Integral Calculus

Antidifferentiation

The Indefinite Integral

Integration Test … Set 3

In problems 1 through 7, find the indicated integral.

1. $\int \sqrt{x}dx$

   Solution.

$$\int \sqrt{x}dx = \int x^{\frac{1}{2}}dx = \frac{2}{3}x^{\frac{3}{2}} + C = \frac{2}{3}x\sqrt{x} + C.$$

2. $\int 3e^x dx$

   Solution.

$$\int 3e^x dx = 3\int e^x dx = 3e^x + C.$$

3. $\int (3x^2 - \sqrt{5x} + 2)dx$

   Solution.

$$\begin{aligned}\int (3x^2 - \sqrt{5x} + 2)dx &= 3\int x^2 dx - \sqrt{5}\int \sqrt{x}dx + 2\int dx = \\ &= 3\cdot\frac{1}{3}x^3 - \sqrt{5}\cdot\frac{2}{3}x\sqrt{x} + 2x + C = \\ &= x^3 - \frac{2}{3}x\sqrt{5x} + 2x + C.\end{aligned}$$

4. $\int \left(\frac{1}{2x} - \frac{2}{x^2} + \frac{3}{\sqrt{x}}\right) dx$

   Solution.

$$\begin{aligned}\int \left(\frac{1}{2x} - \frac{2}{x^2} + \frac{3}{\sqrt{x}}\right) dx &= \frac{1}{2}\int \frac{1}{x}dx - 2\int x^{-2}dx + 3\int x^{-\frac{1}{2}}dx = \\ &= \frac{1}{2}\ln|x| - 2\cdot(-1)x^{-1} + 3\cdot 2x^{\frac{1}{2}} + C = \\ &= \frac{\ln|x|}{2} + \frac{2}{x} + 6\sqrt{x} + C.\end{aligned}$$

5. $\int\left(2e^x+\frac{6}{x}+\ln 2\right)dx$
Solution.

$$\begin{aligned}\int\left(2e^x+\frac{6}{x}+\ln 2\right)dx &= 2\int e^x dx+6\int\frac{1}{x}dx+\ln 2\int dx=\\ &= 2e^x+6\ln|x|+(\ln 2)x+C.\end{aligned}$$

6. $\int\frac{x^2+3x-2}{\sqrt{x}}dx$
Solution.

$$\begin{aligned}\int\frac{x^2+3x-2}{\sqrt{x}}dx &= \int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx-2\int x^{-\frac{1}{2}}dx=\\ &= \frac{2}{5}x^{\frac{5}{2}}+3\cdot\frac{2}{3}x^{\frac{3}{2}}-2\cdot 2x^{\frac{1}{2}}+C=\\ &= \frac{2}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}-4x^{\frac{1}{2}}+C=\\ &= \frac{2}{5}x^2\sqrt{x}+2x\sqrt{x}-4\sqrt{x}+C.\end{aligned}$$

7. $\int(x^3-2x^2)\left(\frac{1}{x}-5\right)dx$
Solution.

$$\begin{aligned}\int(x^3-2x^2)\left(\frac{1}{x}-5\right)dx &= \int(x^2-5x^3-2x+10x^2)dx=\\ &= \int(-5x^3+11x^2-2x)dx=\\ &= -5\cdot\frac{1}{4}x^4+11\cdot\frac{1}{3}x^3-2\cdot\frac{1}{2}x^2+C=\\ &= -\frac{5}{4}x^4+\frac{11}{3}x^3-x^2+C.\end{aligned}$$

8. Find the function f whose tangent has slope $x^3 - \frac{2}{x^2} + 2$ for each value of x and whose graph passes through the point $(1, 3)$.
Solution. The slope of the tangent is the derivative of f. Thus

$$f'(x) = x^3 - \frac{2}{x^2} + 2$$

and so $f(x)$ is the indefinite integral

$$\begin{aligned} f(x) &= \int f'(x)dx = \int \left(x^3 - \frac{2}{x^2} + 2\right) dx = \\ &= \frac{1}{4}x^4 + \frac{2}{x} + 2x + C. \end{aligned}$$

Using the fact that the graph of f passes through the point $(1, 3)$ you get

$$3 = \frac{1}{4} + 2 + 2 + C \quad \text{or} \quad C = -\frac{5}{4}.$$

Therefore, the desired function is $f(x) = \frac{1}{4}x^4 + \frac{2}{x} + 2x - \frac{5}{4}$.

9. It is estimated that t years from now the population of a certain lakeside community will be changing at the rate of $0.6t^2 + 0.2t + 0.5$ thousand people per year. Environmentalists have found that the level of pollution in the lake increases at the rate of approximately 5 units per 1000 people. By how much will the pollution in the lake increase during the next 2 years?

Solution. Let $P(t)$ denote the population of the community t years from now. Then the rate of change of the population with respect to time is the derivative

$$\frac{dP}{dt} = P'(t) = 0.6t^2 + 0.2t + 0.5.$$

It follows that the population function $P(t)$ is an antiderivative of $0.6t^2 + 0.2t + 0.5$. That is,

$$\begin{aligned} P(t) &= \int P'(t)dt = \int (0.6t^2 + 0.2t + 0.5)dt = \\ &= 0.2t^3 + 0.1t^2 + 0.5t + C \end{aligned}$$

for some constant C. During the next 2 years, the population will grow on behalf of

$$\begin{aligned} P(2) - P(0) &= 0.2 \cdot 2^3 + 0.1 \cdot 2^2 + 0.5 \cdot 2 + C - C = \\ &= 1.6 + 0.4 + 1 = 3 \text{ thousand people.} \end{aligned}$$

Hence, the pollution in the lake will increase on behalf of $5 \cdot 3 = 15$ units.

10. An object is moving so that its speed after t minutes is $v(t) = 1+4t+3t^2$ meters per minute. How far does the object travel during 3rd minute?

Solution. Let $s(t)$ denote the displacement of the car after t minutes. Since $v(t) = \frac{ds}{dt} = s'(t)$ it follows that

$$s(t) = \int v(t)dt = \int (1 + 4t + 3t^2)dt = t + 2t^2 + t^3 + C.$$

During the 3rd minute, the object travels

$$\begin{aligned} s(3) - s(2) &= 3 + 2 \cdot 9 + 27 + C - 2 - 2 \cdot 4 - 8 - C = \\ &= 30 \text{ meters.} \end{aligned}$$