3}}{2}$

10) $\int_{-1}^{0} e^x\,dx = e^x\Big|_{-1}^{0}$

$e^0 - e^{-1}$

$1 - \frac{1}{e}$

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Antidifferentiation by Substitution

(Polynomials & Exponentials)

u-Substitution
u-sub

Taking derivatives with differentials	What is a differential? a small change in a variable $dy \quad dx$ Find du for the following functions: $u = 2x$: $du = 2\,dx$ $u = x^2 + 1$: $du = 2x\,dx$ $u = \cos x + 5x$: $du = (-\sin x + 5)\,dx$
The Chain Rule	Remember the chain rule: $\frac{d}{dx}[F(g(x))] = F'(g(x)) \cdot g'(x)$ $= f(g(x)) \cdot g'(x)$ Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is the antiderivative of f on I, then $\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$ If $u = g(x)$, then $du = g'(x)dx$ and $\int f(u)\,du = F(u) + C$ MAIN IDEA: reversing chain rule
Examples	$\int (x^2+1)^2(2x)dx = \int u^2\,du = \frac{1}{3}u^3 + C$ $u = x^2+1$ $du = 2x\,dx$ $= \frac{1}{3}(x^2+1)^3 + C$ check: $\frac{1}{\cancel{3}}(\cancel{3})(x^2+1)^2 \cdot 2x$ $\int 5\cos 5x\,dx = \int \cos u\,du = \sin u + C$ $u = 5x$ $du = 5\,dx$ $= \sin 5x + C$

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What if the *du* doesn't match exactly?

$\int x(x^2+1)^2\,dx = \int u^2 \cdot \frac{1}{2}\,du$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int u^2\,du$

$= \frac{1}{2}\left(\frac{1}{3}u^3\right)+C$

$= \frac{1}{6}(x^2+1)^3 + C$

$\int \sqrt{2x-1}\,dx$

$u = 2x-1$

$du = 2dx$

$\frac{1}{2}du = dx$

$\to \int u^{\frac{1}{2}} \cdot \frac{1}{2}\,du$

$= \frac{1}{2}\int u^{\frac{1}{2}}\,du$

$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}u^{\frac{3}{2}}\right)+C$

$= \frac{1}{3}(2x-1)^{\frac{3}{2}} + C$

What if I want to substitute again?

$\int \sin^2 3x \cos 3x\,dx$

Integrating with exponentials

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{1}{\ln a}\cdot a^x + C$

HELPFUL HINT: u-sub (u = exponent)

Examples

$\int \sin x\, e^{\cos x}\,dx = -\int e^u\,du$

$u = \cos x$

$-du = +\sin x\,dx$

$= -e^u + C = -e^{\cos x} + C$

$\int 5x e^{-5x^2}\,dx = \int 5e^u \cdot -\frac{1}{10}\,du$

$u = -5x^2$

$du = -10x\,dx$

$-\frac{1}{10}du = x\,dx$

$-\frac{1}{2}\int e^u\,du$

$-\frac{1}{2}e^u + C$

$-\frac{1}{2}e^{-5x^2} + C$

$\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = \int e^{\frac{1}{x}} \cdot \frac{1}{x^2}\,dx$

$u = x^{-1}$

$du = -1x^{-2}\,dx$

$-du = \frac{+1}{x^2}\,dx$

$= -\int e^u\,du$

$= -e^u + C = -e^{\frac{1}{x}} + C$

$\int 2^{3x}\,dx = \int 2^u \cdot \frac{1}{3}\,du$

$u = 3x$

$du = 3dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\left(\frac{1}{\ln 2}\cdot 2^u\right) + C$

$= \frac{1}{3\ln 2}\cdot 2^{3x} + C$

$= \frac{1}{\ln 8}\,8^x + C$

$\int (2^3)^x\,dx$

$\int 8^x\,dx$

$\frac{1}{\ln 8}\cdot 8^x + C$