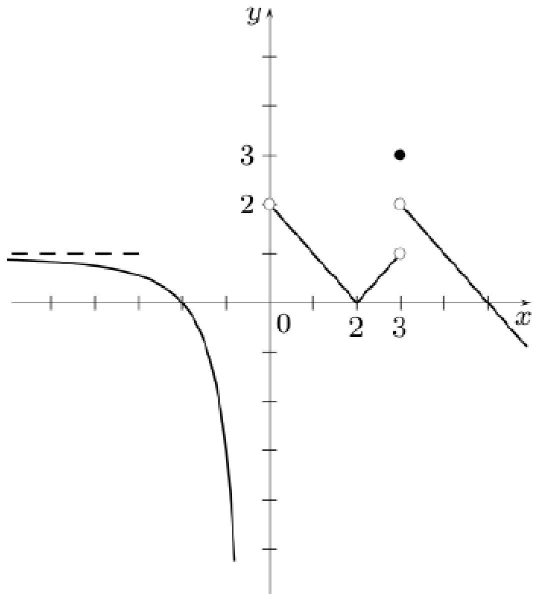
Limits Practice … Set 2

1. Use the graph of the function $f(x)$ to answer each question.
   Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$

(b) $f(2) =$

(c) $f(3) =$

(d) $\lim_{x \to 0^-} f(x) =$

(e) $\lim_{x \to 0} f(x) =$

(f) $\lim_{x \to 3^+} f(x) =$

(g) $\lim_{x \to 3} f(x) =$

(h) $\lim_{x \to -\infty} f(x) =$

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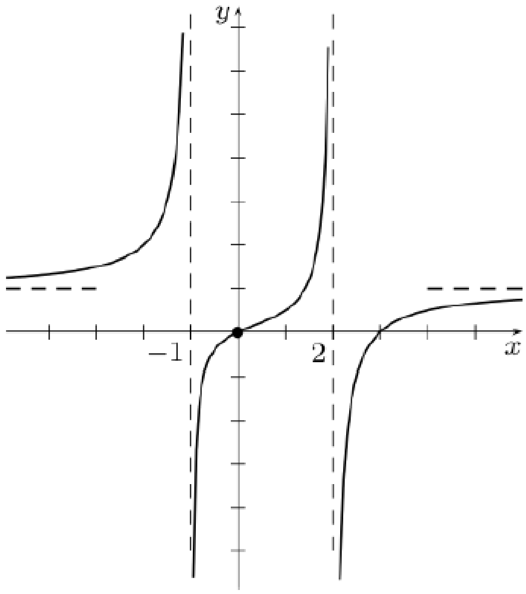
Answers

1. (a) DNE (b) 0 (c) 3 (d) $-\infty$ (e) DNE (f) 2 (g) DNE (h) 1

2. Use the graph of the function $f(x)$ to answer each question.
Use ∞, $-\infty$ or DNE where appropriate.



(a) $f(0) =$

(b) $f(2) =$

(c) $f(3) =$

(d) $\lim_{x \to -1} f(x) =$

(e) $\lim_{x \to 0} f(x) =$

(f) $\lim_{x \to 2^+} f(x) =$

(g) $\lim_{x \to \infty} f(x) =$

Limits Practice … Set 2

Answers

2. (a) 0 (b) DNE (c) 0 (d) DNE (e) 0 (f) $-\infty$ (g) 1

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3. Evaluate each limit using algebraic techniques.
   Use ∞, $-\infty$ or DNE where appropriate.

