Limits Practice … Set 1

Example 5.5. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 2^-} f(x)$

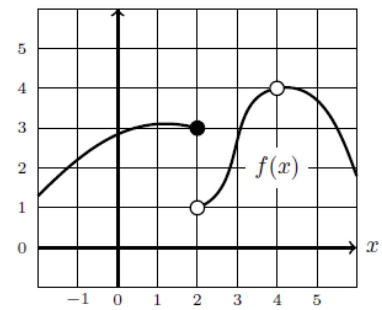
(b) $\lim_{x \to 2^+} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $\lim_{x \to 4} f(x)$

(e) $f(2)$

(f) $f(4)$



Example 5.6. Evaluate the following limits for the function $f(x) = \begin{cases} x^2 + 4 & \text{if } x < 1 \\ 0 & \text{if } x = 1 \\ 3 - x & \text{if } x > 1 \end{cases}$

(a) $\lim_{x \to 1^-} f(x)$

(b) $\lim_{x \to 1^+} f(x)$

(c) $\lim_{x \to 1} f(x)$

(d) $\lim_{x \to 2} f(x)$

Limits Practice … Set 1

Find the vertical asymptotes of the function $y = \dfrac{x^2 + 2x + 1}{x + x^2}$

Evaluate $\lim\limits_{x \to 1^+} \dfrac{5}{1 - x}$

Evaluate the limit $\lim\limits_{x \to 0^-} \dfrac{x^2 + 2x + 1}{x + x^2}$

Limits Practice … Set 1

Theorem (Limit Laws). Suppose that c is a constant and the limits

$\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then:

1. $\lim_{x\to a} [f(x) + g(x)] =$

2. $\lim_{x\to a} [cf(x)] =$

3. $\lim_{x\to a} [f(x)g(x)] =$

4. $\lim_{x\to a} \left[\frac{f(x)}{g(x)}\right] =$ provided $\lim_{x\to a} g(x) \neq 0$

5. $\lim_{x\to a} c =$

6. $\lim_{x\to a} x =$

7. $\lim_{x\to a} x^n =$ when appropriate.

8. $\lim_{x\to a} [f(x)^n] =$ when appropriate.

Limits Practice … Set 1

Evaluate $\lim_{x \to -3} \frac{\sqrt{1-x}}{x^2 - 4x}$

If $\lim_{x \to 2} f(x) = 3$ and $\lim_{x \to 2} g(x) = -2$ then find $\lim_{x \to 2} [5f(x) - xg(x)]$.

Limits Practice … Set 1

Evaluate the limit, if it exists $\lim_{x\to 1} \dfrac{x^2+2x-3}{x^2-1}$

Evaluate the limit, if it exists $\lim_{x\to 2} \dfrac{\dfrac{1}{x-1}-1}{x-2}$

Limits Practice … Set 1

Evaluate the limit, if it exists $\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$

Theorem 6.14 (Squeeze Theorem). If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$$

then

$$\lim_{x\to a} g(x) = L$$

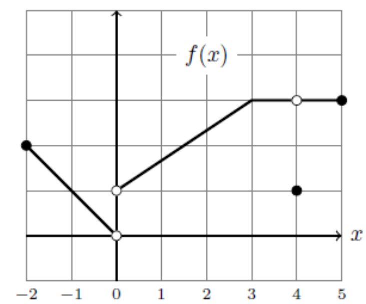
Limits Practice … Set 1

Suppose $2x + 1 \leq g(x) \leq x^2 - 2x + 5$ for all x. Find $\lim_{x \to 2} g(x)$

Use the Squeeze Theorem to find $\lim_{x \to 0} \left[x^2 \cos\left(\frac{2}{x}\right) \right]$

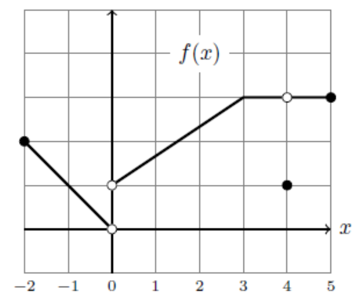
Limits Practice … Set 1

Example 8.1. Consider the graph of $f(x)$ to the right on the domain $[-2, 5]$. Determine where the function is continuous.



Limits Practice … Set 1

Example 8.1. Consider the graph of $f(x)$ to the right on the domain $[-2, 5]$. Determine where the function is continuous.



Example 8.5. Evaluate $\lim_{x \to 0} \frac{x^2 + 3x - 4}{x^2 - 1}$

Example 8.6. Evaluate $\lim_{t \to \pi} 5\cos(t)$

Example 8.12. Consider the function $f(x) = \begin{cases} x+1 & \text{if } x < 0 \\ 2\cos(\pi x) & \text{if } 0 \le x \le 2 \\ 6 - x^2 & \text{if } x > 2 \end{cases}$.

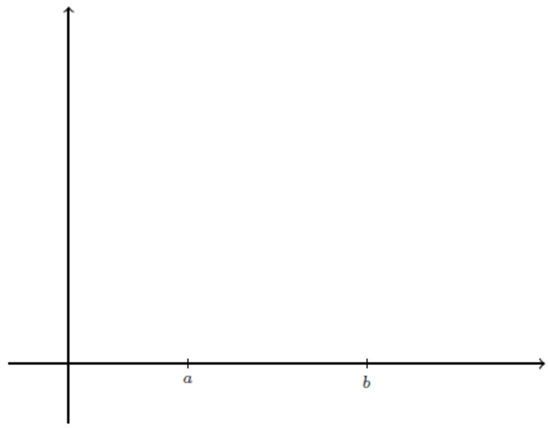
Determine value(s) of x at which $f(x)$ is discontinuous.

Example 8.17. Determine and classify the discontinuities of $f(x) = \dfrac{x^2+5x+6}{x^2-3x-10}$.

Example 8.18. $f(x) = \dfrac{3x^2-3x}{x^2-1}$ has a removable discontinuity at $x = 1$. Find a function $g(x)$ that agrees with $f(x)$ for $x \neq 1$ and is continuous at 1.

Theorem 8.19 (Intermediate Value Theorem (IVT)). Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number $c \in (a, b)$ such that $f(c) = N$.

Picture:



Remark 8.20. The intermediate value theorem states that a continuous function takes on every intermediate value between the function values $f(a)$ and $f(b)$.

Example 8.21. Use the Intermediate Value Theorem to show that there is a root of the function: $f(x) = x^4 + x - 3$ on the interval $(1, 2)$.

Remark 8.22. Notice that it is very difficult to solve $x^4 + x - 3 = 0$!

Limits Practice … Set 1

Example 8.23. Suppose $f(x)$ is a continuous function with values given by the table below.

x	0	1	2	3	4	5
$f(x)$	10.1	3.4	2.9	−1.5	0	0.8

On which interval ***must*** there be a c for which $f(c) = 4$?

A. $(0, 1)$

B. $(1, 2)$

C. $(2, 3)$

D. $(3, 4)$

E. $(4, 5)$

Example 8.24. Prove that the equation $\cos x = x^3$ has at least one solution. What interval is it in?