(a) $\displaystyle \lim_{x\to 0} \frac{x^2-25}{x^2-4x-5}$

(b) $\displaystyle \lim_{x\to 5} \frac{x^2-25}{x^2-4x-5}$

(c) $\displaystyle \lim_{x\to 1} \frac{7x^2-4x-3}{3x^2-4x+1}$

(d) $\displaystyle \lim_{x\to -2} \frac{x^4+5x^3+6x^2}{x^2(x+1)-4(x+1)}$

(e) $\displaystyle \lim_{x\to -3} |x+1|+\frac{3}{x}$

(f) $\displaystyle \lim_{x\to 3} \frac{\sqrt{x+1}-2}{x^2-9}$

(g) $\displaystyle \lim_{x\to 3} \frac{\sqrt{x^2+7}-3}{x+3}$

(h) $\displaystyle \lim_{x\to 2} \frac{x^2+2x-8}{\sqrt{x^2+5}-(x+1)}$

(i) $\displaystyle \lim_{y\to 5} \left(\frac{2y^2+2y+4}{6y-3}\right)^{1/3}$

(j) $\displaystyle \lim_{x\to 0} \sqrt[4]{2\cos(x)-5}$

(k) $\displaystyle \lim_{x\to 0} \frac{\frac{1}{3+x}-\frac{1}{3-x}}{x}$

(l) $\displaystyle \lim_{x\to -6} \frac{\frac{2x+8}{x^2-12}-\frac{1}{x}}{x+6}$

(m) $\displaystyle \lim_{x\to \infty} \sqrt{x^2-2}-\sqrt{x^2+1}$

(n) $\displaystyle \lim_{x\to -\infty} \sqrt{x-2}-\sqrt{x}$

(o) $\displaystyle \lim_{x\to 7} \sqrt[6]{2x-14}$

(p) $\displaystyle \lim_{x\to 1^-} \sqrt{3-3x}$

(q) $\displaystyle \lim_{x\to \infty} \frac{x^4-10}{4x^3+x}$

(r) $\displaystyle \lim_{x\to -\infty} \sqrt[3]{\frac{x-3}{5-x}}$

(s) $\displaystyle \lim_{x\to \infty} \frac{3x^3+x^2-2}{x^2+x-2x^3+1}$

(t) $\displaystyle \lim_{x\to \infty} \frac{x+5}{2x^2+1}$

(u) $\displaystyle \lim_{x\to -\infty} \cos\left(\frac{x^5+1}{x^6+x^5+100}\right)$

(v) $\displaystyle \lim_{x\to 2} \frac{2x}{x^2-4}$

(w) $\displaystyle \lim_{x\to -1} \frac{3x}{x^2+2x+1}$

(x) $\displaystyle \lim_{x\to -1} \frac{x^2-25}{x^2-4x-5}$

(y) $\displaystyle \lim_{x\to 3} \frac{\sqrt{x^2-5}+2}{x-3}$

(z) $\displaystyle \lim_{x\to 0} \frac{2^x+\sin(x)}{x^4}$

(A) $\displaystyle \lim_{x\to 1^-} \frac{1}{x-1}+e^{x^2}$

(B) $\displaystyle \lim_{x\to \infty} 2x^2-3x$

(C) $\displaystyle \lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2-x}}{x}$

(D) $\displaystyle \lim_{x\to 0^+} \frac{e^x}{1+\ln(x)}$

(E) $\displaystyle \lim_{x\to \infty} \sqrt{x^2+1}-2x$

(F) $\displaystyle \lim_{x\to 1} \frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

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Answers

3.

(a) 5

(b) $\frac{5}{3}$

(c) 5

(d) 1

(e) 1

(f) $\frac{1}{24}$

(g) $\frac{1}{6}$

(h) -18

(i) $\frac{4}{3}$

(j) DNE

(k) $-\frac{2}{9}$

(l) $\frac{1}{36}$

(m) 0

(n) DNE

(o) DNE

(p) 0

(q) ∞

(r) -1

(s) $-\frac{3}{2}$

(t) 0

(u) 1

(v) DNE

(w) $-\infty$

(x) DNE

(y) DNE

(z) ∞

(A) $-\infty$

(B) ∞

(C) $\frac{1}{\sqrt{2}}$

(D) 0

(E) $-\infty$

(F) $\frac{2}{3}$

Limits Practice … Set 2

4. Find the following limits involving absolute values.

(a) $\lim_{x\to 1} \frac{x^2-1}{|x-1|}$ (b) $\lim_{x\to -2} \frac{1}{|x+2|} + x^2$ (c) $\lim_{x\to 3^-} \frac{x^2|x-3|}{x-3}$

5. Find the value of the parameter k to make the following limit exist and be finite. What is then the value of the limit?

$$\lim_{x\to 5} \frac{x^2+kx-20}{x-5}$$

6. Answer the following questions for the piecewise defined function $f(x)$ described on the right hand side.

(a) $f(1) =$

(b) $\lim_{x\to 0} f(x) =$

(c) $\lim_{x\to 1} f(x) =$

$$f(x) = \begin{cases} \sin(\pi x) & \text{for } x < 1, \\ 2^{x^2} & \text{for } x > 1. \end{cases}$$

7. Answer the following questions for the piecewise defined function $f(t)$ described on the right hand side.

(a) $f(-3/2) =$

(b) $f(2) =$

(c) $f(3/2) =$

(d) $\lim_{t\to -2} f(t) =$

(e) $\lim_{t\to -1^+} f(t) =$

(f) $\lim_{t\to 2} f(t) =$

(g) $\lim_{t\to 0} f(t) =$

$$f(t) = \begin{cases} t^2 & \text{for } t < -2 \\ \dfrac{t+6}{t^2-t} & \text{for } -1 < t < 2 \\ 3t-2 & \text{for } t \geq 2 \end{cases}$$

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Answers

4. (a) DNE (b) ∞ (c) -9
5. $k = -1$, limit is then equal to 9
6. (a) DNE (b) 0 (c) DNE
7. (a) DNE (b) 4 (c) 10 (d) DNE (e) $\frac{5}{2}$ (f) 4 (g) DNE
8. (a) 0 (b) 0 (c) $\frac{5}{3}$

Limits Practice … Set 2

For each of the rational functions find:

a. domain
b. holes
c. vertical asymptotes
d. horizontal asymptotes
e. y-intercept
f. x-intercepts

1. $f(x)=\dfrac{x^2+x-2}{x^2-x-6}$

2. $f(x)=\dfrac{2x^2}{x^2-1}$

3. $f(x)=\dfrac{3}{x-2}$

4. $f(x)=\dfrac{2x-1}{x}$

5. $f(x)=\dfrac{x^2+x-12}{x^2-9}$

6. $f(x)=\dfrac{x^2-4}{x+3}$

Limits Practice … Set 2

7. $f(x)=\frac{x^2-x}{x+1}$

8. $f(x)=\frac{x^2-x-2}{x-1}$

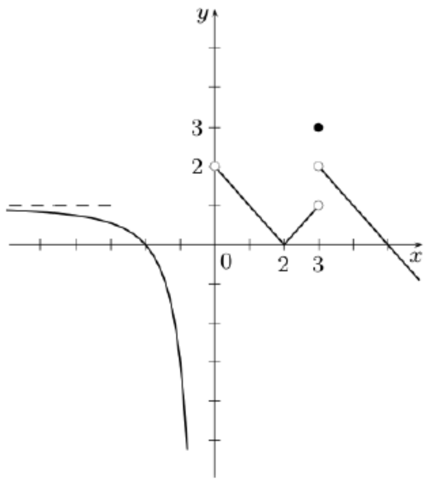
9. $f(x)=\frac{x+1}{x^2+3x+2}$

10. $f(x)=\frac{x^2-9}{x^2-2x-3}$

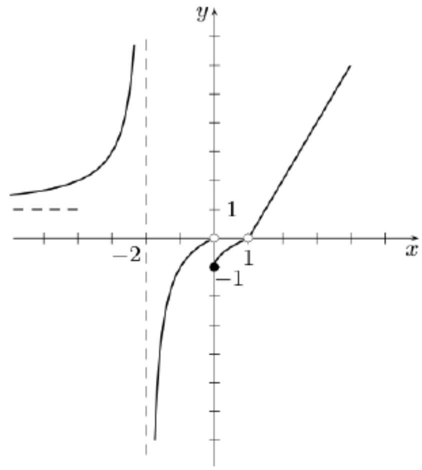
Limits Practice … Set 2

1. For each graph, determine where the function is **discontinuous**. Justify for each point by: (i) saying which condition fails in the definition of continuity, and (ii) by mentioning which type of discontinuity it is.

(a)



(b)



2. For each function, determine the interval(s) of continuity.

   (a) $f(x) = x^2 + e^x$

   (b) $f(x) = \dfrac{3x+1}{2x^2-3x-2}$

   (c) $f(x) = \sqrt[4]{5-x}$

   (d)* $f(x) = \dfrac{2}{4-x^2} + \dfrac{1}{\sqrt{x^2-x-12}}$

3. For each piecewise defined function, determine where $f(x)$ is continuous (or where it is discontinuous). Justify your answer in detail.

(a) $f(x) = \begin{cases} 2^x - 3x^2 & \text{for } x \le 1 \\ \log_{10}(x) + x & \text{for } x > 1 \end{cases}$

(b) $f(x) = \begin{cases} \frac{2x}{3-x} & \text{for } x \le 0 \\ x^2 - 3x & \text{for } 0 < x < 2 \\ \frac{x^2-8}{x} & \text{for } x > 2 \end{cases}$

4. Find all the value(s) of the parameter c (if possible), to make the given function continuous everywhere.

(a) $f(x) = \begin{cases} c \cdot 3^x - x^2 + 2c & \text{for } x \le 0 \\ 2x^5 + c(x+1) + 16 & \text{for } x > 0 \end{cases}$

(b) $f(x) = \begin{cases} 2(cx)^3 + x - 1 & \text{for } x \le 1 \\ 2cx + (x-1)^2 & \text{for } x > 1 \end{cases}$

(c) $f(x) = \begin{cases} 3x + c & \text{for } x < -1 \\ x^2 - c & \text{for } -1 \le x \le 2 \\ 3 & \text{for } x > 2 \end{cases}$

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Answers

1. (a) $x = 0, 3$ (b) $x = -2, 0, 1$

2. (a) $\mathbb{R}$ (b) $\mathbb{R} \backslash \{-1/2, 2\}$ (c) $(-\infty, 5]$ (d) $(-3, 2) \cup (-2, 2) \cup (2, 4)$

3. (a) discontinuous only at $x = 1$ (b) discontinuous only at $x = 2$

4. (a) $c = 8$ (b) $c = -1, 0, 1$ (c) no solution possible

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5. Consider the function $f(x) = \lfloor x \rfloor$, the greatest integer function (also called the floor function or the step function). Where is this function discontinuous?

6. Find an example of a function such that the limit exists at every x, but that has an infinite number of discontinuities. (You can describe the function and/or write a formula down and/or draw a graph.)

Answers

5. discontinuous at every integer, $x = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

6. many answers are possible, show me your solution!